Gauge Fields, Scalars, Warped Geometry, & Strings

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Schematically, there are many systems which reduce to warped solutions of

$$S = \int d^d x \, dy \left(a(\phi) R + b(\phi)(\partial\phi)^2 + c(\phi) F^2 - \Lambda(\phi) \right)$$

e.g.

Many have non-gravitational duals

- AdS / CFT a, b, c, Λ const
  + relevant deformations, R-S
- linear dilaton / little string theory

Most do not have (known) non-gravitational duals

- branes of M-theory
- Horava-Witten; type I'
- Other warped compactifications
- undiscovered solutions

Most have curvature singularities and/or strong coupling in GR approximation

Consider cases where

d-dimensional Poincare invariant

warped metric $$ds_E^2 = e^{2A(y)} dx_{\parallel}^2 + dy^2$$

"trapped gravity" $$M_d^{d-2} \sim M_{d+1}^{d-1} \int dy\, e^{(d-2)A(y)} < \infty$$

Scalar(s)

$$\phi = \phi(y)$$

gauge fields $A_{\mu_1 \cdots \mu_p} = 0 + \delta A(y,x)$

trivial in the background

N. Kaloper, E.S., L. Susskind

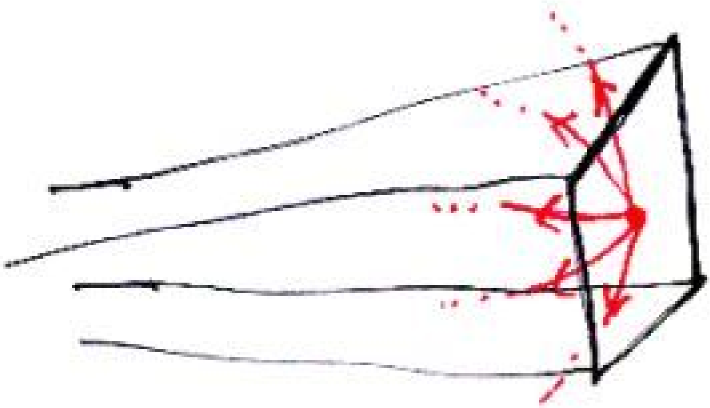
Consider the gauge field kinetic term

$$\int d^d x \, dy \sqrt{g} \; c(\alpha) \, \frac{F^2}{e_{d+1}^2}$$

Dimensionally reducing,

$$\frac{1}{e_d^2} \sim \frac{1}{e_{d+1}^2} \int dy \, \sqrt{g(y)} \; g^{\cdot\cdot} g^{\cdot\cdot} \; c(\alpha(y))$$

There are lots of cases for which this integral diverges



Gravity "trapped", but gauge fld not?

d d+1

This would be surprising given Black Hole no-hair theorems

global symmetry in d-dimensional description ?!

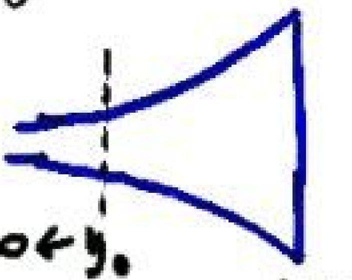
In several basic examples in M-theory, we find a more conventional result:

screening or Higgs at low energies

Simplest example : Randall-Sundrum background (cut off AdS_5)

$$ds^2 = e^{-\frac{2|y|}{L}} dx_{\parallel}^2 + dy^2$$

$$\alpha = \text{const}$$
$$\Lambda(\alpha) = -|\text{const}'|$$



$$\frac{1}{e_4^2} \sim \frac{1}{e_5^2}\int_0^{y_0} dy \; \cancel{\overbrace{e^{-\frac{4y}{L}}}^{\sqrt{g}}} \; \cancel{\overbrace{e^{\frac{2y}{L}}}^{g^{\cdot\cdot}}} \; \cancel{\overbrace{e^{\frac{2y}{L}}}^{g^{\cdot\cdot}}}$$

$$\sim \frac{1}{e_5^2} y_0 \sim \frac{1}{e_5^2} L \log(L p_0)$$

the charge is <u>screened</u> at long distance $\Leftarrow$ IR cutoff in 4d (CFT + gravity) description

(E. Witten)

This agrees with a calculation of the electrostatic potential from a point source Q at $y = 0$:

$$A_0(p, y=0) = -\frac{Q\, K_1(pL)}{2p\, K_0(pL)}$$

$$\xrightarrow[p \to 0]{} \frac{Q}{p^2 \log p}$$

As a function of dimension d, for q-form gauge field strength, we find

$$\frac{1}{e^2_{d,q}} \propto \begin{cases} \dfrac{1}{p_0^{2q-d}} & 2q \neq d \\ \log p_0 & 2q = d \end{cases}$$

generalization of screening to higher-d, higher-q

NS 5-brane backgrounds

String frame:

$$ds^2 = \overbrace{dx_6^2 + dy^2}^{\text{flat}} + l_s^2 N \, d\Omega_3^2$$

$$\phi = \frac{1}{l_s\sqrt{N}}\, y \qquad \leftarrow \text{linear dilaton}$$

$$S = \underbrace{\int d^6x\, dy}_{\text{Gauge fld propagates in 7 flat dimensions}} d\Omega_3 \left[e^{-2\phi}\left(R + (\partial\phi)^2\right) + K_{RR}^2 \right]$$

$$M_6^4 \sim m_A^5 \int^{\infty} dy\, e^{-\frac{2}{l_s\sqrt{N}}\, y}$$

finite for $l_s\sqrt{N} < \infty$

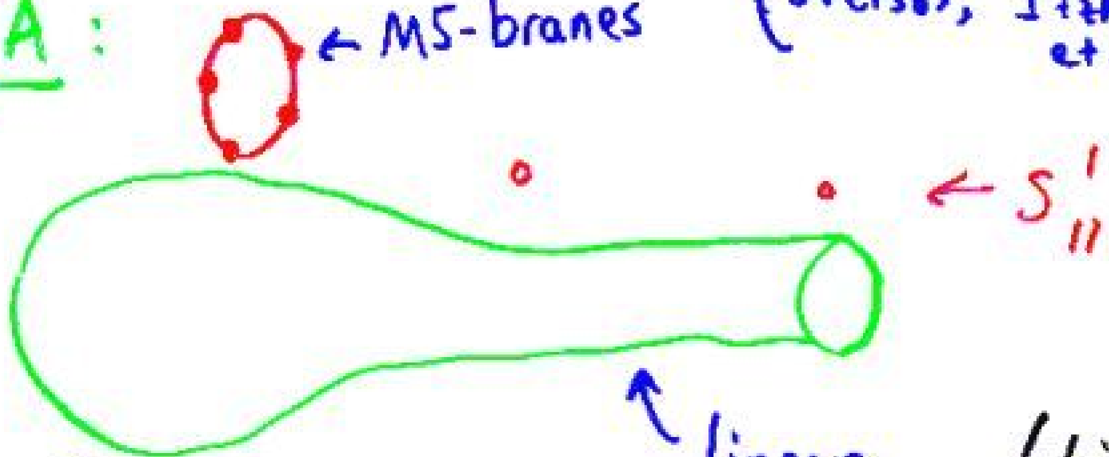
$$\frac{1}{e_6^2} = \infty$$

No screening

But the solutions are corrected:

(Sfetsos, Itzhaki et al)

IIA:



linear dilaton / Little string theory

(Aharony, Berkooz, Kutasov, Seiberg)

$AdS_7 \times S^4$

& relevant deformations

(Coulomb branch)

Our RR 1-form gauge field came from translations on S^1_{11}; these are broken in the microscopic solution $\Rightarrow$ $\vec{E} \sim e^{-m|x_{11}|}$

IIB

RR 2-form gauge potential B_{RR} is lifted by Stuckelberg coupling:

$$S = \int d^{10}x \, |dB|^2 + \int d^6x \, (B-F)^2$$

→ Other linear dilaton backgrounds (e.g. conifolds, $\Lambda_{tree} \neq 0$) ?

→ More general warped geometries not dual to world-volume theory → d-dimensional effective QFT ?

Scalars & Λ_d

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Now consider

bulk SUSY to leading approx.

$$S_5 = \int d^4x\, dy \sqrt{G}\left(R - \frac{4}{3}(\nabla\phi)^2\right.$$

(cf Cvetic de Wolfe et al)

$$\left. + \int d^4x \sqrt{-g}\,(-f(\phi))\right)$$

← Thin domain wall source

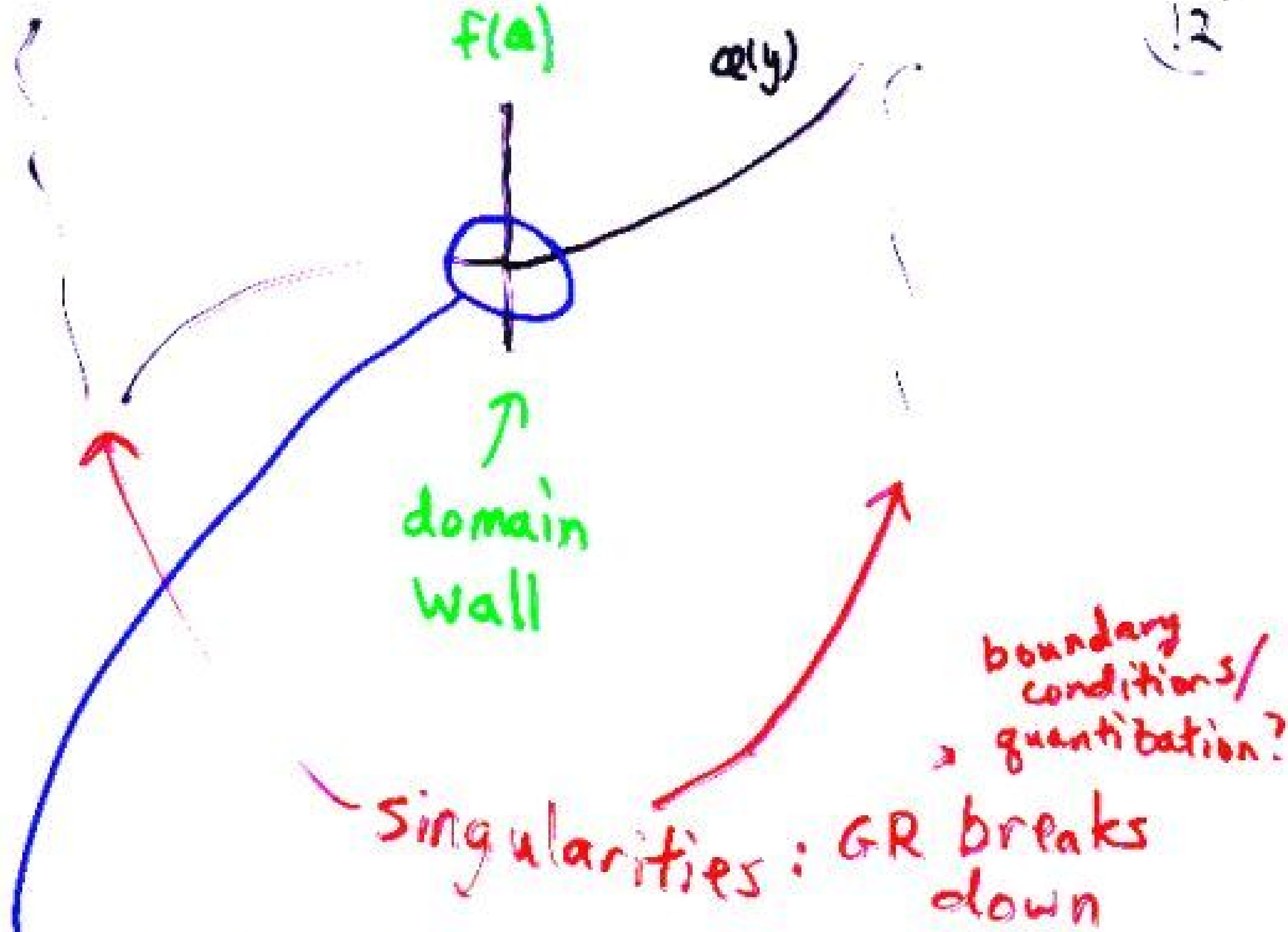
In contrast to RS case, here find 4d Poincaré-invariant solutions for <u>generic</u> brane tension $f(\phi)$:

cf [illegible]-Shaposhnikov et al

$$\phi(y) = \pm\frac{3}{4}\log\left|\frac{4}{3}y + c\right| + d$$

$$A(y) = \frac{1}{4}\log\left|\frac{4}{3}y + c\right| + \bar{d}$$

in bulk



The jump conditions (Einstein eqs) at the domain wall are solved by adjusting integration constants c, d; <u>not</u> by tuning parameters (like the brane tension $f(a)$) in the action.

In M-theory, cannot choose the parameters in the "Lagrangian"; we only have the freedom to choose solutions.

Horowitz, Low, Zee:

stringy cosmological solutions

Find solution **independent** of brane tension, with $\lambda_{eff} = 0$

singularities **null**: no issue of bdry conditions from interior

Generic expectation:

$$\Lambda_4 \sim \mathrm{TeV}^4 \left(\cancel{1} + \frac{\mathrm{TeV}^2}{M_5^2} + \cdots \right)$$

0

bulk quantum effects suppressed by G_N

corrections to bulk action; possible quantization of integration constants c, d

Questions:

1. Are these particular singularities resolved by the quantum theory?
2. Is there a standard 4d effective QFT?

The answers probably depend on the specific microscopic embedding.

(Girardello, Petrini, Porrati, Zaffaroni, Polchinski-Strassler, Gubser, Pilch, Warner, Aharony et al)

AdS/CFT analogies:

Couple domain wall to large-N gauge theory obtained by relevant deformation of $N=4$ SYM:

singularity
↕
mass gap

$f(Q_I)$

cf Bousso-Polchinski

[illegible]

1. Singularities resolved, to good enough precision ($e^{\sqrt{N}}$ vacua fit in 0–1 TeV range)
2. Standard effective QFT

Unlike the AdS cases, our original bulk solution is not holographically defined by a dual QFT

$$\leftarrow \; e^{2A} \sim (y-y_s)^k \qquad \overrightarrow{\text{flat space}}$$

y_s

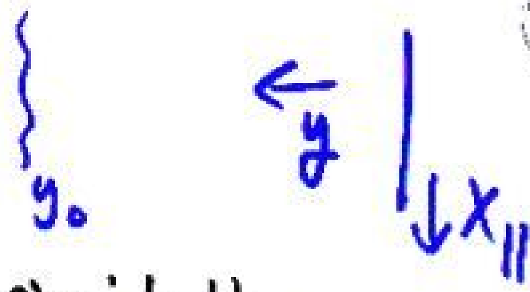
since the warp factor g_{00} decreases toward the singularity, it is possible that this IR region is dual to QFT (Maldacena–Nunez)

It is interesting to try to understand cases with no known QFT dual (as can happen in type I′ & compactified Horava-Witten theory).

→ Can the gravity description still be useful?

Yes, for example in characterizing States' size as a function of energy: see IR/UV results characteristic of gravity vs QFT

In general,



the energies of excitations are

$$M(y) \sim \sqrt{g_{00}(y)}\, M_0(Q(y))$$

redshift factor from warped metric

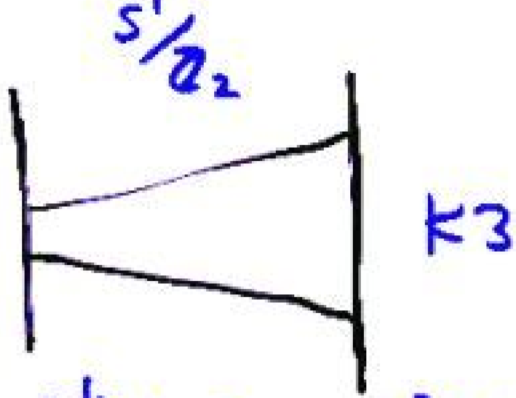
moduli-dependent bare mass

Some modes get heavier toward the core y_0

$$\left\{\begin{array}{l} \underline{\text{size}} \sim \delta x_\parallel \sim e^{-A(y)} \\ \underline{\text{mass}} \sim m(y) \end{array}\right.$$

$$\Rightarrow \quad \delta x_\parallel \sim M^{q}$$

e.g. Horava-Witten



$$ds^2 = w^{-\frac{1}{3}}\, dX_6^2 + w^{\frac{2}{3}}\left(dK^2 + dw^2\right)$$

SUGRA modes:

$$\delta X_6 \sim \frac{1}{\text{mass}} \sim w^{\frac{1}{2}}$$

wrapped M2:

$$M_{eff} \sim \underbrace{\sqrt{g_{00}}}_{w^{-\frac{1}{6}}}\ \underbrace{m(w)}_{w^{2/3}} \sim w^{\frac{1}{2}}$$

$$\underset{R}{\text{size}} \sim \frac{1}{\sqrt{g_{00}}} \sim w^{\frac{1}{6}}$$

$$M_{eff} \sim R^3$$

Many open questions

Non-AdS warped geometries:

- Dual description (gravitational or non-gravitational)?
- Physics of singularities
- Growth of states at high energies

Is there ever breakdown of effective QFT at low energies?

(cf Banks ...)