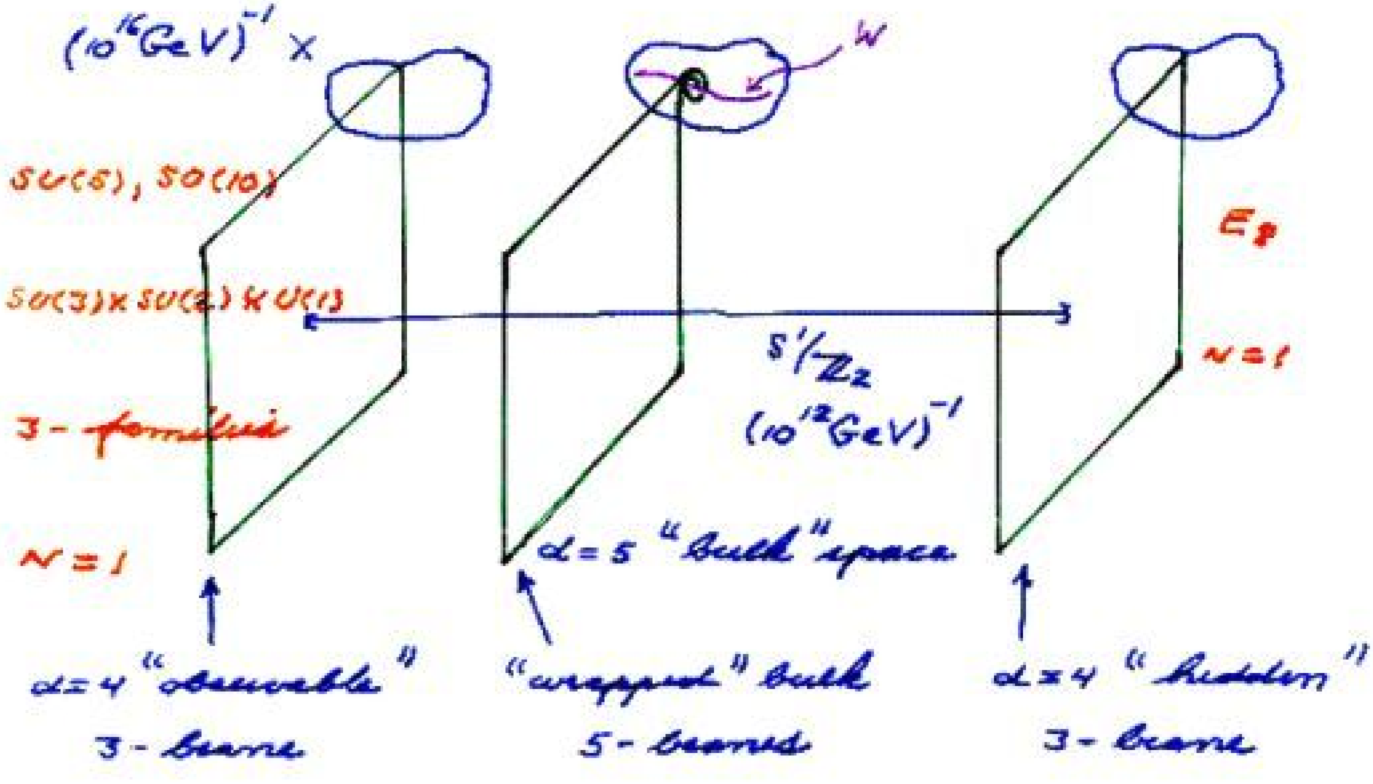
M-Theory compactified on $S^1/\mathbb{Z}_2 \times CY \Rightarrow$

a $d = 5$ brane Universe – "Heterotic M-Theory"

Lukas, Ovrut, Waldram + Stelle + Donagi, Pantev

General Discussion:



A) Supergravity + Branes :

- "discovery" of the $d = 5$ phase of M-Theory.

- most gen'l compactification of M-theory on $S^1/\mathbb{Z}_2 \times CY$ ($d=5$, $S^1/\mathbb{Z}_2$, $N=2$) consistent with anomaly cancellation (G-flux).
- supports BPS 3-branes with "warped" geometry - not RS branes.

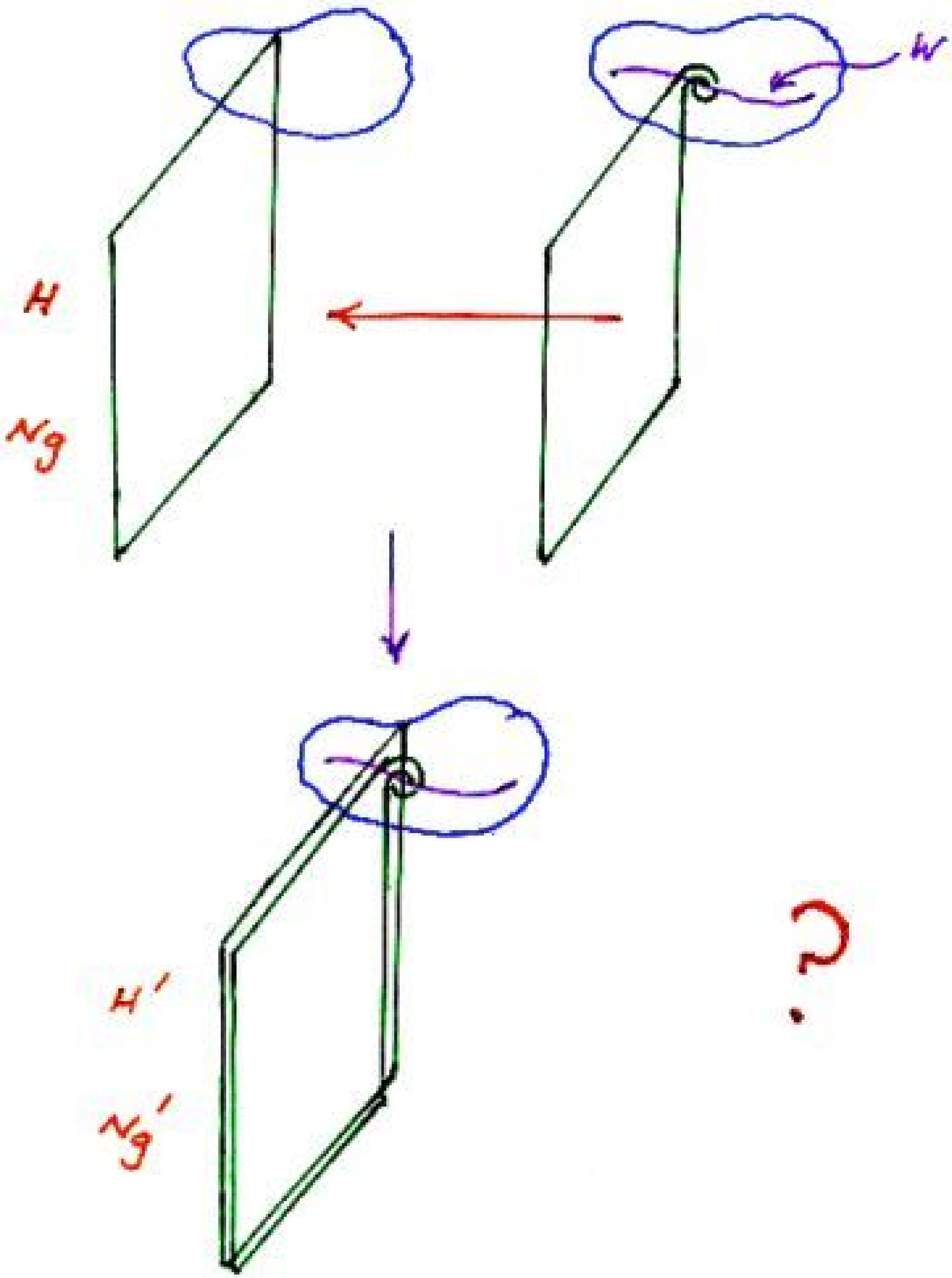
B) Particle Physics:

- softly broken $N=1$, 3 families, GUT and standard model gauge groups.
- suppressed nucleon decay
- hierarchy pattern of Yukawa couplings.

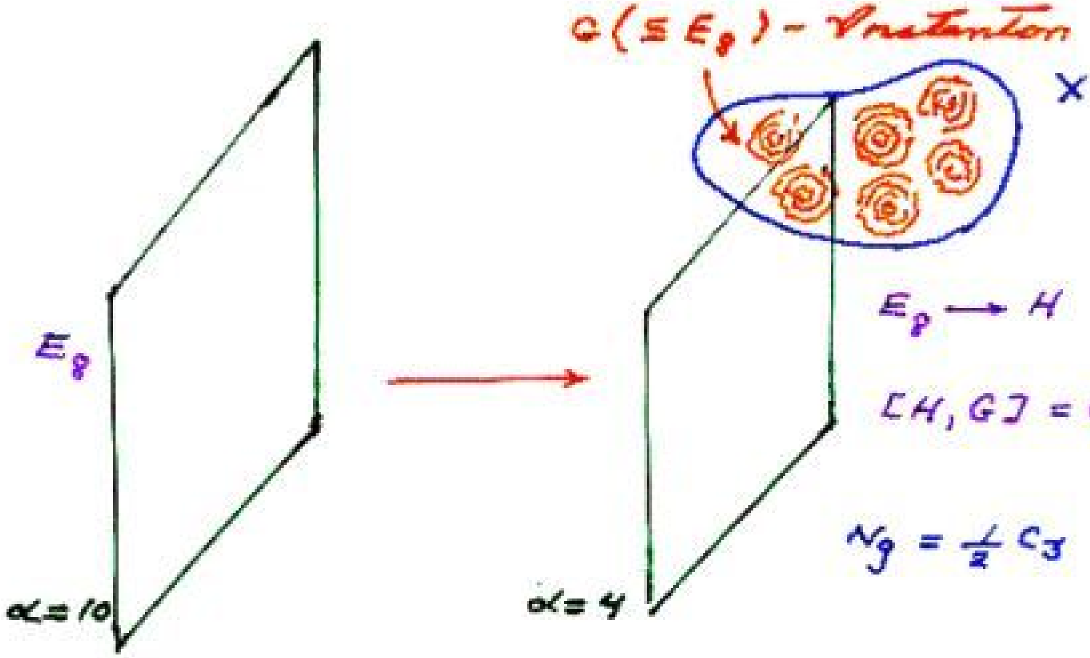
Arnowitt

Non-Perturbative Phase Transitions : BAO, Pantev, Park

What happens when



I) G-Instantons and Vector Bundles



$E_8 \longrightarrow H$

$[H, G] = 0$

$N_g = \frac{1}{2} c_3$

Examples:

a) "standard" embedding $F \wedge F = R \wedge R \Rightarrow$

$G = SU(3) \Rightarrow H = E_6$

b) non-standard embeddings $F \wedge F \neq R \wedge R \Rightarrow$

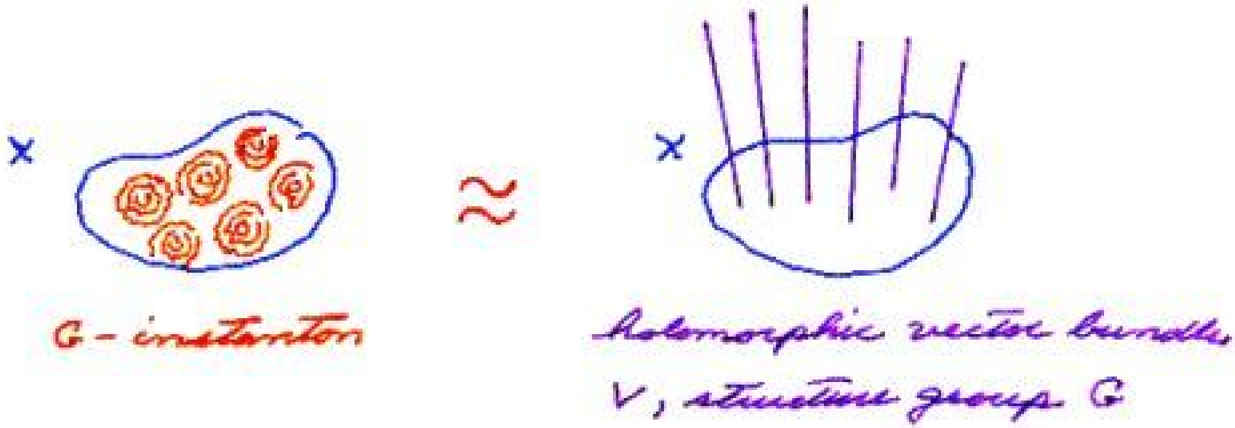
$G = SU(4) \Rightarrow H = SO(10)$

$G = SU(5) \Rightarrow H = SU(5)$

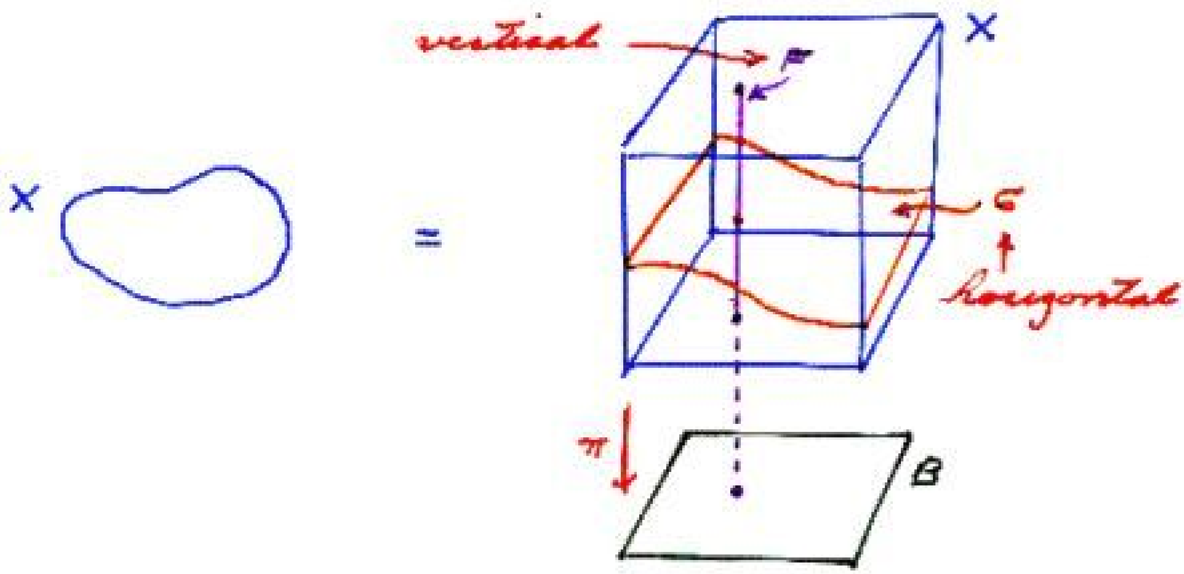
$G = SU(5) \times \mathbb{Z}_2 \Rightarrow H = SU(3)_C \times SU(2)_L \times U(1)_Y$

All embeddings are "natural" in Heterotic M-Theory! $\Longrightarrow$ How does one classify and construct G-instantons?

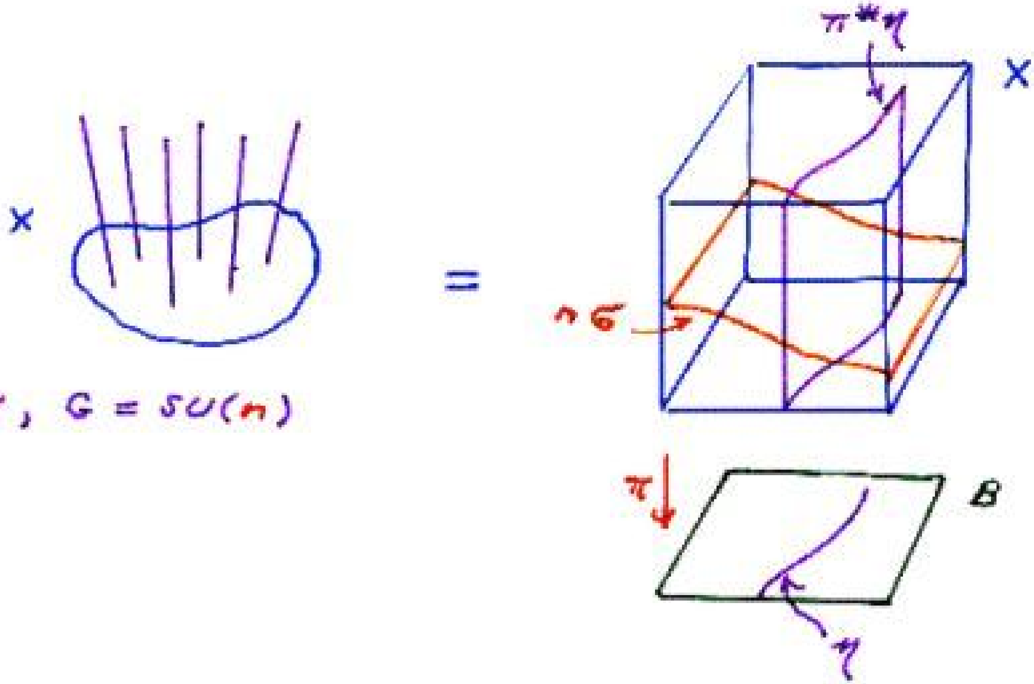
- Donaldson, Uhlenbeck, Yau Theorem:



Take CY X to be "elliptically fibered".

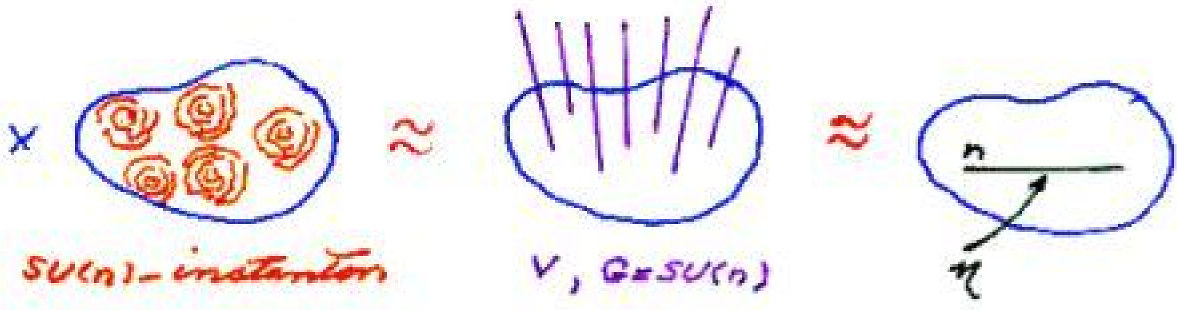


Then FMW ⟹



Conclusion:

An SU(n)-instanton is equivalent to a horizontal curve η in B. Pictorially



II) Bulk Space 5-Branes :

Anomaly cancellation $\Rightarrow$

$$W = c_2(TX) - c_2(V)$$

$\Rightarrow$

$$W = \pi^* z \cdot \sigma + a_f F$$

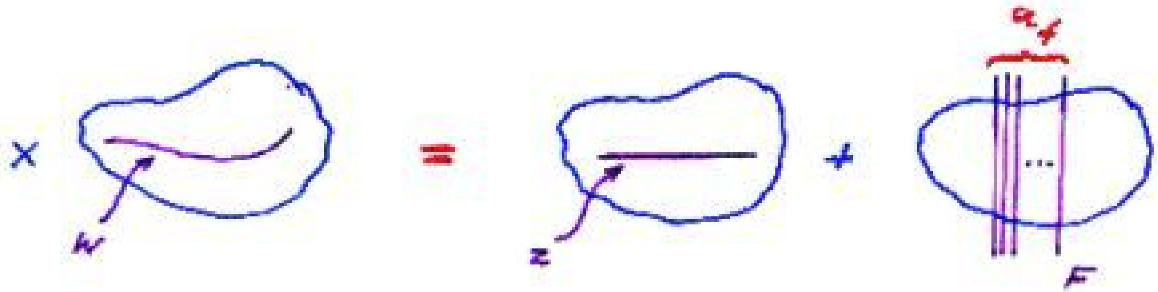
where

$$z = 12\, c_1(B) - \eta$$

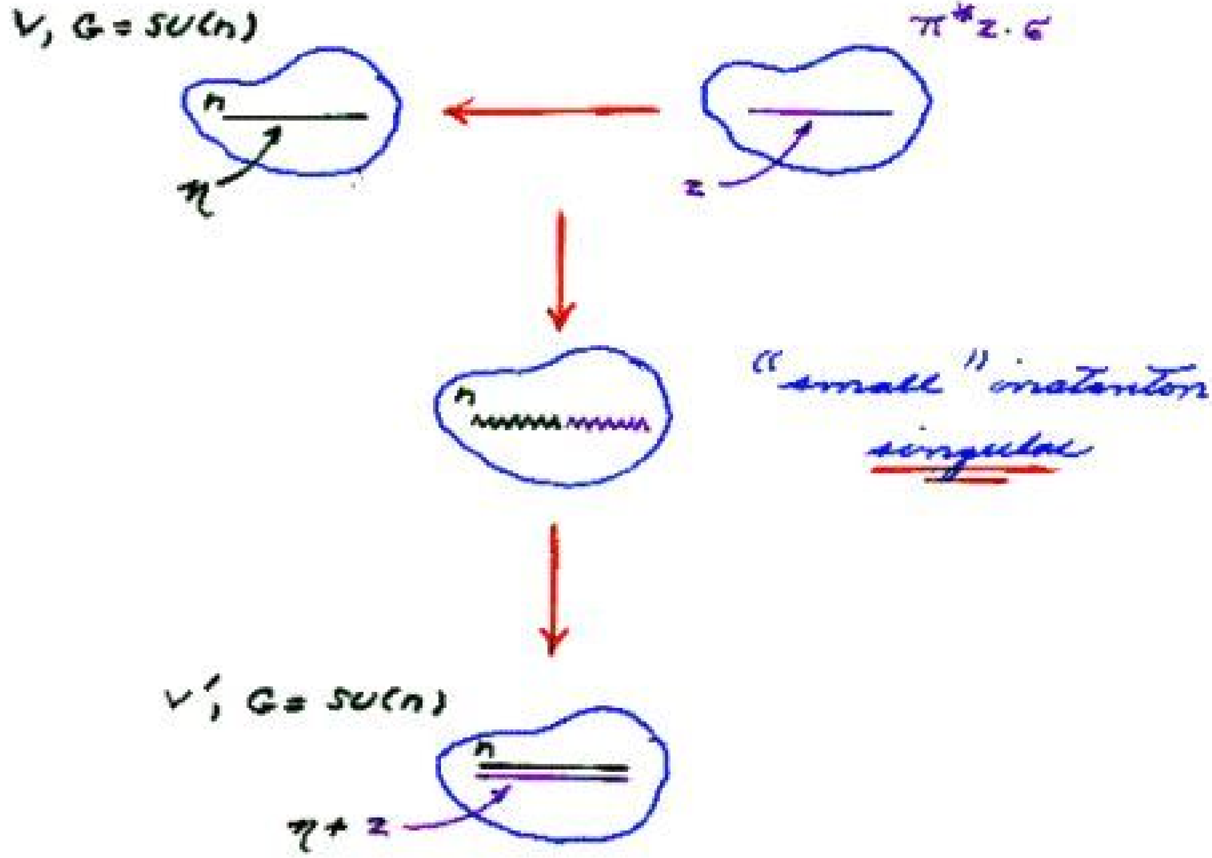
is a curve in B and

$$a_f = f(n, B, \eta)$$

is a non-negative integer. Pictorially



4) Collision of the pure horizontal 5-brane component:



Note: n unchanged, but $\eta \longrightarrow \eta + z$ $\Rightarrow$

- structure group $SU(n)$ is unchanged $\Rightarrow$

$$H' = H$$

$\Rightarrow$ observable 3-brane gauge group is unchanged during the phase transition

- vector bundle changes since $\eta \to \eta + z \Rightarrow$

a) $$c_2(V') = c_2(V) + \pi^* z \cdot \sigma$$

Physically

$$c_2(V) + (\pi^* z \cdot \sigma + a_f F) = c_2(TX)$$

$$\Rightarrow (c_2(V) + \pi^* z \cdot \sigma) + a_f F = c_2(TX)$$

$$\Rightarrow c_2(V') + a_f F = c_2(TX)$$

$\Rightarrow$ horizontal part of W "absorbed" by the bundle.

b) $$c_3(V') = c_3(V) + (2\eta + z - n c_1(B)) \cdot z$$

Physically

$$N_g = \frac{1}{2} c_3(V)$$

$$\Rightarrow N_g' = N_g + \frac{1}{2}(2\eta + z - n c_1(B)) \cdot z$$

$\Rightarrow$ the number of quark / lepton generations changes during the phase transition.

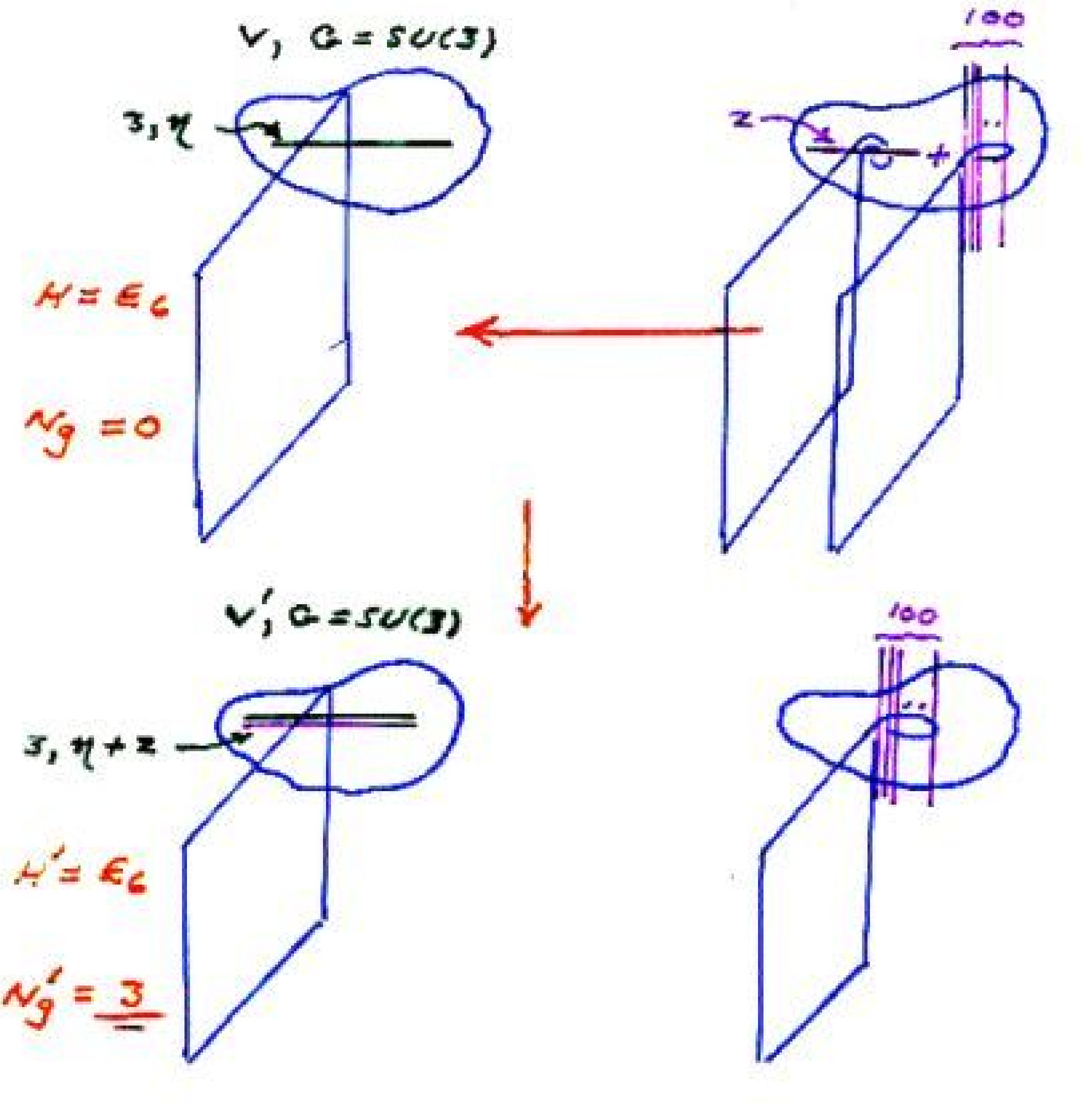
Example:

Choose $B = \mathbb{F}_0$. Define the instanton by taking

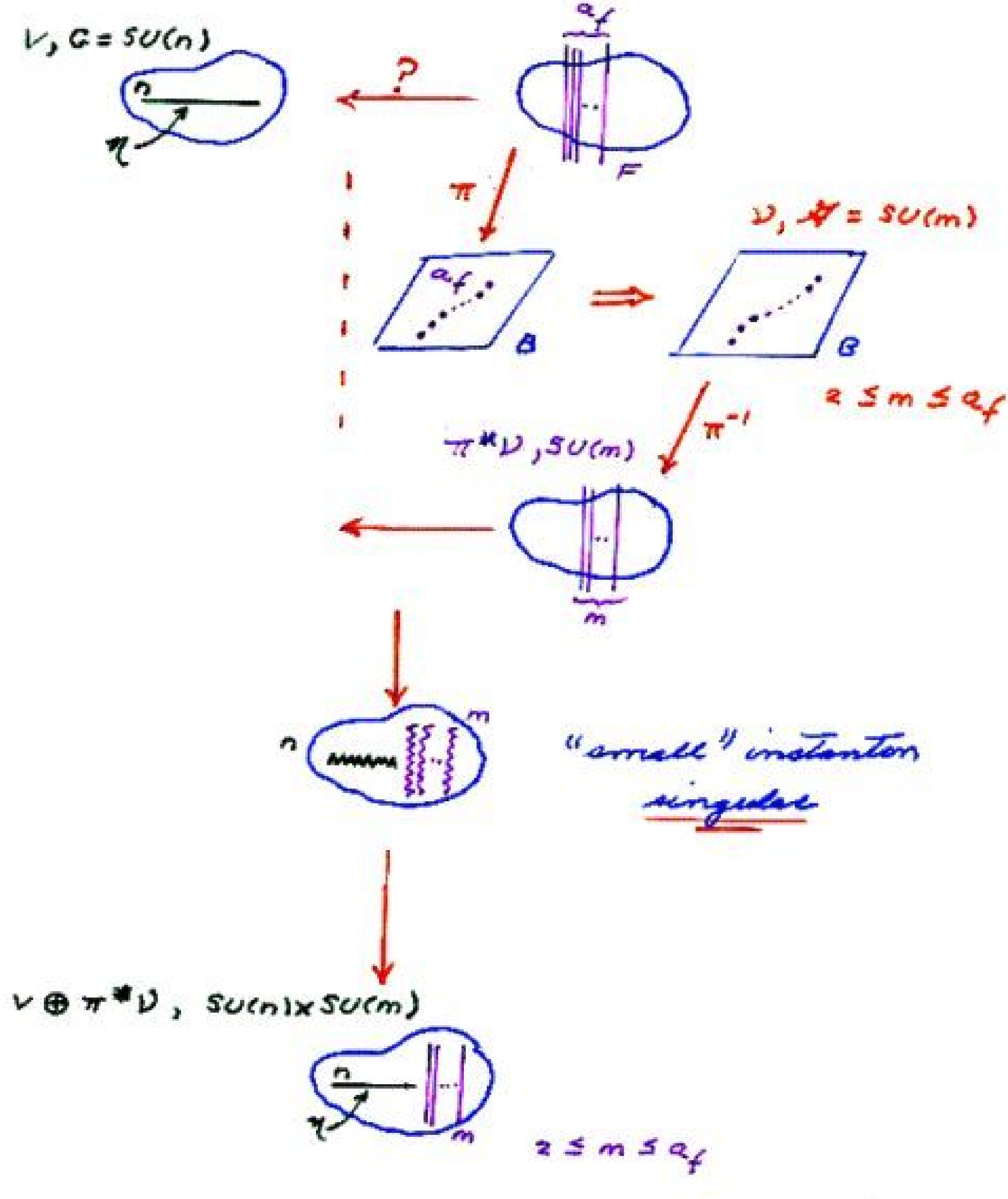
$G = SU(3) \Rightarrow H = E_6$ and $\eta = 6S + 6C \Rightarrow N_g = 0$.

Zero anomaly $\Rightarrow W \neq 0$ with $z = 18S + 10C$, $a_f = 100$.

Pictorially



B) Collision of the pure fiber 5-brane component



- structure group

$$SU(n) \longrightarrow SU(n)\times SU(m) \qquad 2 \le m \le a_f$$

$\Rightarrow$ observable 3-brane gauge group changed

$$H \to H'$$

during the phase transition.

- vector bundle changed $\Rightarrow$

a) $\quad c_2(V + \pi^* \mathcal{V}) = c_2(V) + a_f F$

Physically

$$c_2(V) + (\pi^* z \cdot \sigma + a_f F) = c_2(TX)$$

$$\Rightarrow (c_2(V) + a_f F) + \pi^* z \cdot \sigma = c_2(TX)$$

$$\Rightarrow c_2(V + \pi^* \mathcal{V}) + \pi^* z \cdot \sigma = c_2(TX)$$

$\Rightarrow$ fiber part of W "absorbed" by the vector bundle.

6) $c_3(V \oplus \pi^* \nu) = c_3(V)$

Physically

$$N_g' = N_g$$

$\Rightarrow$ The number of quark / lepton generations is <u>unchanged</u> during the phase transition.

Example:

Choose $B = dP_8$. Define the instanton by taking

$G = SU(4) \Rightarrow H = SO(10)$ and η. $\Rightarrow N_g = 3$.

Zero anomaly $\Rightarrow W \neq 0$ with $z \neq 0$, $a_f = 132$

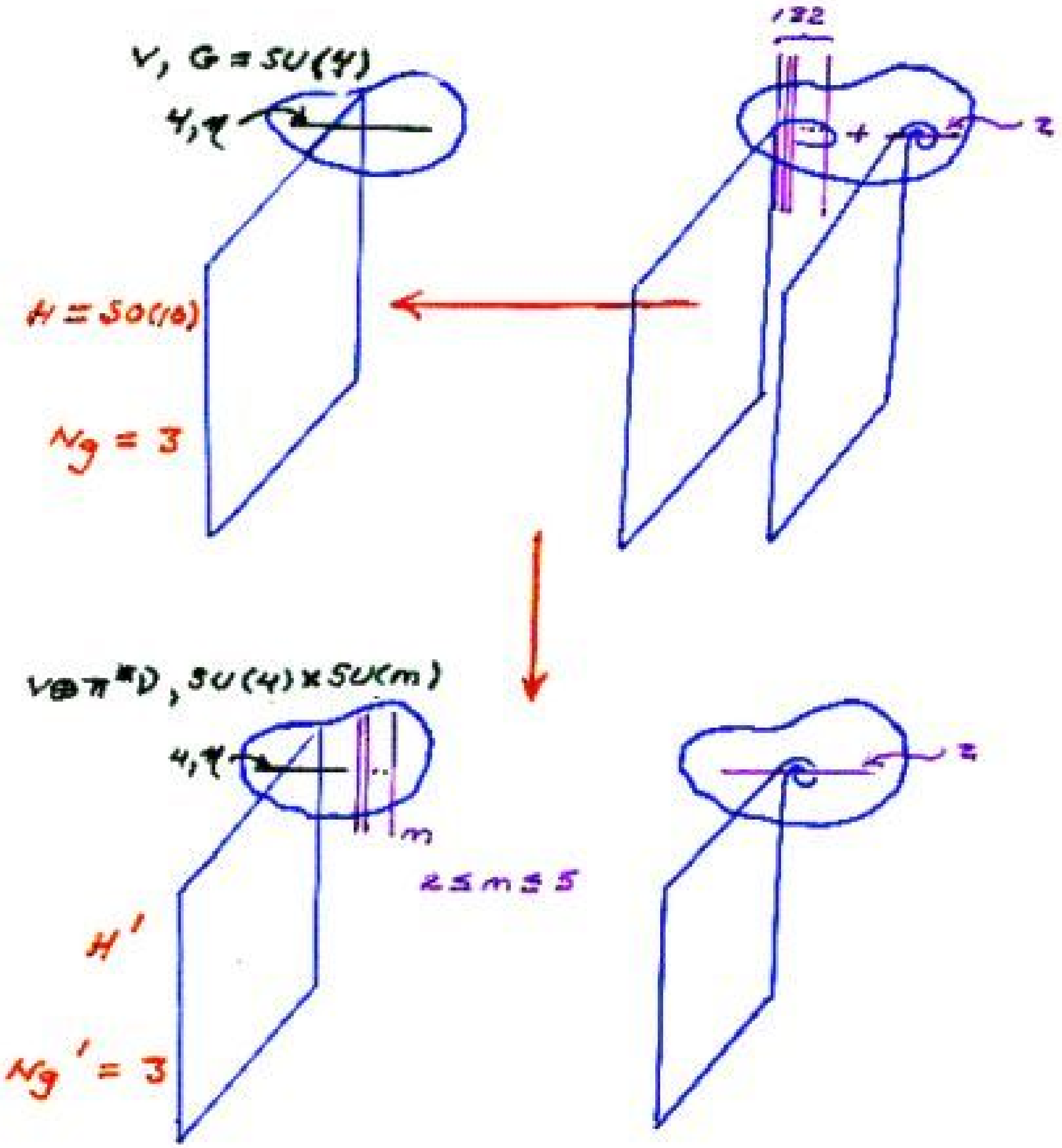
Note that after the phase transition the structure group is $SU(4) \times SU(m)$. Hence $[SU(m), SU(4)] = 0 \Rightarrow$

$$SU(m) \subseteq SO(10)$$

$\Rightarrow$

$$2 \leq m \leq 5 \quad (\underline{\text{not}}\ a_f = 132)$$

Pictorially



What is H'?

$m = 2 \implies H' = SO(7),\ SU(2)\times SU(4)$

$m = 3 \implies H' = \mathbb{1}$

$m = 4 \implies H' = SU(2)\times SU(2)$

$m = 5 \implies H' = U(1)$