RG Flows

and the

Supergravity Brane World

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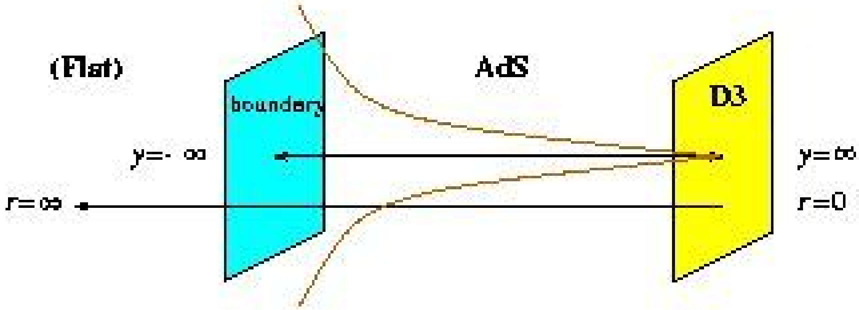
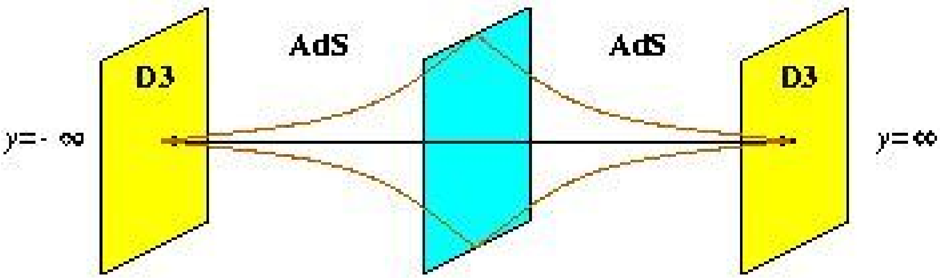
Anatomy of the Randall-Sundrum brane world

- AdS_5 in Poincaré coordinates: the near horizon geometry of D3 branes



$$ds^2 = e^{-2gy}(dx^\mu)^2 + dy^2$$

- The Randall-Sundrum metric: AdS_5 cut off before reaching the boundary

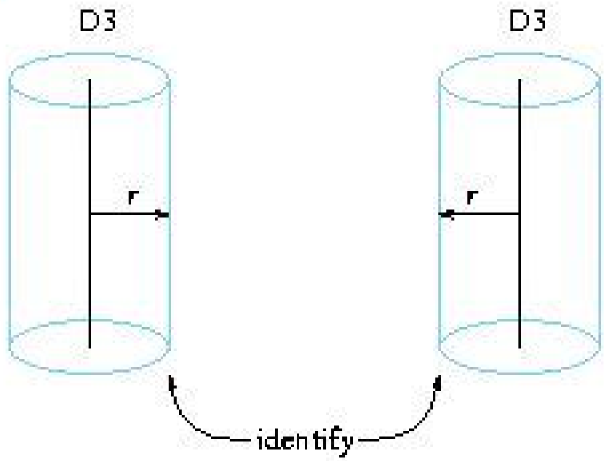


$$ds^2 = e^{-2g|y|}(dx^\mu)^2 + dy^2$$

The brane world from D3 branes

The brane world can be examined in either five or ten dimensions

- $D = 10$: Doubled D3 brane configuration



- $D = 5$: Breathing mode domain wall (as sphere compactification)

$$ds^2 = H_{D3}^{-\frac{1}{2}}(dx^\mu)^2 + H_{D3}^{\frac{1}{2}}dr^2 + \boxed{H_{D3}^{\frac{1}{2}}r^2}\, d\Omega_5^2$$

These objects are supersymmetric

The brane world is compatible with supersymmetry

Supergravity domain walls

- Avoid thin branes by introducing a scalar field to support the domain wall

$$e^{-1}\mathcal{L} = R - \tfrac{1}{2}\partial\phi^2 - V(\phi)$$

- For a scalar of mass m in gauged supergravity ($m^2 \geq -4g^2$)

$$V = -12g^2 + \tfrac{1}{2}m(\phi - \phi_*)^2 + \cdots$$

- Choose a domain wall ansatz

$$ds^2 = e^{2A(y)}(dx^\mu)^2 + dy^2$$

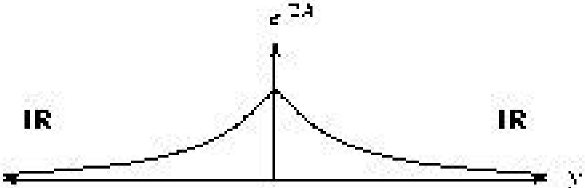
- The resulting equations of motion

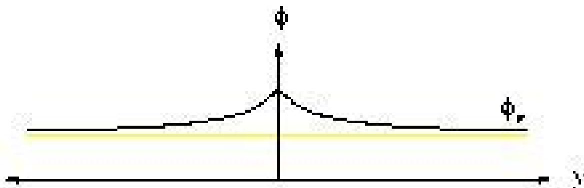
$$\begin{aligned} A'' &= -\tfrac{1}{6}\phi'^{\,2} \\ A'^{\,2} &= \tfrac{1}{24}\phi'^{\,2} - \tfrac{1}{12}V \\ \phi'' + 4A'\phi' &= \partial_\phi V \end{aligned}$$

(well studied from holographic flows)

Stability of the brane world and flows to the IR

- We seek domain wall solutions with a decaying 'warp-factor'





- Expand ϕ near its fixed value ϕ_* and use the form of the warp factor $A(y) \approx \pm gy$

$$\phi'' \pm 4g\phi' - m^2(\phi - \phi_*) \approx 0$$

- This is solved by

$$(\phi - \phi_*) \sim e^{-E_0 A(y)} \text{ or } e^{-(4-E_0)A(y)}$$

$E_0 = 2 + \sqrt{(m/g)^2 + 4}$ (scalar in AdS_5)

- Stability:

$E_0 > 4$	IR stable
$E_0 = 4$	marginal
$2 \leq E_0 < 4$	UV stable

Perception: IR stable flows are impossible in supergravity

Kallosh & Linde, JHEP02 (2000) 005

- Scalars in $N = 2$ vector multiplets (very special geometry)

$$V = (\partial_\phi W)^2 - \frac{2}{3} W^2$$

- Using $\partial_\phi^2 W_* = \frac{1}{3} W_*$, we expand

$$W = \pm 3\sqrt{2} g (1 + \frac{1}{6}(\phi - \phi_*)^2 + \cdots)$$

- The resulting potential is

$$V = -12g^2 - 2g^2(\phi - \phi_*)^2 + \cdots$$

- This is IR unstable

$$m^2 = -4g^2 \qquad (E_0 = 2)$$

- Furthermore, $N = 8$ scalars (round S^5 spectrum) cannot come to the rescue

$$D(2,0,0,\mathbf{20'}) \qquad D(3,0,0,\mathbf{10} + \overline{\mathbf{10}})$$
$$D(4,0,0,\mathbf{1} + \mathbf{1})$$

(fits into $N = 2$ vector, tensor and chiral multiplets)

Reality: No go result does not exclude **massive** supergravity scalars

- IR stable provided $E_0 > 4$ and proper relative sign in the superpotential
- The massive breathing mode satisfies these conditions
- In terms of the $N = 2$ decomposition of the $N = 8$ model

$$\begin{aligned}\mathcal{D}(6,0,0,0) = D(7,\tfrac{1}{2},\tfrac{1}{2},0) + D(6.5,\tfrac{1}{2},0,-1) \\ + D(6.5,0,\tfrac{1}{2},1) + D(6,0,0,0) \\ +D(7,0,0,-2) + D(7.5,0,\tfrac{1}{2},-1) \\ +D(7,0,0,2) + D(7.5,\tfrac{1}{2},0,1) + D(8,0,0,0)\end{aligned}$$

(massive vector + LH chiral + RH chiral)

Breathing mode:	$E_0 = 8$
Squashing mode:	$E_0 = 6$

(these scalars are also $U(1)$ singlets)

Supersymmetry for the breathing mode

- Sphere reduction of IIB yields

$$\delta\psi_\mu = [\nabla_\mu - \tfrac{1}{6\sqrt{2}} W \gamma_\mu]\epsilon$$
$$\delta\lambda = \tfrac{1}{2}[\gamma \cdot \partial\varphi + \sqrt{2}\partial_\varphi W]\epsilon$$

where

$$W = \sqrt{2}m[2e^{\frac{5}{\sqrt{15}}\varphi} - 5\sqrt{\frac{R_5}{20m^2}}e^{\frac{2}{\sqrt{15}}\varphi}]$$

$$m = \text{Freund Rubin flux}$$
$$R_5 = \text{Ricci scalar of } S^5$$

Bremer *et al.*, NPB543 (1999) 321

- Killing spinor equations (in general)

$$A' = \tfrac{1}{3\sqrt{2}} W \qquad \varphi' = -\sqrt{2}\partial_\varphi W$$

$$\epsilon = e^{A/2}(1 + \gamma^{\overline{y}})\epsilon$$

(or sign flipped)

Perception: Kinks break supersymmetry

We present two arguments:

1) Because of the kink, A' changes sign at the RS brane

$$y > 0: \; A' = \frac{1}{3\sqrt{2}} W \qquad \epsilon_> = e^{A/2}(1+\gamma^{\overline{y}})\epsilon$$

$$y < 0: \; A' = -\frac{1}{3\sqrt{2}} W \qquad \epsilon_< = e^{A/2}(1-\gamma^{\overline{y}})\epsilon$$

- $\epsilon_>$ and $\epsilon_<$ cannot be joined at $y = 0$ (without introducing δ-function terms)

Reality: W also changes sign so that $A' = \frac{1}{3\sqrt{2}} W$ is valid everywhere

- For a RS brane at $y = 0$, the Z_2 symmetry $y \leftrightarrow -y$ must be accompanied by an orientation reversal of S^5 in the underlying IIB theory
- This flips Freund Rubin $\leftrightarrow$ anti-Freund Rubin, or $m \leftrightarrow -m$

2) Supersymmetry (Killing spinor equations) implies equations of motion

EOM (δ-function sources)

$\updownarrow$

Killing spinor equations
(supersymmetry broken at sources)

- Start from $A' = \frac{1}{3\sqrt{2}} W$

$$A'' = \frac{1}{3\sqrt{2}} W' = \frac{1}{3\sqrt{2}} \partial_\varphi W \varphi' = -\frac{1}{6} \varphi'^{\,2}$$

Reality: Supersymmetry only implies a subset of the equations of motion

- $W' \neq \partial_\varphi W \varphi'$ if W is discontinuous

$$W' = \partial_\varphi W \varphi' + 2W_+ \delta(y)$$

for $W \leftrightarrow -W$ at $y = 0$

- In general, the second order 'harmonic function condition' must be imposed in addition to the first order BPS conditions.

- Note that Killing spinors may 'exist' for non-physical backgrounds