1 + 1 Dimensional NCOS and its U(N) Gauge Theory Dual

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Based mainly on

I. K., J. Maldacena,

hep-th/0006085

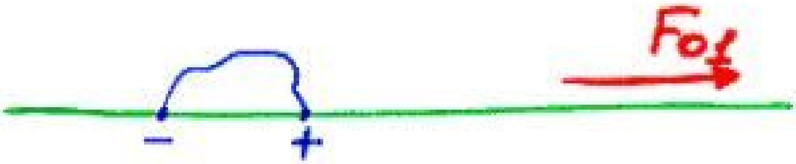
1+1 d NCOS gives a new example of gauge field/string duality.

For earlier work, see papers by H. Verlinde; Gubser, IK, Polyakov

The NCOS limit was defined by Seiberg, Susskind, Toumbas; Gopakumar, Maldacena, Minwalla, Strominger

1+1 d case is special: the theory is invariant under Lorentz boosts.

Start with a D1-brane with electric field.



For $F_{01} = \frac{1}{2\pi\alpha'}$ the open string becomes TENSIONLESS relative to the closed strings.

The Born-Infeld lagrangian

$$\mathcal{L}_{BI} = -\frac{1}{2\pi\alpha' g_s}\sqrt{1-(2\pi\alpha' F_{01})^2}$$

In the $A_0 = 0$ gauge, $F_{01} = \dot{A}_1$;

The electric field is the momentum conjugate to A_1:

$$E_1 = \frac{\partial \mathcal{L}_{BI}}{\partial \dot{A}_1} = \frac{2\pi\alpha' \dot{A}_1}{\sqrt{1-(2\pi\alpha'\dot{A}_1)^2}}\,\frac{1}{g_s} = \underset{\Uparrow}{N}$$

the member of flux units.

$$\tilde{F} = 2\pi\alpha' \dot{A}_1 \ ;$$

$$\frac{\tilde{F}}{\sqrt{1-\tilde{F}^2}} = N g_s \implies \tilde{F} = \frac{N g_s}{\sqrt{(N g_s)^2 + 1}} .$$

Callan, IK

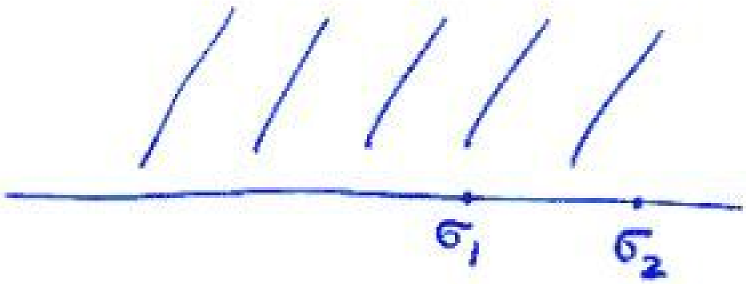
We are interested in the critical field limit $\tilde{F} \to 1$; i.e. $N g_s \to \infty$,

of the bound state of N fundamental and 1 D-string.

As a check, calculate the Energy/Length:

$$\mathcal{H} = E_1 \dot{A}_1 - \mathcal{L}_{BI} = \frac{1}{2\pi\alpha'}\sqrt{\frac{1}{g_s^2} + N^2}$$

Get correct tension of the $(N,1)$ bound state.



The Green function for X^0, X^1 is

$$\langle X^\alpha(\sigma_1) X^\beta(\sigma_2)\rangle = -2\alpha' \frac{1}{1-\tilde{F}^2}\eta^{\alpha\beta} \ln|\sigma^1-\sigma^2|$$

$$+ i\pi \frac{\tilde{F}}{1-\tilde{F}^2}\alpha' \epsilon^{\alpha\beta} \operatorname{sgn}(\sigma^1-\sigma^2)$$

Define $\alpha'_e = \frac{\alpha'}{1-\tilde{F}^2} \implies T_e = T(1-\tilde{F}^2)$

Sending $\tilde{F}\to 1$ have $\Theta = 2\pi\alpha'_e$;

$$\langle X^\alpha(\sigma_1) X^\beta(\sigma_2)\rangle = -2\alpha'_e \eta^{\alpha\beta} \ln|\sigma_1-\sigma_2|$$

$$+ i\frac{\Theta}{2}\epsilon^{\alpha\beta} \operatorname{sgn}(\sigma_1-\sigma_2)$$

Open string coupling is also rescaled (GKP)

$G_o^2 = g_s\sqrt{1-\tilde{F}^2}$: each hole $\bigcirc$ picks up a BI factor.

$= \dfrac{\tilde{F}}{N} \rightarrow \dfrac{1}{N}$

$\dfrac{1}{G_o^2}$ is quantized!

To keep G_o^2 fixed, $g_s \rightarrow \infty$ in the NCOS limit.
(SST, GMMS)

$\alpha' = \alpha'_e(1-\tilde{F}^2) \rightarrow \dfrac{\alpha'_e}{(Ng_s)^2}$

Apply S-duality : $(N, 1) \longrightarrow (1, N)$

Get $U(N)$ $\mathcal{N}=8$ gauge theory with

ONE unit of electric flux : $\operatorname{Tr} E_1 = 1$.

Now have $\tilde{g}_s = \dfrac{1}{g_s} \longrightarrow 0$,

$\tilde{\alpha}' = g_s\alpha' = \dfrac{\alpha'_e}{N^2 g_s} \longrightarrow 0$.

$g_{YM}^2 = \dfrac{\tilde{g}_s}{\tilde{\alpha}'} = \dfrac{N^2}{\alpha'_e} \Leftarrow$ finite.

This is the decoupling limit for N D1-branes.

Gopakumar, Minwalla, Seiberg, Strominger.

Make N large to have weakly interacting open strings.

Since $\frac{1}{\alpha'_e} = \frac{g_{YM}^2}{N^2}$, and NOT $g_{YM}^2 N$,

this is NOT a 't Hooft limit.

Get a new connection between gauge fields and OPEN strings.

I will discuss the following aspects of the connection:

1. Decoupling of the massless $U(1)$ modes.
2. Stable massive particles.
3. High energy behavior.
4. Higgs branch and closed strings.
5. Hagedorn transition.

Free the $U(1)$!

The Gauge group is $U(N) = U(1) \times SU(N)/Z_N$.

The $SU(N)/Z_N$ theory has mass gap due to the 1 unit of flux (Witten).

Overall oscillations of the $(N,1)$ string decouple from its internal dynamics:

we have 8 massless bosonic + 8 fermionic modes that are FREE.

They are dual to the massless modes of the NCOS created by vertex operators

$$\epsilon_j e^{-\phi} \psi^j e^{ik\cdot X} \Leftarrow \text{8 scalars}$$

$$\epsilon^\alpha e^{-\phi/2} \Sigma_\alpha e^{ik\cdot X} \Leftarrow \text{8 fermionic partners.}$$

Consider a 4-point function for scalars.

In the COM frame:

$$p_1 = \begin{pmatrix} p \\ p \end{pmatrix}; \quad p_2 = \begin{pmatrix} p \\ -p \end{pmatrix}; \quad p_3 = -p_1; \quad p_4 = -p_2.$$

Due to the special kinematics,

$$s = -t = 4p^2\alpha'_e \, ; \qquad u = 0 \, .$$

The complete 4-pt amplitude is

$$A = -\frac{G_0^2}{\alpha'_e}\,\delta^2(\Sigma p_i)\;\epsilon_1\cdot\epsilon_3\,\epsilon_2\cdot\epsilon_4\;\pi s \times$$

$$\left[\;\underbrace{\frac{\cos\left(s\,\frac{\theta}{2\alpha'_e}\right)}{\sin(\pi s)}}_{\text{s-channel}} \;-\; \underbrace{\frac{\cos(\pi s)}{\sin(\pi s)}}_{\text{t and u channels}}\;\right]$$

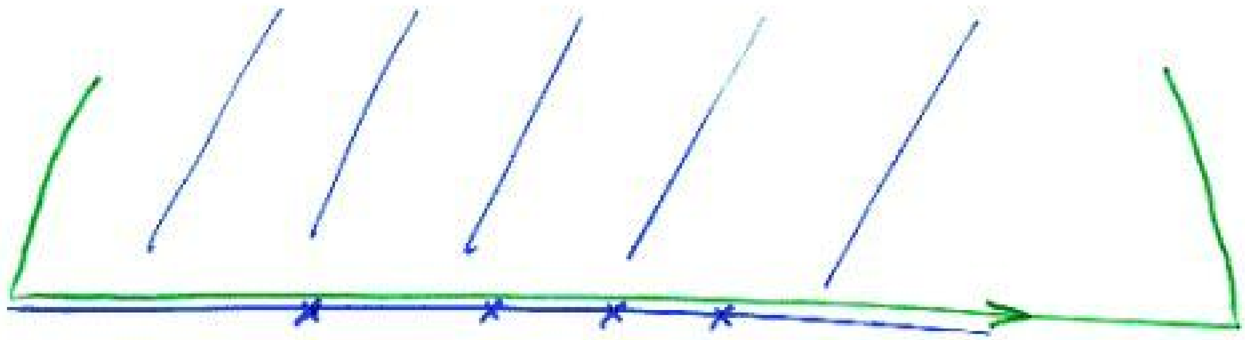
In the NCOS limit, $\theta = 2\pi\alpha'_e$ and A indeed vanishes!

All amplitudes with at least 1 massless state vanish since

$$\langle X^+(t)\,X^-(0)\rangle = 4\alpha'_e\left(\log|t| + i\frac{\pi}{2}\,\mathrm{sgn}(t)\right)$$

$$\rightarrow 4\alpha'_e \log t$$

where t is a complex variable.



If <u>one</u> of the vertex operators has $e^{ip^- X^+}$ or $e^{ip^+ X^-}$ factor then the contour may be deformed into the interior of the disk.

We believe this generalizes to higher genus.

Moyal phases + 2-d kinematics conspire to cancel all massless amplitudes, consistent with the duality.

Stable Massive Particles (with C. Herzog)

Each massive state is characterized by the level $k = \alpha_e' m^2$ and the $SO(8)$ irrep.

For an n-body decay energy conservation gives

$$\sqrt{k} \geq \sum_{i=1}^{n} \sqrt{k_i} \; ;$$

The only allowed decay modes have $k \geq 1$ (the massless modes are free).

<u>All</u> massive particles with $k = 1, 2, 3, 4$ are stable!

There is an ∞ class of absolutely stable particles.

For instance: leading Regge trajectory whose $SO(8)$ Young tableaux is $\underbrace{\boxed{}\boxed{}\boxed{}\cdots\boxed{}\boxed{}\boxed{}}_{k+1}$

Decays into such states are most likely (they have the most angular momentum / energy).

$$\sum_{i=1}^{n} (k_i + 1) \geq k + 1 \quad (J \text{ conservation}).$$

Combining with energy conservation, we need

$$\sum_{i<j} \sqrt{k_i k_j} < \frac{n-1}{2}$$

This cannot be satisfied $\Rightarrow$ the leading SO(8) Regge trajectory is stable.

There are lots of other stable states.

Prediction: the gauge theory has stable states with energies $\sqrt{k}\,\frac{g_{YM}}{N}$ (for large N)

They should look like stable lumps of energy (non-topological solitons?).

High Energy Behavior

4-pt function of level 1 states $\left(m^2 = \frac{1}{\alpha'_e}\right)$.

$$V = \epsilon_{ij} e^{-\phi} \psi^i \partial X^j e^{ik\cdot X}$$

$$A = -\pi G_0^2 \delta^2\left(\sum p_i\right)(s-2) \frac{\cos\left(\frac{\theta}{2\alpha'_e}\sqrt{s(s-4)}\right) - \cos(\pi s)}{\sin(\pi s)} \times$$

$$\mathrm{tr}(\epsilon_1 \cdot \epsilon_3)\, \mathrm{tr}(\epsilon_2 \cdot \epsilon_4).$$

For $\theta = 2\pi\alpha'_e$ there is a large s cancellation:

$$A \longrightarrow -2\pi^2 \frac{G_0^2}{\alpha'_e} \frac{\sin\left(\pi s - \frac{\pi}{s-2}\right)}{\sin(\pi s)}$$

Away from the poles $A \sim \frac{G_0^2}{\alpha'_e}$ (E independent).

The high energy cross-section is

$$\sigma \sim \frac{G_0^4}{\alpha'^2_e E^4};$$

This behavior is generic for massive $2 \to 2$ process.

For SYM quanta have $A \sim g_{YM}^2$;

$$\sigma \sim \frac{g_{YM}^4}{E^4}.$$ Same energy dependence!

Higgs Branch and Closed Strings

SU(N) with 1 unit of flux may be Higgsed to

$SU(N-K) \times U(1)^K$

Now the electric field is

$$E_1 = \frac{1}{N-K} \begin{pmatrix} 1 & & & & & & \\ & 1 & & & & & \\ & & \ddots & & & & \\ & & & 1 & & & \\ & & & & 0 & & \\ & & & & & \ddots & \\ & & & & & & 0 \end{pmatrix}$$

$$\underbrace{\qquad\qquad}_{K}$$

If $X^1 \sim X^1 + 2\pi R$ (compactify on a circle)

then the energy of the flux is

$$\mathcal{E} = \frac{R g_{YM}^2}{2(N-K)} = \frac{R g_{YM}^2}{2} \left(\frac{1}{N} + \frac{K}{N^2} + \frac{K^2}{N^3} + \ldots \right)$$

This is the state with K free strings.

$$\text{String tension} = \frac{1}{4\pi} \frac{g_{YM}^2}{N^2} = \frac{1}{4\pi \alpha'_e}$$

There are no anti-strings $(K>0)$.

A fixed string has U(1) gauge theory living on it => its energy formula is

$$\Delta \mathcal{E} = \frac{R}{2\alpha'_e} + \frac{1}{R}\left(N_L + N_R + \alpha'_e \frac{k_\perp^2}{2}\right).$$

In the dual string theory this is nicely reproduced by the on-shell condition for a wound closed string with m units of winding:

$$-\alpha' p_0^2 - 2 p_0 B_{01} m R + \frac{(mR)^2}{\alpha'}(1 - B_{01}^2) + \alpha'\left(\frac{n}{R}\right)^2 +$$
$$+ 2(N_L + N_R) + \alpha'_e k_\perp^2 = 0;$$

In the limit $\alpha' = \alpha'_e (1 - B_{01}^2) \to 0$,

<u>Non-relativistic</u> mass formula

$$p_0 = \frac{mR}{2\alpha'_e} + \frac{\alpha'_e}{2mR} k_\perp^2 + \frac{N_L + N_R}{mR};$$

Since $p_0 > 0$, $m > 0$ => strings may wind only in one direction.

Their tension is indeed $\frac{1}{4\pi\alpha'_e}$.

The Hagedorn Transition

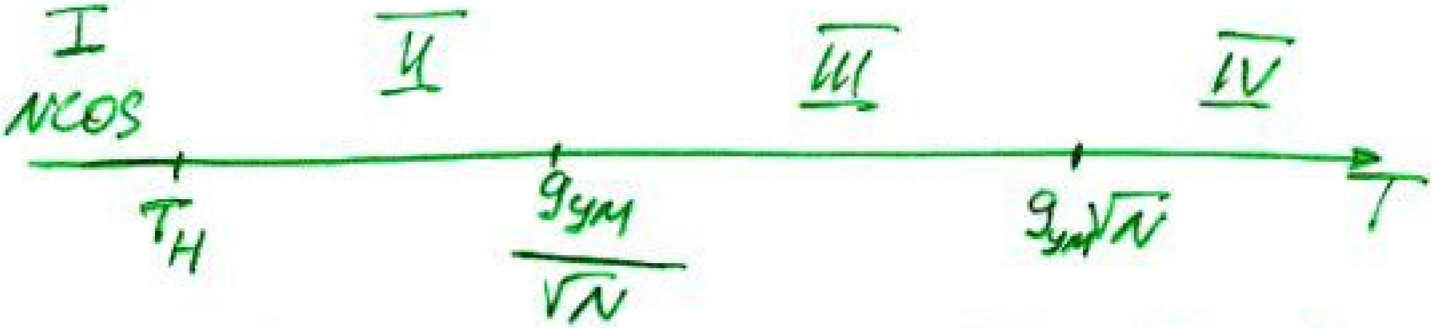
Preliminary results developed in discussions with Gubser, Gukov, Witten, ...

The open string density of states is

$$\rho(m) \sim m^a e^{m/T_H};$$

$$T_H = \frac{1}{\pi\sqrt{8\alpha'_e}} = \frac{1}{\pi\sqrt{8}}\,\frac{g_{YM}}{N};$$

Phase structure of U(N) with 1 flux unit.



IV is the free phase where $F \sim N^2 L T^2$;

III is the strongly interacting phase dual to SUGRA where $F \sim \frac{N^2 L T^3}{g_{YM}\sqrt{N}}$;

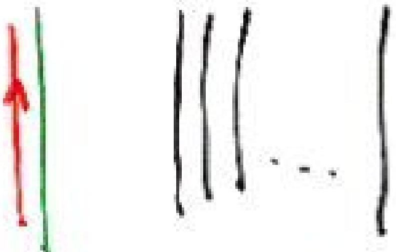
L. Susskind, IK; IMSY

What is phase II?

The free energy needed to split off a wound closed string, i.e. to Higgs $SU(N) \to SU(N-1) \times U(1)$ is $\Delta F = L\left(\frac{1}{4\pi\alpha'_e} - 2\pi T^2\right)$

$\Delta F = 0$ precisely at $T = T_H$.

For $T > T_H$ a fraction ν of closed strings has been freed.



Phase II is the Higgs phase

$SU(N-K) \times U(1)^K$

where K is of order N.

To calculate $\nu = \frac{K}{N}$, set

$$L\left(\frac{1}{4\pi}\frac{g_{YM}^2}{(N-K)^2} - 2\pi T^2\right) = 0$$

Find $\nu = \frac{T - T_H}{T}$;

$$-\frac{\partial F}{\partial \tau} = S = 4\pi N \nu T = 4\pi N(\tau - \tau_H)$$

$$F(\tau > \tau_H) = F(\tau = \tau_H) - 2\pi N(\tau - \tau_H)^2,$$

$F(\tau = \tau_H)$ is of order 1 (it is free energy of weakly interacting open strings).

Comments

1. A T-dual of the $(N, 1)$ bound state is a D0-brane with N units of KK momentum. Critical F_{01} is T-dual to the speed of light. Hence NCOS is T-dual to DLCQ.

Winding states of closed strings transform into momentum states.

Do they decouple?

2. We may start with M D1-branes. Then $G_0^2 = \frac{M}{N}$; $\frac{1}{\alpha_e'} = g_{YM}^2 \frac{M^2}{N^2}$.

3. Some of our conclusions, such as the presence of wound closed strings around the electric direction, carry over to NCOS in arbitrary dimension $p+1$.

U(1) modes moving along the E-field always decouple.