SUPERSYMMETRY IN SINGULAR SPACES

R. Kallosh

Strings2000

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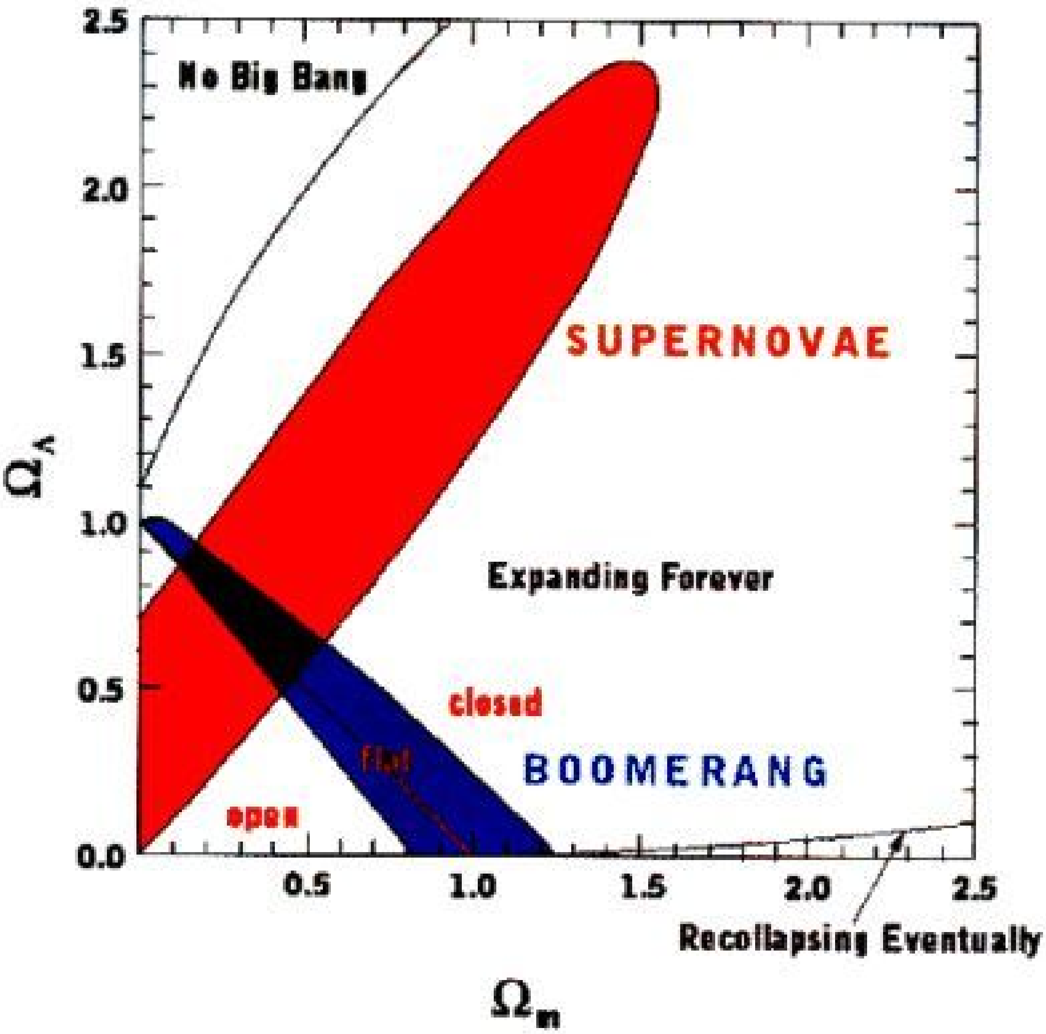
Based on

E. Bergshoeff, R.K., A. Van Proeyen,

hep-th/0007044

BRANES AS SINKS FOR FLUXES



Cosmological Results from BOOMERANG compared with Type 1a Supernovae Results. Shown here are limits from BOOMERANG and Type Ia supernovae on the values of the average density of space in matter (Ω_M, on the horizontal axis) which slows the expansion of the Universe, and the density of the so-called Dark Energy of empty space (Ω_Λ, on the vertical axis) which causes the expansion of the universe to accelerate, preventing re collapse. The BOOMERANG results are consistent with cosmological models whose parameters lie within the blue region. This curve is concentrated near the diagonal red line. From this we learn that, according to BOOMERANG data, the Universe is cosmologically flat. As an excellent complement to this, recent results from the study of S1a supernovae are consistent with cosmological models which lie inside of the yellow region. If both measurements are correct, then allowed models lie in the gree overlap region. This overlap region indicates that our universe is cosmologically flat, started with a Big Bang, and will not collapse again

COSMOLOGICAL CONSTANT:

WHY IT IS ZERO

AND WHY IT IS NOT...

NATURAL VALUE

$$\Lambda \sim M_{PL}^4 \sim 10^{94} g/cm^3$$

EXPERIMENTALLY

$$\Lambda \sim \Omega_\Lambda \cdot 10^{-29} g/cm^3$$

FINE-TUNING BY 10^{-123}

SUPERSYMMETRY AND THE BRANE WORLD

R. Kallosh

SUSY2K, CERN

Geneva, June 26, 2000

DO WE LIVE ON THE BRANE?

Fascinating part: if brane world is realized in Nature, one would be forced eventually to change the fundamentals. Experimental tests seems to be possible. New cosmology?

Worrisome part: extremely speculative and controversial field. Fine tuning, naked singularities, sometimes low standards...

Outline:

- NO-GO theorem: no smooth supersymmetric RS one-brane scenario as alternative to Kaluza-Klein compactification in a certain class of 5D supergravities.

R.K., Linde, Shmakova hep-th/9910021
R.K., Linde, hep-th/0001071

Behrndt, Cvetič hep-th/99909058 hep-th/0001159

- SUPERSYMMETRY IN SINGULAR SPACES

New Consistent and Complete Set of Rules (in example of 3-branes in 5D on a $S_1/\mathbf{Z}_2$ orbifold.)

No fine-tuning for brane-worlds!

Bergshoeff, R.K., Van Proeyen, work in progress

Outline of Part 1:

- Randall-Sundrum one-brane scenario in 5D
  Confinement of gravity; 4D Newton Law; $\Lambda_{brane} = 0$

- Attempts to relax the fine tuning using supergravities with ADS vacua: (No Singular sources)

  I) Massless

  Günaydin Sierra Townsend → a) N=2 gauged supergravity with vector multiplets with any number of moduli

  Günaydin Zagermann → b) N=2 gauged supergravity with vector and tensor multiplets

  c) N=4, N=8

  II) Massive: breathing mode of S^5, CLP versus DLS

  Ceresole Dall'Agata → III) Most general N=2 supergravity with vector, tensor and hypermultiplets

  NO-GO FOR SMOOTH BPS SOLUTIONS!

  hypers ?

SUSY OF BULK & BRANE

The concept of supersymmetry in singular spaces.

New features: (D-1)-form field, promotion of the gauge coupling (gravitino mass) to the status of the supersymmetry singlet field

An example of 3-branes in D=5 (The same mechanism is expected to work for 8-branes in D=10): a consistent definition of the supersymmetry on a $S_1/\mathbf{Z}_2$ orbifold (compare with HW).

The new actions: bulk & brane are separately invariant under supersymmetry forming an algebra.

DISCONTINUITIES ARE TAKEN CARE BY SUPERSYMMETRY OF THE BULK & BRANE ACTION

PRACTICAL APPLICATIONS

Randall-Sundrum solutions come out without fine tuning.

For fixed scalars, the doubling of unbroken supersymmetries takes place and the negative tension brane can be pushed to infinity

In more general BPS domain walls with 1/2 of unbroken supersymmetries, the distance between branes in some cases may be restricted by the collapsing cycles of the Calabi–Yau manifold.

Singular Space: The curvature has some δ-function singularities, for example as in case of orbifold $\mathbf{M} = \mathbf{M}_4 \times \frac{S^1}{\mathbf{Z}_2}$.

Supersymmetric Theory: Bosons And Fermions

$$S_{bulk}(b, f) = \int_{\mathbf{M}} \mathcal{L}_{bulk}(b, f)$$

$$S_{brane}(b, f) = \int_{\mathbf{M}_4} \mathcal{L}_{brane}$$

Off-shell Supersymmetry transformations:

$$\delta_\epsilon f = \delta_\epsilon f(b, f) \qquad \delta_\epsilon b = \delta_\epsilon b(b, f)$$

$$\delta_\epsilon S_{bulk}(b, f) = 0 \qquad \delta_\epsilon S_{brane}(b, f) = 0$$

On-shell Closed Algebra

$$\{\delta_{\epsilon_1}, \delta_{\epsilon_2}\} = \text{all symmetries}$$

Misconception: Consistent and Complete Supersymmetry of the Theory with BOSONS AND FERMIONS is replaced by Unbroken Supersymmetry of the Bosonic Background

- All fermions vanish for the bosonic configuration, $f = 0$

- Bosonic fields solve equations of motion (background)

- Bosonic fields solving equations of motion admit Killing spinors:

$$\delta_\epsilon f(b, f = 0)$$

where $\delta_\epsilon f(b, f = 0)$ is taken from the usual supergravity or with the step functions

$$m_\psi \Rightarrow m_\psi \, \epsilon(x^5)$$

inserted by hand.

$$\epsilon(x^5) = \begin{matrix} +1 & x^5 > 0 \\ -1 & x^5 < 0 \end{matrix}$$

If the fermion mass changes the sign across the wall, the standard supergravity action is not supersymmetric!

$$\delta_{susy}\psi_\mu \sim m\ \epsilon(x^5)\ \gamma_\mu\epsilon$$

$$\delta\partial_5\psi_\mu \sim m\ \delta(x^5)\ \gamma_\mu\epsilon$$

$$\delta S_{sugra} \sim m\ \delta(x^5)\ \bar{\psi}_\mu\gamma^{\mu\nu5}\psi_\nu\epsilon$$

Previous attempts to define supersymmetry in singular spaces

Hořava-Witten: D=11 on $S_1/\mathbf{Z}_2$

The supersymmetry in HW construction includes the contribution from anomalies and requires quantum consistency. Okay at $\kappa^{2/3}$, problems with supersymmetry at the order $\kappa^{4/3}$, problems with $\delta(0)$: incomplete, so far.

Lukas, Ovrut, Stelle, Waldram D=11 $\rightarrow$ D=5

RECENT 5D Supergravity

Altendorfer, Bagger, Nemeschansky: m even under $\mathbf{Z}_2$

Falkowski, Lalak, Pokorski: m odd under $\mathbf{Z}_2$

All versions are incomplete from the point of view of consistent supersymmetry.

Only odd m works for complete and consistent supersymmetry!

PHYSICS OF D-2 BRANES

Objects of codimension $d = 1$, are associated with D-form fluxes.

Such forms do not fall off with distance.

$$F \sim \frac{1}{r^{d-1}}$$

In an infinite volume, they may lead to an infinite energy and would be unphysical.

Therefore branes of codimension $d = 1$ need some planes which serve as sinks for the fluxes. (Polchinski)

Note that the fluxes in the branes of higher codimension do vanish at infinity and this problem can be avoided.

SUSY OF BULK & BRANE

3-brane in D=5 gauged supergravity + vector multiplets: $C_{IJK}X^I X^J X^K = 1$, Günaydin, Sierra, Townsend

ungaged: $g = 0$ gauged: $g \neq 0$

$m_\psi = gW$, superpotential $W = X^I q_I$

$S_{sugra}(g) = S_0 + gS_1 + g^2 S_2$

NEW

1. gauge coupling g is promoted to a status of the field $G(x)$ which is supersymmetry singlet

2. 4-form $A_{\mu\nu\lambda\sigma}(x)$ added, supersymmetry and gauge symmetry transformations are established to make the bulk action supersymmetric

3. Supersymmetric Brane action added

1. and 2. impossible in D=11 (which explains a more complicated HW construction), but possible in D=10 for the supersymmetric theory of the 8-brane

Consistent and Complete Supersymmetry of

BULK & BRANE ACTION ON $\mathrm{M}_4 \times \frac{S^1}{\mathbf{Z}_2}$

$$\mathcal{L}_{bulk} = \mathcal{L}_0 + 6e\,G^2(x)\left(W^2 - \frac{3}{4}W_{,x}^2\right) - e\,G(x)\,F$$
$$+e\,iG(x)Q_{ij}\overline{\lambda}^{ix}\lambda^{jy}\left(-\frac{1}{4}g_{xy}W + \sqrt{\frac{3}{2}}T_{xyz}W^{,z}\right).$$

The covariant flux F is defined as follows:

$$F \equiv \tfrac{1}{4!}e^{-1}\varepsilon^{\mu\nu\rho\sigma\tau}\partial_\mu A_{\nu\rho\sigma\tau} + \tfrac{1}{2}\overline{\psi}^i_\mu\gamma^{\mu\nu\rho}A^{(R)}_\nu Q_{ij}\psi^j_\rho$$
$$+\tfrac{1}{2}\overline{\lambda}^i_x\gamma^\mu A^{(R)}_\mu Q_{ij}\lambda^{jx}$$
$$+\tfrac{3}{2}\left[\overline{\lambda}^i_x\gamma^\mu\psi^j_\mu W^{,x} - i\tfrac{1}{2}\overline{\psi}^i_\mu\gamma^{\mu\nu}\psi^j_\nu W\right]Q_{ij}$$

$$\mathcal{L}_{brane} = -2g\,\left(\delta(x^5) - \delta(x^5 - \tilde{x}^5)\right)\left(e_{(4)}3W + \frac{1}{4!}\varepsilon^{\underline{\mu\nu\rho\sigma}}A_{\underline{\mu\nu\rho\sigma}}\right)$$

The new supersymmetry rules

$$
\begin{aligned}
&\delta(\epsilon)\{e_\mu^m, A_\mu^I, \varphi^x\} = \delta_0\{e_\mu^m, A_\mu^I, \varphi^x\}\,,\\
&\delta(\epsilon)\psi_{\mu i} = \delta_0(\epsilon)\psi_{\mu i} + G(x) A_\mu^{(R)} Q_{ij}\epsilon^j + i\tfrac{1}{2} G(x)\,\gamma_\mu \epsilon^j W Q_{ij}\,,\\
&\delta(\epsilon)\lambda_i^x = \delta_0(\epsilon)\lambda_i^x - \tfrac{3}{2} G(x)\,\epsilon^j W^{,x} Q_{ij}\,,\\
&\tfrac{1}{4!}\delta(\epsilon)A_{\mu\nu\rho\sigma} = [-6\bar{\epsilon}^i \gamma_{[\mu\nu\rho}\psi_{\sigma]}^j W + i\bar{\epsilon}^i \gamma_{[\mu\nu} A_\rho^{(R)} \psi_{\sigma]}^j\\
&-i\tfrac{3}{2}\bar{\epsilon}^i \gamma_{\mu\nu\rho\sigma}\lambda_x^j W^{,x}] Q_{ij}\,,\\
&\delta(\epsilon)G = 0\,.
\end{aligned}
$$

The bulk and the brane actions are separately invariant

$$
\delta(\epsilon)S_{bulk} = 0 \qquad \delta(\epsilon)S_{brane} = 0
$$

The new gauge R-symmetry transformations are

$$
\begin{aligned}
&\delta_R A_\mu^{(R)} = \partial_\mu \Lambda_R\,, \delta_R \psi_\mu^i = G(x)\Lambda_R Q^{ij}\psi_{\mu j}\,,\\
&\delta_R \lambda^{xi} = G(x)\Lambda_R Q^{ij}\lambda_{\mu j}^x\,,\\
&\varepsilon^{\mu\nu\rho\sigma\tau}\delta_R A_{\mu\nu\rho\sigma} - \tfrac{1}{2}\bar{\psi}_\mu^i \gamma^{\mu\tau\rho}\Lambda_R Q_{ij}\psi_\rho^j\\
&-\tfrac{1}{2}\bar{\lambda}_x^i \gamma^\tau \Lambda_R Q_{ij}\lambda^{jx}\,.
\end{aligned}
$$

The field equation for the 4-form field is

$$\partial_5 G(x^5) = 2g\left(\delta(x^5) - \delta(x^5 - \tilde{x}^5)\right),$$

which has as solution

$$G(x) = g\varepsilon(x^5),$$

The closure of the algebra on shell is due to a G field equation

$$\begin{aligned}\hat{F} &= 12G(x)\left(W^2 - \frac{3}{4}\left(\frac{\partial W}{\partial \varphi^x}\right)^2\right) \\ &\quad + i\bar{\lambda}^{ix}\lambda^{jy}\left(-\tfrac{1}{4}g_{xy}W + \sqrt{\tfrac{3}{2}}T_{xyz}W^{,z}\right)Q_{ij}\end{aligned}$$

This equation when considered on shell for the G-field shows that the bosonic part of the flux will change the sign when passing through the wall. This means:

THE WALL IS A SINK FOR THE FLUX

VANISHING ENERGY

Orbifold : $\tilde{x}^5$ is identified with $-\tilde{x}^5$

Analogy with the Hamiltonian of the spatially closed universe (it vanishes in absence of boundaries as it is given by diffeomorphism constraint)

For x^5-dependent configurations, $ds^2 = a^2(x^5)dx^2 + (dx^5)^2$

The BPS form of the energy

$$\frac{1}{2}a^4\left\{[\varphi^{x\prime} - 3\alpha G W^{,x}]^2 - 12[\frac{a'}{a} + \alpha G W]^2\right\} + 3\alpha[a^4 G W]'$$
$$+ \left[2g\left(\delta(x^5) - \delta(x^5 - \tilde{x}^5)\right) - G'\right]\left(3a^4\alpha W + \tfrac{1}{4!}\epsilon^{\underline{\mu\nu\rho\sigma}}A_{\underline{\mu\nu\rho\sigma}}\right)$$

The energy of the susy singlet cancels the tension contributions at each brane separately since $G' = 2g\left(\delta(x^5) - \delta(x^5 - \tilde{x}^5)\right)$ due to the field equations for the 4-form.

$$E(x^5) = 0$$

PRACTICAL APPLICATIONS: Randall-Sundrum brane world, no fine tuning required

Technical tool Supersymmetric attractor equation (Ferrara, R.K., Strominger)

$$C_{IJK}\tilde{h}^J\tilde{h}^K = q_I$$

- Fixed scalars, doubling of supersymmetries includes an alternative to compactification world brane (double-extreme Calabi-Yau black holes)
  example: STU wall

- x^5-dependent scalars, 1/2 of unbroken supersymmetry, Calabi-Yau Walls (extreme Calabi-Yau black holes), the distance between two branes restricted sometimes by the collapsing cycles of CY
  example: Candelas,Font,Katz,Morrison model

Double-extreme STU Wall

$$C_{IJK}h^I h^J h^K = STU = 1$$

$$W = Sq_S + Tq_T + Uq_U$$

$$S_{crit} = \left(\frac{q_T q_U}{q_S^2}\right)^{1/3} \quad T_{crit} = \left(\frac{q_S q_U}{q_T^2}\right)^{1/3} \quad U_{crit} = \left(\frac{q_T q_S}{q_U^2}\right)^{1/3}$$

$$W_{crit} = 3(q_S q_T q_U)^{1/3}$$

The Randall-Sundrum metric

$$ds^2 = e^{-2gW_{crit}|x^5|} dx^{\underline{\mu}} dx^{\underline{\nu}} \eta_{\underline{\mu\nu}} + (dx^5)^2$$

For $gW_{crit} > 0$ the negative tension brane can be pushed to ∞

Calabi-Yau Wall (Base $\mathbf{P}_2$ Vacuum)

$$\frac{3}{8}U^3 + \frac{1}{2}UT^2 = 1\,.$$

The basic cycles are $h^1(U)$ and $h^2(U,T)$
The metric is

$$ds^2 = a^2(y)dx^\mu dx^\nu \eta_{\mu\nu} + a^{-4}(y)dy^2$$

where $a^2(|y|)$ is

$$\left(H_U \frac{2}{\sqrt{3}}\sqrt{H_U - \sqrt{H_U^2 - \frac{9}{4}H_T^2}} + H_T\sqrt{3}\sqrt{H_U + \sqrt{H_U^2 - \frac{9}{4}H_T^2}}\right)^{2/3}$$

$$H_U = c_U - 2gq_U|y|\,, \qquad H_T = c_T - 2gq_T|y|\,,$$

are harmonic functions

Collapsing Cycles: $h^1, h^2 \to 0$ at $|y|_{sing}$

$$|\tilde{y}| < |y|_{sing} = \frac{c_T}{2gq_T}\,, \qquad |\tilde{y}| < |y|_{sing} = \frac{2c_U - 3c_T}{2g(2q_U - 3q_T)}$$

Examples with restriction on the distance between the branes

Historical info

June 12 1980

Duff and van Nieuwenhuizen

„ Quantum inequivalence of different field representations

Nothing ⟷ $A_{\mu_1 \ldots \mu_{D-1}}$ + ghosts for ghosts

Anomalies

Topological 1-loop counterterms

June 20 1980

Aurilia, Nicolai, Townsend

„ Hidden constants: The theta parameter of QCD and the cosmological constant of N=8 supergravity "

$\partial F_{\mu_1 \ldots \mu_D} = 0$ $F_{\ldots} =$ constant

surprising physics

Back to General Issues

Same Supersymmetric Bulk&Brane mechanism is expected to work for

8-brane in D=10 : IIA Massive Supergravity

(Romans; Bergshoeff, Green, Papadopoulos, Townsend)

$$S = S_{bulk} + S_{brane}$$

$$S_{bulk} = S_{Romans}(m \to M(x)) + S_A = S_{BGPT} + \text{fermions}$$

To get the piecewise constant M we have to add the supersymmetric brane action

$$\mathcal{L}_{brane} \sim -2m\,(\delta(y) - \delta(y - \tilde{y}))\left(e_{(9)} e^{\frac{5\phi}{4}} + \frac{1}{9!}\epsilon^{\underline{\mu}_1 \cdots \underline{\mu}_9} A_{\underline{\mu}_1 \cdots \underline{\mu}_9}\right)$$

Supersymmetric Version of Polchinski-Witten?

The reason why in D=11 one has to use instead of a Supersymmetric Bulk&Brane a more complicated Horava-Witten mechanism based on anomalies is that it is impossible to introduce a 10-form potential: D=11 Supergravity does not admit a Cosmological constant!

STRATEGY

- Improved understanding of 8-brane of string theory into the M-theory

- Supersymmetry and chiral fermions on the brane