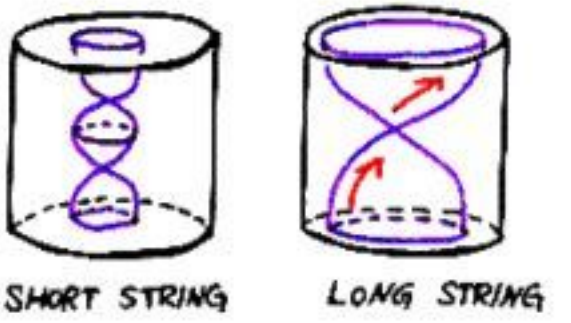
STRINGS IN AdS_3 AND THE $SL(2,\mathbb{R})$ WZW MODEL

HIROSI OOGURI (CALTECH)

BASED ON A WORK WITH J. MALDACENA.

(1) HEP-TH / 0001053

SPECTRUM OF THE MODEL



(2) HEP-TH / 0005183 (ALSO WITH J. SON)

ONE-LOOP FREE ENERGY

$$\int d^2\tau \quad = \sum_n \log\left(\frac{1}{1-e^{-\beta E_n}}\right)$$

(3) IN PROGRESS

FOUR-POINT AMPLITUDE

$$\int d^2z \quad = \langle O_1(0)\, O_2(x)\, O_3(1)\, O_4(\infty)\rangle$$

THE WORLDSHEET OF STRING IN AdS_3 (NS BACKGROUND)
IS DESCRIBED BY THE $SL(2,\mathbb{R})$ WZW MODEL.

$$S = \frac{k}{8\pi}\int d^2z\, \mathrm{Tr}[g^{-1}\partial g\, g^{-1}\bar{\partial} g] + k\,\Gamma_{WZ}$$

k : LEVEL OF $\widehat{SL}(2,\mathbb{R})$

THE MODEL IS INTERESTING SINCE

- SOLVABLE CASE WITH NON-TRIVIAL g_{00}
- RELEVANT TO AdS_3/CFT_2 CORRESPONDENCE

THERE HAVE BEEN PUZZLES (SINCE 1989)
ABOUT THE SPECTRUM OF THE MODEL.

IN THIS TALK, I WILL

(1) DESCRIBE THE HILBERT SPACE STRUCTURE
OF THE LORENTZIAN MODEL.
⇒ RESOLUTION TO THE PUZZLES.

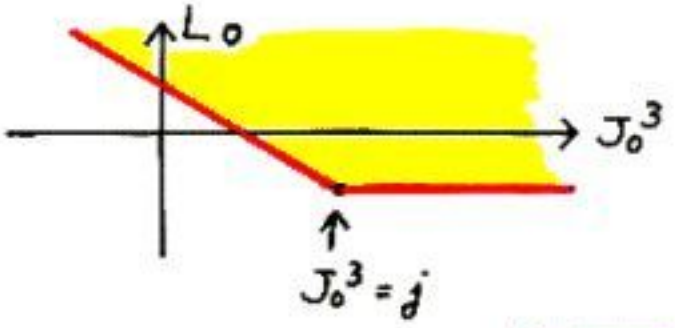
(2) SHOW THAT IT AGREES WITH
THE EUCLIDEAN PATH INTEGRAL COMPUTATIONS OF

- ONE-LOOP FREE ENERGY
  ⇒ REPRODUCES THE SPECTRUM OF (1).
- FOUR-POINT AMPLITUDE
  ➡ TARGET SPACE OPE CONSISTENT WITH (1).

THE HILBERT SPACE OF THE $SL(2,\mathbb{R})$ WZW MODEL CONSISTS OF

(1) DISCRETE REPRESENTATION $D_j^{\pm} \otimes D_j^{\pm}$ OF $\widehat{SL}(2,\mathbb{R}) \otimes \widehat{SL}(2,\mathbb{R})$.

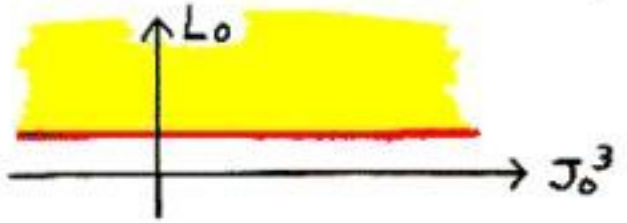
D_j^+ :



WITH THE CONSTRAINT $\boxed{\frac{1}{2} < j < \frac{k-1}{2}}$

(2) CONTINUOUS REPRESENTATION $C_j^{\alpha} \otimes C_j^{\alpha}$ $\left(j = \frac{1}{2} + iS\right)$

C_j^{α} :



(3) SPECTRAL FLOW OF (1) AND (2).

$$\begin{cases} J_n^3 \to J_n^3 - \frac{k}{2}\omega\,\delta_{n,0} \\ J_n^{\pm} \to J_{n\pm\omega}^{\pm} \end{cases}$$

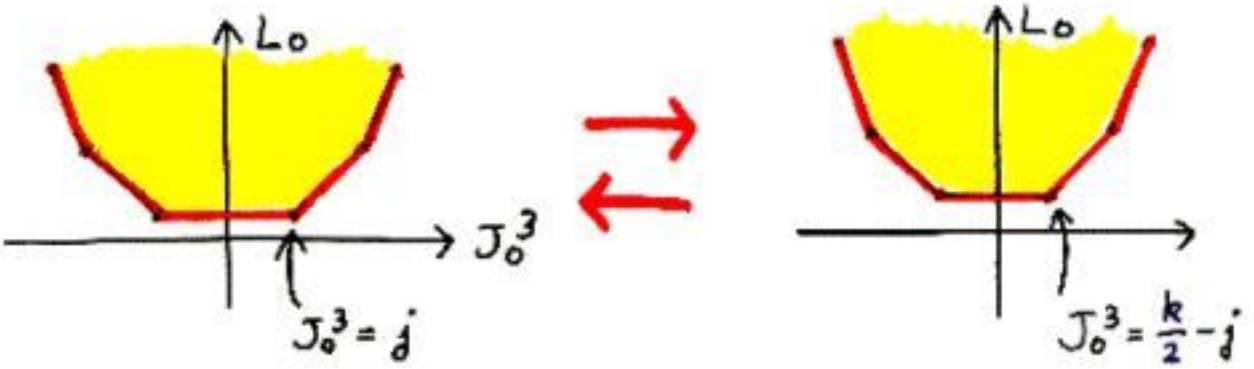
$\omega = 0, \pm 1, \pm 2, \cdots$

NO GHOST THEOREM HOLDS IN THIS HILBERT SPACE.

THE SPECTRAL FLOW IS A SYMMETRY OF THE WZW MODEL.

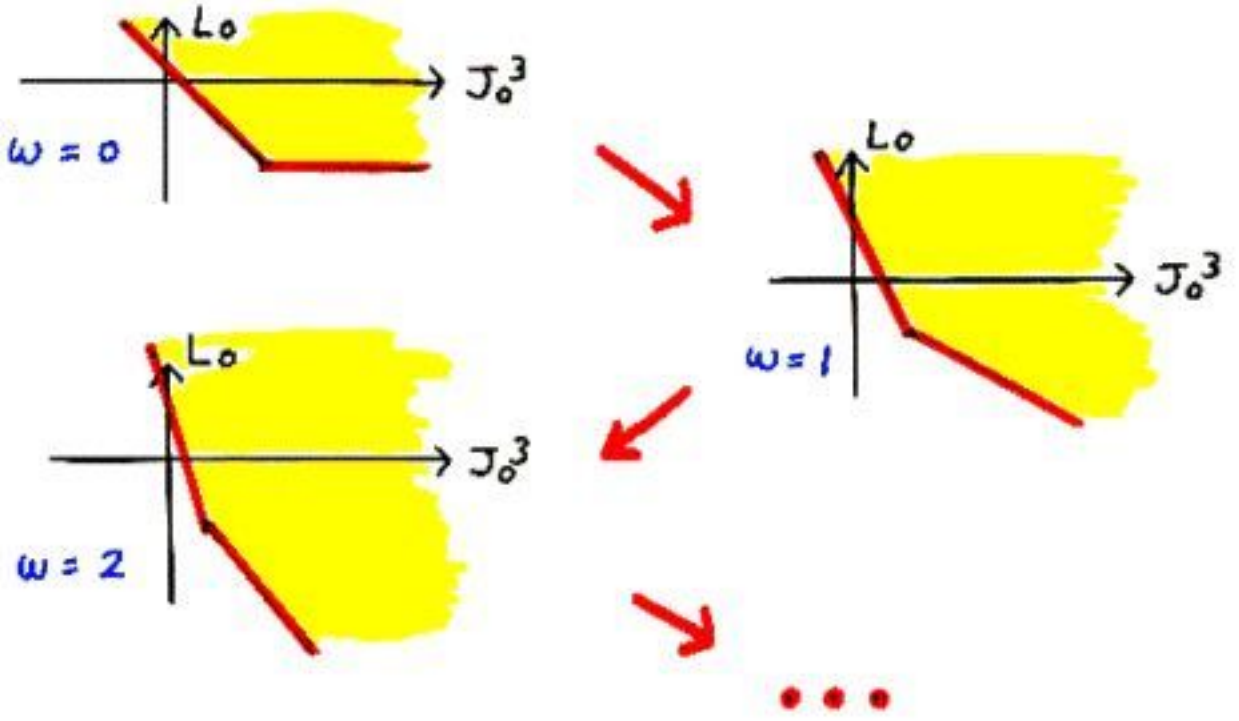
- IN THE $SU(2)$ WZW MODEL,

  IT MAPS A STANDARD REPRESENTATION INTO ANOTHER.



- IN THE $SL(2,\mathbb{R})$ WZW MODEL,

  IT GENERATES A NEW TYPE OF REPRESENTATIONS.



THIS IS RELATED TO THE FACT

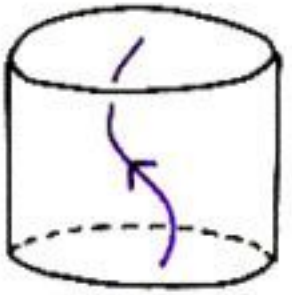
THAT $\pi_1(SL(2,\mathbb{R})) = \mathbb{Z}$.

IN THE SEMI-CLASSICAL APPROXIMATION,

THE AMOUNT ω OF THE SPECTRAL FLOW

$$\begin{cases} J_m^3 \to J_m^3 - \frac{k}{2}\omega\,\delta_{m,0} \\ J_m^\pm \to J_{m\pm\omega}^\pm \end{cases}$$

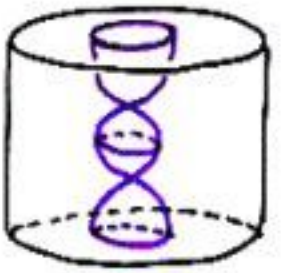
CORRESPONDS TO THE WINDING NUMBER OF STRING.



MASSIVE PARTICLE

$\in D_j^+ \otimes D_j^+$

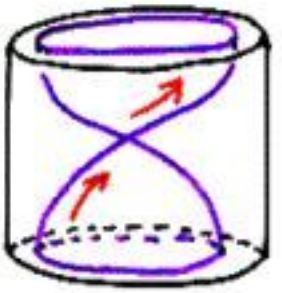
WITH $\omega = 0$.



SHORT STRING

$\in D_j^+ \otimes D_j^+$

WITH $\omega = 1, 2, 3, \cdots$



LONG STRING

$\in C_j^\alpha \otimes C_j^\alpha$

WITH $\omega = 1, 2, 3, \cdots$

CONTINUOUS ENERGY SPECTRUM

One-Loop Free Energy

Thermal AdS_3 : Consider the Euclidean AdS_3,

and periodically identify the imaginary time,

$$t \to t + \beta$$

The worldsheet theory for the Euclidean AdS_3,

$$S = \frac{k}{\pi} \int d^2z \left(\partial\phi \bar{\partial}\phi + (\partial + \partial\phi)\bar{v} \cdot (\bar{\partial} + \bar{\partial}\phi) v \right),$$

is exactly solvable by the iterative Gaussian integral.

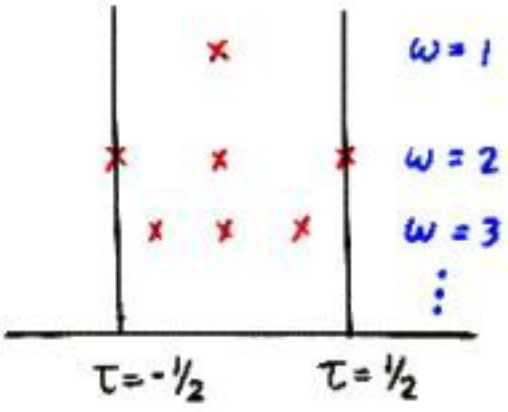
(Gawedzki, 1989)

Path Integral

worldsheet torus → thermal AdS_3

The amplitude $Z(\tau, \beta)$ has singularities

at $\tau = \frac{i\beta}{2\pi\omega}, \frac{i\beta \pm 1}{2\pi\omega}, \frac{i\beta \pm 2}{2\pi\omega}, \ldots$



$\Updownarrow$

$\exists$ Holomorphic map

The worldsheet can grow indefinitely large.

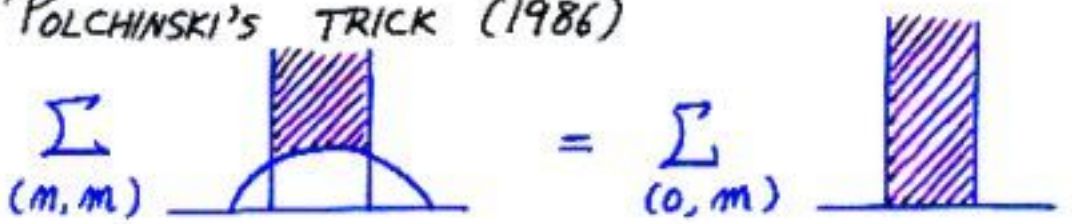
$$\int d^2\tau \underbrace{Z(\tau,\beta)} = \sum_i \log\left(\frac{1}{1-e^{-\beta E_i}}\right)$$

GAWEDZKI, 1991

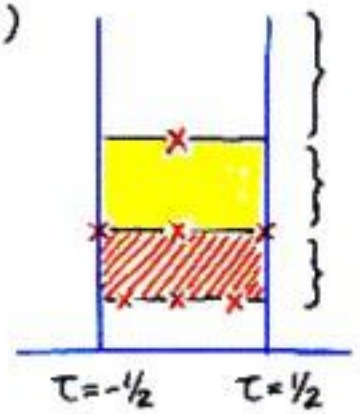
E_i : SINGLE STRING SPECTRUM GIVEN IN HEP-TH/0001053.

TO VERIFY THIS,

(1) POLCHINSKI'S TRICK (1986)



(2)



→ SHORT STRING WITH $\omega = 0$

→ SHORT STRING WITH $\omega = 1$

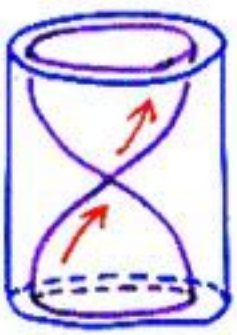
→ " " WITH $\omega = 2$

WITH THE CONSTRAINT

$$\frac{1}{2} < j < \frac{k-1}{2}$$

THE POLES AT × ⇒ LONG STRING WITH CONTINUOUS SPECTRUM

THE DENSITY OF STATES : $\rho(E) \sim L_{IR} + \frac{d}{dE}\delta(E)$



$\delta(E)$ = PHASE SHIFT

AGREES WITH THE 2-POINT FUNCTION COMPUTED BY TESCHNER, (ZAMOLODCHIKOV)^2.

FOUR-POINT AMPLITUDE

$\mathcal{F}(z,x)$ GIVEN BY THE PATH INTEGRAL OVER

WORLDSHEET SPHERE WITH 4 PUNCTURES

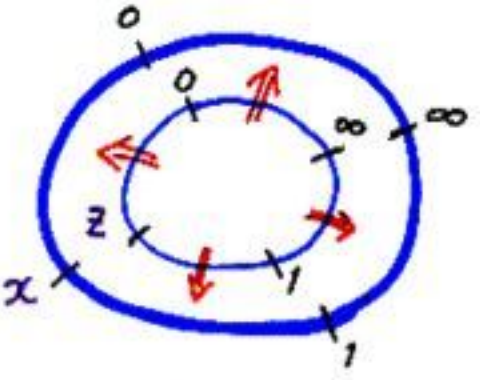
EUCLIDEAN AdS_3 WITH 4 BOUNDARY POINTS FIXED

FOUR-POINT AMPLITUDE OF THE TARGET SPACE CFT_2 :

$$\langle \mathcal{O}_1(0)\mathcal{O}_2(x)\mathcal{O}_3(1)\mathcal{O}_4(\infty)\rangle = \int d^2z\, \mathcal{F}(z,x)$$

THE KNIZHNIK-ZAMOLODCHIKOV EQUATION IMPLIES

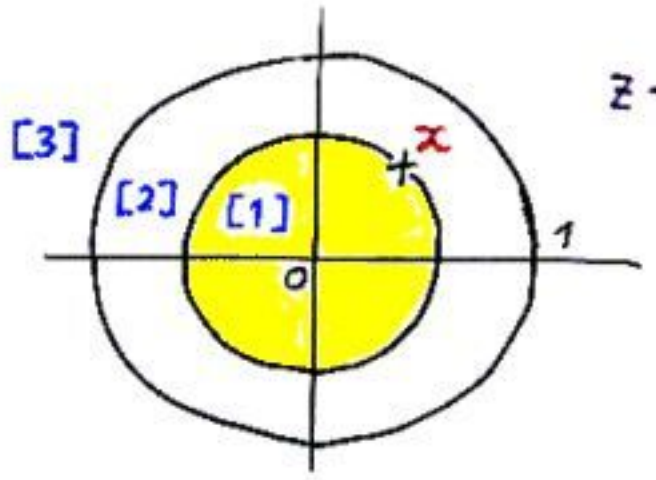
$\mathcal{F}(z,x)$ HAS A SINGULARITY AT $z=x$,
AS WELL AS AT $z=0,1,\infty$.



AT $z=x$,
THERE IS A HOLOMORPHIC MAP
FROM THE WORLDSHEET
TO THE BOUNDARY OF AdS_3.

THE WORLDSHEET CAN GROW
INDEFINITELY LARGE.

$$\langle O_1(0)\, O_2(x)\, O_3(1)\, O_4(\infty)\rangle = \int d^2z\, \mathcal{F}(z,x)$$



- THE INTEGRAL OVER [1]

$$\Rightarrow\ O_1(0)\, O_2(x) \sim \sum_{(h,\bar{h})} x^{h-h_1-h_2}\, \bar{x}^{\bar{h}-\bar{h}_1-\bar{h}_2}\, O_{h\bar{h}}(0)$$

$(h,\bar{h})$: SHORT STRING WITH $\omega = 0$

AND $\frac{1}{2} < j < \frac{k-1}{2}$

- SINGULARITY AT $z = x$

$\Rightarrow$ LONG STRING WITH $\omega = 1$

WITH THE CORRECT INFRARED SINGULARITY.

- THE INTEGRAL OVER [2] AND [3] (PRELIMINARY)

$\Rightarrow$ CONTAINS
- MIXING WITH 2 PARTICLE STATES
- $\log x$ CORRESPONDING TO SHIFTS OF ANOMALOUS DIMENSIONS.

SUMMARY

(1) THE $SL(2,\mathbb{R})$ WZW MODEL MAKES SENSE.

- NO GHOST THEOREM HOLDS.
- NEW SECTORS
  - WINDING STATES $\Leftarrow$ SPECTRAL FLOW
  - LONG STRING WITH CONTINUOUS SPECTRUM

(2) THE EUCLIDEAN ROTATION OF THE TARGET SPACE WORKS FOR PHYSICAL AMPLITUDES, AFTER INTEGRATING OUT THE WORLDSHEET MODULI.

$$SL(2,\mathbb{R}) \iff SL(2,\mathbb{C})/SU(2)$$

(3) EXPLICIT EVALUATIONS OF EUCLIDEAN PATH INTEGRAL ARE POSSIBLE, AND

- THE RESULTS ARE CONSISTENT WITH (1).
- SINGULARITIES IN THE INTERIOR OF THE WORLDSHEET MODULI SPACE.

  $\Rightarrow$
  - LONG STRING
  - CONSTRAINT $\frac{1}{2} < j < \frac{k-1}{2}$ ON SHORT STRING SPECTRUM
- $\rho(E) \sim L_{IR} + \frac{d\delta}{dE}$ FOR LONG STRING
- MIXING WITH 2 PARTICLE STATES / $\log x$