Vertex Operators for the Supermembrane

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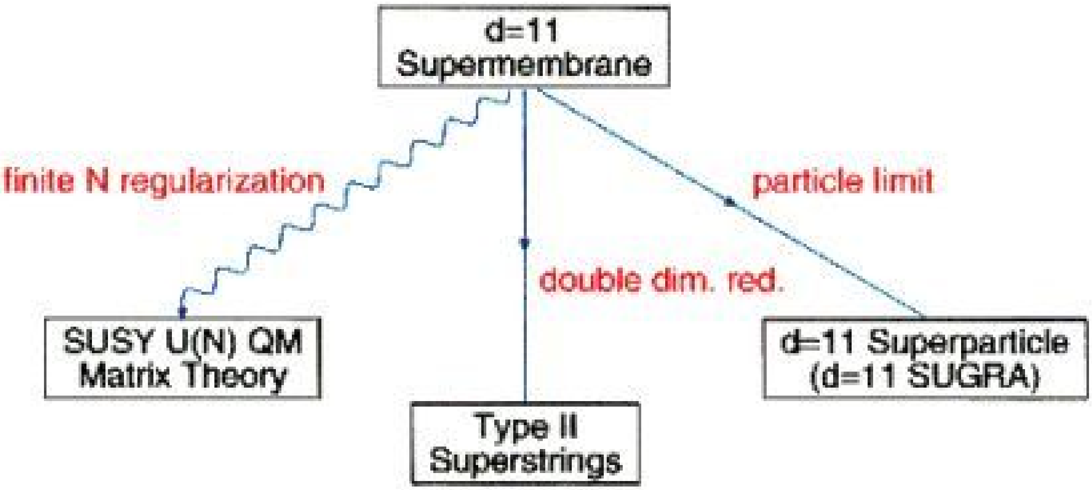
What is the role of the d=11 Supermembrane in M-theory?



Problems:

- Nonlinear theory, no well defined perturbative scheme
- Continuous spectrum $\rightarrow$ multiparticle interpretation
- What are the sensible quantities/operators to compute?

Nevertheless: How far can one go?

- Embarrassing situation: Higher derivative corrections to d=11 SUGRA known (Green,Gutperle,Vanhove)

$$S_{R^4} = \int d^{11}x \sqrt{-g}\, t_8\, t_8\, R^4 \left(\frac{2\pi^2}{3} + \frac{\zeta(3)}{R_{11}^3}\right)$$

via duality arg.

but NO model to reproduce it!

- Matrix Theory seems to fail (Helling,P,Serone,Waldron)

This talk:

- Construct vertex operators for the membrane in light cone gauge

- Govern emission of massless $d = 11$ supermultiplet $(h_{ab}, C_{abc}, \psi_a^\alpha)$

$\rightarrow$ $\boxed{\text{makes the supermembrane more ``computable''}}$

Applications:

- Define scattering amplitudes (a la superstring operator approach)

- 3 point tree level amps agree with d=11 SUGRA

- Semiclassical quantization on $R^9 \times S_1 \times S_1$: 4-gravitons at one loop $\leftrightarrow$ M-Theory R^4-terms

As a by-product:

- Construct Matrix theory action in weak background fields, upon compactification: IKKT-model, Matrix Strings.

Covariant approach?

$\boxed{\text{vertex ops}} \leftrightarrow$ linear coupling of background fields to world-volume (X^μ, θ)

- Full background field action known in superspace:
  (Bergshoeff, Sezgin, Townsend)

$$S = \int d^3\xi \sqrt{-g[Z(\xi)]} + \tfrac{1}{6}\epsilon^{ijk}\,\pi_i^A \pi_j^B \pi_k^C\, B_{CBA}[Z(\xi)]$$

where $Z^A = (X^\mu(\xi), \theta^\alpha(\xi)) \quad \pi_i^A = \frac{\partial Z^M}{\partial \xi^i} E_M^A$
$g_{ij} = \pi_i^r\, \pi_j^s\, \eta_{rs}$

- but one needs action in components!
  $\to$ Gauge completion (Cremmer, Ferrara)

$$\begin{aligned} E_\mu^r &= e_\mu^r + 2\,\bar\theta\Gamma^r \psi_\mu + \ldots + O(\theta^{32}) \\ E_\mu^\alpha &= \psi_\mu^a - \tfrac{1}{4}\hat\omega_\mu{}^{rs}\,(\Gamma_{rs}\theta)^\alpha + (T_\mu \cdot \hat F\,\theta)^\alpha + \ldots \\ B_{\mu\nu\rho} &= C_{\mu\nu\rho} - 6\,\bar\theta\Gamma_{[\mu\nu}\psi_{\rho]} + \ldots + O(\theta^{32}) \end{aligned}$$

explicitly known only up to $O(\theta^3)$ (de Wit, Peeters, P)

$\boxed{\text{covariant vertex ops via superspace beyond reach}} \to \boxed{\text{Go to light cone gauge!}}$

The Light-Cone Supermembrane (de Wit,Hoppe,Nicolai)

imposing $X^+ = p^+ \tau$, $\Gamma^+\Theta = 0$ yields flat space action

$$\boxed{\mathcal{L} = \tfrac{1}{2}(DX^a)^2 - \tfrac{1}{4}\{X^a, X^b\}^2 - i\theta\, D\theta - i\theta\,\gamma_a\{X^a, \theta\}}$$

$$\begin{array}{lll} \text{with} & DX^a := \partial_\tau X^a - \{\omega, X^a\} & X^a = X^a(\tau, \sigma_1, \sigma_2) \\ & \{A, B\} := \epsilon^{ij}\,\partial_{\sigma_i} A\, \partial_{\sigma_j} B & \theta_\alpha = \theta_\alpha(\tau, \sigma_1, \sigma_2) \\ & a = 1, \ldots, 9 & \alpha = 1, \ldots, 16 \end{array}$$

- residual symmetry under **A**rea **P**reserving **D**iffeomorphisms (Goldstone,Hoppe)

$$\delta X^a = \{\xi, X^a\}, \qquad \delta\theta = \{\xi, \theta\}, \qquad \delta\omega = \partial_\tau + \{\xi, \omega\}$$

- Supersymmetry:

$$\boxed{\begin{array}{rcll} \delta X^a & = & -2\,\epsilon\,\gamma^a\theta & \qquad \delta\omega = -2\,\epsilon\,\theta \\ \delta\theta & = & i\,DX^a\,\gamma_a\,\epsilon - \frac{i}{2}\{X^a, X^b\}\,\gamma_{ab}\,\epsilon + \eta & \end{array}}$$

with $32 = 16 + 16$ components.

$\rightarrow$ $\boxed{\text{SUSY quantum mechanics with } \infty\text{-dim gauge group}}$

Vertex Operator Construction:

We seek:

$$
\begin{aligned}
V_h &= h_{ab} \int d\tau d^2\sigma\, \mathcal{O}^{ab}[X^a(\tau,\sigma_i), \theta(\tau,\sigma_i)]\, e^{ik\cdot X} \\
V_C &= C_{abc} \int d\tau d^2\sigma\, \mathcal{O}^{abc}[X^a, \theta]\, e^{ik\cdot X} \\
V_\psi &= \psi_a^\alpha \int d\tau d^2\sigma\, \mathcal{O}_\alpha^a[X^a, \theta]\, e^{ik\cdot X}
\end{aligned}
$$

subject to on-shell constraints $k^a h_{ab} = h_{aa} = 0 = k^a C_{abc}$.

Strategy: $\delta\mathcal{L}_{\text{full}}[X,\theta; h, C, \psi] = \mathcal{L}_{\text{full}}[X,\theta; \delta h, \delta C, \delta\psi]$

$\leftrightarrow$ Require covariance under SUSY

$$
\boxed{
\begin{aligned}
\delta V_h &= V_{\delta\psi[h]} \\
\delta V_C &= V_{\delta\psi[C]} \\
\delta V_\psi &= V_{\delta h} + V_{\delta C}
\end{aligned}
}
$$

δ : APD SUSY of X^a, θ, ω

δ : light-cone SUGRA variations of h, C, ψ

e.g. linear SUSY $\quad \delta X^a = 0 \qquad \delta h_{ab} = -\bar{\psi}_{(a}\gamma_{b)}\eta$

$\qquad\qquad\qquad\quad \delta\theta = \eta \qquad\quad \delta C_{abc} = \frac{3}{2}\bar{\psi}_{[a}\gamma_{bc]}\eta$

Results:

- Graviton vertex

$$V_h = h_{ab}\Big[DX^a\,DX^b - \{X^a,X^c\}\{X^b,X^c\} - i\theta\gamma^a\{X^b,\theta\}$$
$$-2DX^a\,R^{bc}\,k_c - 6\{X^a,X^c\}\,R^{bcd}\,k_d + 2R^{ac}\,R^{bd}\,k_c\,k_d\Big]$$

- 3-Form vertex

$$V_C = -C_{abc}\,DX^a\{X^b,X^c\} + F_{abcd}\Big[(DX^a - \tfrac{2}{3}R^{ae}\,k_e)\,R^{bcd}$$
$$-\tfrac{1}{2}\{X^a,X^b\}\,R^{cd} - \tfrac{1}{96}\{X^e,X^f\}\,\theta\gamma^{abcdef}\theta\Big]$$

- Gravitino vertex

$$V_\psi = \psi_a\Big[\Big(DX^a - 2R^{ab}\,k_b + \gamma_c\{X^c,X^a\}\Big)\theta\Big]$$
$$+\tilde{\psi}_a\Big[\gamma\cdot DX\Big(DX^a - 2R^{ab}\,k_b + \gamma_c\{X^c,X^a\}\Big)\theta$$
$$+\tfrac{1}{2}\gamma_{bc}\{X^b,X^c\}(DX^a - \{X^a,X^d\}\gamma^d)\theta$$
$$+8\gamma_b\theta\{X^b,X^c\}\,R^{cad}\,k_d + \tfrac{5}{3}\gamma_{bc}\theta\{X^b,X^c\}\,R^{ad}\,k_d$$
$$+\tfrac{4}{3}\gamma_{bc}\theta\Big(\{X^a,X^b\}\,R^{cd} + \{X^c,X^d\}\,R^{ab}\Big)k_d$$
$$+\tfrac{2}{3}i\Big(\gamma_b\theta\{X^a,\theta\}\gamma^b\theta - \theta\{X^a,\theta\}\theta\Big)$$
$$+\tfrac{8}{9}\gamma^b\theta\,R^{ac}\,R^{bd}\,k_ck_d\Big]$$

where $R^{ab} = \frac{1}{4}\theta\gamma^{ab}\theta \qquad R^{abc} = \frac{1}{12}\theta\gamma^{abc}\theta$
$(h_{+a}, h_{++}, C_{+ab}, \psi_+, \tilde{\psi}_+)$ also known.

Stringent Consistency Checks:

- Background field symmetries: vertex ops. invariant under

$$
\begin{aligned}
\delta h_{ab} &= k_{(a}\,\xi_{b)} && \text{(coordinate transf.)} \\
\delta C_{abc} &= k_{[a}\,\xi_{bc]} && \text{(tensor gauge transf.)} \\
\delta \psi_a &= k_a \epsilon && \text{(field indep. SUSY)}
\end{aligned}
$$

- Point particle limit: Drop $\{.,.\}$ terms
  $\leftrightarrow$ d=11 superparticle vertex ops (Green,Gutperle,Kwon)

- Double Dimensional Reduction (Duff,Howe,Inami,Stelle)

$$
X^a(\tau,\sigma_1,\sigma_2) \to \begin{pmatrix} X^i(\tau,\sigma_1) \\ X^9 = \sigma_2 \end{pmatrix} \quad i = 1,\ldots,8
$$

$$
\theta_\alpha = \begin{pmatrix} S_a(\tau,\sigma_1) \\ \tilde{S}_{\dot{a}}(\tau,\sigma_1) \end{pmatrix} \quad a,\dot{a} = 1,\ldots,8
$$

  vertex ops should reduce to Type IIA superstring vertices

  $\rightarrow$ factorization into left/right movers
  $\rightarrow$ yields both R & NS type vertices

e.g. double dim reduction of graviton vertex:

$$\{X^i, X^j\} = 0 \qquad \{X^i, X^9\} = \partial_{\sigma_1} X^i \qquad \omega = 0$$

$$\theta\gamma^{ij}\theta = S\Gamma^{ij}S + \tilde{S}\Gamma^{ij}\tilde{S} \qquad \underbrace{\theta\gamma^{ij9}\theta}_{SO(9)} = \underbrace{S\Gamma^{ij}S - \tilde{S}\Gamma^{ij}\tilde{S}}_{SO(8)}$$

then

$$\begin{aligned} V_h\big|_{\text{DDR}} = \; & h_{ab}\Big[DX^a\, DX^b - \{X^a, X^c\}\,\{X^b, X^c\} \\ & -i\theta\gamma^a \cancel{\{X^b, \theta\}}^{\,0} - \tfrac{1}{2} DX^a\, \theta\gamma^{bc}\theta\, k_c \\ & -\tfrac{1}{2}\{X^a, X^c\}\,\theta\gamma^{bcd}\theta\, k_d + 2R^{ac}\, R^{bd}\, k_c\, k_d\Big] \\ \to \; & h_{ij}\Big[\partial_0 X^i \partial_0 X^j - \partial_1 X^i \partial_1 X^j \\ & -\tfrac{1}{2}\partial_0 X^i (S\Gamma^{jm}S + \tilde{S}\Gamma^{jm}\tilde{S})k_m \\ & +\tfrac{1}{2}\partial_1 X^i (S\Gamma^{jm}S - \tilde{S}\Gamma^{jm}\tilde{S})k_m \\ & +\tfrac{1}{4} S\Gamma^{im}S\, \tilde{S}\Gamma^{jn}\tilde{S} k_m k_n\Big] \end{aligned}$$

$$= h_{ij} \underbrace{\left(\partial_+ X^i - \tfrac{1}{2} S\Gamma^{im}S k_m\right)}_{\text{left moving part}} \underbrace{\left(\partial_- X^j - \tfrac{1}{2}\tilde{S}\Gamma^{jn}\tilde{S}k_n\right)}_{\text{right moving part}}$$

Appl. to Matrix Theory in Background Fields:

$\boxed{\text{APD} = \lim_{N\to\infty} SU(N)}$ $\qquad \{.,.\} \to i\,[.,.]$

$$X^a(\tau,\sigma^1,\sigma^2) \to \mathbf{X}^a_{mn}(\tau) \qquad m,n = 1,\ldots,N$$

$$\tfrac{1}{4\pi}\int d^2\sigma(\ldots) \to \tfrac{1}{N}\,\mathsf{STr}[\ldots]$$

Sym. Trace to maintain transformation properties

"Weak" background field action now known to *all* orders in θ.

$$\mathcal{S}_{\mathsf{MT}} = \int d\tau \Big(\mathcal{L}_0 + V_{h(X)} + V_{C(X)} + V_{\Psi(X)}\Big)$$

where e.g.

$$\begin{aligned} V_{h(X)} \;&=\; \mathsf{STr}\Big[\Big\{\dot{\mathbf{X}}^a\dot{\mathbf{X}}^b + [\mathbf{X}^a,\mathbf{X}^c]\,[\mathbf{X}^b,\mathbf{X}^c] + \\ &+\Theta\gamma^a\,[\mathbf{X}^b,\Theta] - 2\dot{\mathbf{X}}^a\,\mathbf{R}^{bc}\frac{\partial}{\partial\mathbf{X}^c} - 6i[\mathbf{X}^a,\mathbf{X}^c]\,\mathbf{R}^{bcd}\frac{\partial}{\partial\mathbf{X}^d} \\ &\qquad +2(\Theta\gamma^{ac}\Theta)\,(\Theta\gamma^{bd}\Theta)\frac{\partial}{\partial\mathbf{X}^c}\frac{\partial}{\partial\mathbf{X}^d}\Big\}\,h_{ab}(\mathbf{X})\Big]\end{aligned}$$

$\to$ agrees with Matrix current calculations ($o(\theta^2)$)
(Taylor,V.Raamsdonk)

Tree Level Amplitudes:

- Split off center of mass dynamics:

$$\mathbf{X}^a = x^a\,\mathbb{1} + \hat{\mathbf{X}}^a \qquad \Theta^a = \theta^a\,\mathbb{1} + \hat{\Theta}^a \qquad \mathrm{Tr}\begin{pmatrix} \hat{\mathbf{X}}^a \\ \hat{\Theta} \end{pmatrix} = 0$$

- Asymptotic 1-particle state:

$$|\mathrm{IN}\rangle\rangle = |k_1, h_1\rangle_{x,\theta} \otimes |\mathrm{GS}\rangle^{\mathrm{SU(N)}}_{\hat{\mathbf{X}},\hat{\Theta}}$$

- 3-point amp: $\boxed{\langle\langle\, 1\,|\,V_2\,|\,3\,\rangle\rangle}$

$$V_2 = h^{(3)}_{ab}\,\mathrm{STr}\Big[(p^a\,p^b + 2\,p^a\,\hat{\mathbf{P}}^b + \hat{\mathbf{P}}^a\,\hat{\mathbf{P}}^b + [\hat{\mathbf{X}}^a, \hat{\mathbf{X}}^c]\,[\hat{\mathbf{X}}^b, \hat{\mathbf{X}}^c]\,)e^{ik\cdot\hat{\mathbf{X}}}\Big]\,e^{ik\cdot x}$$

insert into amplitude:

$$\underbrace{\langle k_1, h_1 | p^a\,p^b e^{ik\cdot x} | k_3, h_3\rangle h^{(2)}_{ab}}_{\text{superparticle result}}\ \underbrace{\langle \mathrm{GS}|\ \mathrm{STr}\,e^{ik\cdot\hat{\mathbf{X}}}\ |\mathrm{GS}\rangle}_{=N \text{ as } k^2=0} + (\dots)$$

$$h_{ab}\,\langle \mathrm{GS}|\mathrm{STr}\,\hat{\mathbf{P}}^b\,e^{ik\cdot\hat{\mathbf{X}}}|\mathrm{GS}\rangle \quad \sim \quad k^b\,h_{ab} = 0$$

$$h_{ab}\,\langle \mathrm{GS}|\mathrm{STr}\Big[(\hat{\mathbf{P}}^a\,\hat{\mathbf{P}}^b + [\hat{\mathbf{X}}^a, \hat{\mathbf{X}}^c]\,[\hat{\mathbf{X}}^b, \hat{\mathbf{X}}^c]\,)\,e^{ik\cdot\hat{\mathbf{X}}}\Big]|\mathrm{GS}\rangle$$

$$\sim \quad (k^a\,k^b + c\,\delta^{ab})\,h_{ab} = 0$$

Hence $\boxed{\langle\langle\, 1\,|\,V_2\,|\,3\,\rangle\rangle = \langle\, 1\,|\,V_2\,|\,3\,\rangle_{x,\theta} \cdot N\,\langle \mathrm{GS}|\mathrm{GS}\rangle}$

→ 3 point amps agree with tree level SUGRA!

One Loop Amplitudes: (work in progress)

- Proposal: $\boxed{\int d^{11}p_0\, \mathbf{Tr}(\Delta V_1\, \Delta V_2\, \Delta V_3\, \Delta V_4 \ldots)}$

  with propagator

$$\Delta = \frac{1}{\frac{1}{2}{p_0}^2 + \hat{\mathbf{H}}}$$

- compactify on $S_1 \times S_1$ (Duff, Inami, Pope, Sezgin, Stelle; Russo, Tseytlin)

$$\hat{\mathbf{H}} = (\underbrace{l_1 l_2}_{\text{winding numbers}} R_1 R_2)^2 + \sum_{m^2+n^2\neq 0} \alpha^\dagger_{m,n}\cdot\alpha_{m,n} + \lambda^{\dagger\,\alpha}_{m,n}\,\lambda^{\alpha}_{m,n} + \hat{\mathbf{H}}_{\text{Int}}$$

- 2 and 3 particles amps vanish as

$$\mathbf{Tr}(\theta_{\alpha_1}\ldots\theta_{\alpha_N})_{\theta_0} = \delta_{N,16}\,\epsilon^{\alpha_1\ldots\alpha_{16}}$$

- 4 gravitons:

$$\mathcal{A}_{4h} = \epsilon^{\alpha_1\ldots\alpha_{16}}\,\gamma^{a_1 a_2}_{\alpha_1\alpha_2}\ldots\gamma^{a_{15} a_{16}}_{\alpha_{15}\alpha_{16}}\; R^{(1)}_{a_1 a_2 a_3 a_4}\ldots R^{(4)}_{a_{13}a_{14}a_{15}a_{16}}$$

$$\cdot \sum_{\text{windings, KK}} \int d^9\, p_0\, \mathbf{Tr}\,\Delta^4 \longrightarrow ?$$

$\leftrightarrow$ (Green, Gutperle, Vanhove; Russo, Tseytlin; de Wit, Lüst)

Summary:

- constructed membrane vertex ops in light cone gauge
- reduce to d=11 superparticle/d=10 type IIA string vertices
- yields Matrix theory action in weak background fields
  (→ IKKT model, Matrix Strings ?)
- 3-point tree level amps agree with SUGRA
- 1-loop amps ↔ M theory R^4 corrections (?)

Open Questions:

- Where is the multiparticle picture?
- (Why) should vertex ops correspond to asymptotic states?

Outlook:

- higher point tree level amps
- R^4 SUSY partners, R^6 ?
- Reduction to IKKT model $\leftrightarrow$ MC simulations