K.S. Stelle, Strings 2000

Randall – Sundrum Geometry from D3 branes

- How to realize the geometry as a supergravity brane solution
- Some of its characteristic properties

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A. Lukas, B. Ovrut, KSS + D. Waldram
hep-th/9803235, 9806051
Hořava – Witten as 3-branes in D=5

M.S. Bremer, M.J. Duff, H. Lü, C.N. Pope + KSS
hep-th/9807051
II B reduced on spheres, 3-branes in D=5

M.J. Duff, J.T. Liu + KSS
– current work

—

Some other highlights in the litterature:

L. Randall + R. Sundrum
I: hep-ph/9905221
II: hep-th/9906064
cause of all this fuss ...

R. Kallosh + A. Linde
hep-th/0001071
K. Behrndt + M. Cvetič
hep-th/0001159
} RG flow issues

M. Cvetič, H. Lü, C.N. Pope
hep-th/0001002
importance of S^5 breathing mode

P. Kraus
hep-th/9910149
S. de Alwis, A.T. Flournoy + N. Irges
hep-th/0004125
Z_2 structure in supergravity RS realizations
orbifold configurations

Punchline:

To realize RS as a BPS supergravity solution, copy the 3-brane $D=5$ realization of Hořava-Witten in M-theory, making the following substitutions:

M-theory in $D=11$ ⟶ IIB theory in $D=10$

Calabi-Yau 3-fold compactification ⟶ S^5 compactification

$G_{[4]}$ flux ⟶ $H_{[5]}$ flux

- pay attention to details, like Z_2 behavior
- then everything works in strict analogy

⇒ RS is a $N=4, D=4$ supersymmetric configuration of two 3-branes with opposite tensions + magnetic charges in IIB theory with $S^5 \times S^1$ compactification

- note: compactification, but not full Kaluza-Klein reduction
  - ∴ solution makes specific use of several higher dimensional features (breathing mode, Z_2 properties)

- Concerns about RS sharp edges not adressed here: δ-function sources for D3 branes are taken as acceptable here ⟶ "kinks" in solution

Review: D=5 3-branes in M-theory

Scheme:
dimensionally reduce M-theory on a CY 3-fold with $G_{[4]}$ fluxes turned on

- deformations of CY complex structure:

$$\omega_{AB} = a^i \omega_{iAB} \qquad i = 1, \dots, h^{(1,1)}$$

ω_{iAB}: basis of harmonic (1,1) forms

$a^i \longrightarrow a^i(x)$ scalar modulus fields

$$V = \frac{1}{6} d_{ijk} a^i a^j a^k$$

CY volume modulus

$$d_{ijk} = \int \omega_i \wedge \omega_j \wedge \omega_k$$

intersection numbers

- $G_{[4]}$ fluxes:

generalized Kaluza-Klein reduction turns on $G_{[4]}$ in CY directions

$$G_{ABCD} = \alpha_i v^i_{ABCD}$$

α_i: constants

$v^i \in H^{(2,2)}$ harmonic (2,2) forms

preliminary ansatz

yields a D=5 theory with a potential

$$I_5^M = \frac{-1}{2\kappa_5^2} \int d^5x \sqrt{-g} \Big[R + G_{ij}(b)\, \partial_\mu b^i \nabla^\mu b^j + \frac{1}{2} V^{-2} \partial_\mu V \nabla^\mu V + \frac{1}{2} V^{-2} G^{ij}(b)\, \alpha_i \alpha_j + \text{more} \dots \Big]$$

G_{ij}: moduli space metric

$$G_{ij} = \frac{1}{2V} \int \omega_i \wedge (* \omega_j)$$

$$b^i = V^{-\frac{1}{3}} a^i$$

shape-determining moduli

- potential rules out flat space as a solution, but with a subtle modification, the theory admits magnetically-charged BPS 3-brane solutions
  - change $G_{[4]}$ ansatz to

$$G_{ABCD} = \alpha_i v^i_{ABCD}\, \theta(y)$$

final ansatz

$$y = x^5 \qquad \theta = \pm 1 \quad y \gtrless 0$$

- change in the $G_{[4]}$ ansatz introduces <u>magnetic charge</u> $dG_{[4]} \sim \delta(y)$
  - change also <u>restores</u> a $\mathbb{Z}_2$ symmetry otherwise violated by the preliminary ansatz:

$\int A_{[3]} \wedge G_{[4]} \wedge G_{[4]}$ Chern-Simons term in $D=11$

$\Rightarrow A_{[3]}, G_{[4]}$ are pseudo quantities
  require extra (-1) factor under parity

→ so $G_{[4]} = \alpha_i Y^i$ violates $\mathbb{Z}_2$

but $G_{[4]} = \alpha_i Y^i \theta(y)$ restores $y \to -y$ $\mathbb{Z}_2$

- resulting "patched" $D=5$ theory has BPS 3-brane solutions:

$$ds_5^2 = a(y)^2 dx^\mu dx^\nu + b(y)^2 dy^2 \qquad \mu,\nu = 0,1,2,3$$

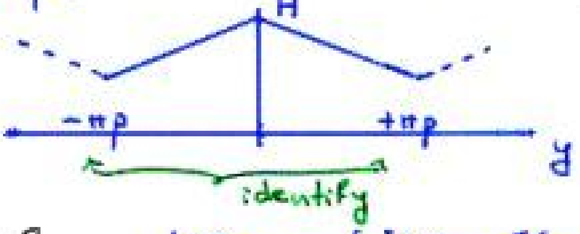
$$a(y) = \tilde{k} V^{1/6}(y) \qquad b(y) = k V^{2/3}(y) \qquad k, \tilde{k} : \text{constants}$$

$$V(y) = \left(\tfrac{1}{6} d_{ijk} f^i f^j f^k\right)^2 \qquad b^i(y) = V^{-\frac{1}{6}} f^i$$

$$d_{ijk} f^j f^k = H_i(y) = 2\sqrt{2}\, k\, \alpha_i |y| + k_i$$

codimension one harmonic function – <u>linear</u>

- complete the Hořava-Witten-like scenario by declaring the y dimension to be a <u>circle</u> S^1: <u>identify</u> $y = \pm \pi\rho$; this introduces a <u>second</u> 3-brane



- magnetic charges from $dG_{[4]} \sim (\delta(y) - \delta(y - \pi\rho))$ are proportional to $\Delta\alpha^i$ at each brane
  - y dimension is compact $S^1 \Rightarrow \sum_{\text{branes}} \Delta\alpha^i = 0$

i.e. magnetic charges are <u>equal + opposite</u> — sources, sinks of flux

Residual supersymmetry

Substituting the background bosonic fields, one has fermionic variations

$$\delta\psi_\alpha{}^i = D_\alpha\epsilon^i - \frac{\sqrt{2}}{12} V^{-1} b^i\alpha_i \gamma_\alpha \theta(y)(\tau_3)^i{}_j \epsilon^j \quad \text{gravitino}$$
$$\delta\lambda^i = \frac{-i}{2} b_i \left(\gamma^\alpha \partial_\alpha b^i \epsilon^i + \frac{1}{\sqrt{2}} V^{-1}\alpha^i \theta(y)(\tau_3)^i{}_k \epsilon^k\right) \quad \text{gaugino}$$
$$\delta\zeta^i = \frac{i}{2} V^{-1}\partial_\alpha V \gamma^\alpha (\tau_3)^i{}_j \epsilon^j - \frac{i}{\sqrt{2}} b^i\alpha_i V^{-1}\theta(y)\epsilon^i \quad \text{hyperino}$$

which vanish for
$\delta\psi_\alpha{}^i = \delta\lambda^i = \delta\zeta^i = 0$
for all y

$$\epsilon^i(x^\mu, y) = a(y)^{1/2}\epsilon_0^i$$
$$\gamma_5 \epsilon_0^i = (\tau_3)^i{}_j \epsilon_0^j$$

- note the crucial rôle of the $\theta(y)$ in making this work: needed to balance signs coming from $\partial_5 a(|y|)$
- result is a continuous Killing spinor $a(y)^{1/2}\epsilon_0^i$ for the surviving supersymmetry (here: $N=1, D=4$ susy)

Brane sources

- "kinks" in the double 3-brane solution require sources on RHS of field equations
  - these sources arise from

$$I_{\text{branes}} = \frac{\sqrt{2}}{\kappa_5^2}\int_{M_4^{(1)}} \sqrt{-g}\, V^{-1}\alpha_i b^i - \frac{\sqrt{2}}{\kappa_5^2}\int_{M_4^{(2)}} V^{-1}\alpha_i b^i$$

Source terms for opposite kinks necessarily have opposite signs

- apparent negative potential is canceled by ∂_5 gradients of the background: 3-branes are flat

3-branes in Type IIB theory

BDLPS

Consider $D=10$ IIB field equations, keeping just the metric and the $H_{[5]}$ field strength:

$$R_{\mu\nu} = \frac{1}{96}\left(H_{[5]}\right)^2_{\mu\nu}$$

$$H_{[5]} = * H_{[5]} \qquad\qquad dH_{[5]} = 0$$

self-duality $\qquad\qquad$ Bianchi identity

– all other bosonic fields (dilaton ϕ, axion χ, $F^{1,2}_{[3]}$) will not play a rôle and have been set to zero

- Kaluza-Klein ansatz for S^5 reduction:

$$ds^2_{10} = e^{2\alpha\varphi} ds^2_5 + e^{2\beta\varphi} ds^2(S^5) \qquad \alpha = \frac{1}{4}\sqrt{\frac{5}{3}}$$

φ: S^5 "breathing mode" $\qquad \beta = -\frac{3\alpha}{5}$

$$H_{[5]} = 4m e^{8\alpha\varphi}\epsilon_{[5]} + 4m\,\epsilon_{[5]}(S^5)$$

(preliminary ansatz)

– this reduction yields

$$\mathcal{L}_5 = eR - \frac{1}{2}e(\partial\varphi)^2 - 8m^2 e\, e^{8\alpha\varphi} + R_5\, e\, e^{\frac{16\alpha\varphi}{5}}$$

from $H^2_{[5]}$ $\qquad$ from $D=10$ Ricci

- because there are now two potential terms of opposite signs, one does have a maximally symmetric solution, with constant $\varphi = \varphi_*$:

$$e^{\frac{24\alpha}{5}\varphi_*} = \frac{R_5}{20m^2} \qquad R_{\mu\nu} = -4m^2 e^{8\alpha\varphi_*} g_{\mu\nu}$$

– this is the $AdS_5 \times S^5$ "vacuum" of the $D=5$ theory

Z_2 structure

Following the M-theory pattern as closely as possible, change the $H_{[5]}$ ansatz to

$$H_{[5]} = 4m\,\theta(y)\,e^{8\alpha\varphi}\epsilon_{[5]} + 4m\,\theta(y)\,\epsilon_{[5]}(S^5)$$

final ansatz

- this restores a certain $y \to -y$ Z_2 that otherwise was broken by the preliminary ansatz
  - but which Z_2? Must be a symmetry of IIB
    - ∴ must combine $y \to -y$ with an orientation-reversing transformation in S^5: net transformation must respect $H_{[5]} = {}^*H_{[5]}$
  - bottom line: must flip $m \to -m$, $y \to -y$

3-brane

BDLPS

Starting from a 3-brane ansatz, find

$$ds_5^2 = e^{2A}dx^\mu dx^\nu \eta_{\mu\nu} + e^{2B}dy^2 \qquad B = -4A$$

$$e^{-\frac{7}{\sqrt{15}}\varphi} = H = e^{-\frac{7}{\sqrt{15}}\varphi_0} + k|y|$$

$$e^{4A} = \tilde{b}_1 H^{2/7} + \tilde{b}_2 H^{5/7} \qquad \tilde{b}_1 = \pm\frac{28m}{3k}$$

$$\tilde{b}_2 = \pm\frac{14}{15k}\sqrt{5R_5}$$

(also considered by Cvetič, Lü, Pope; de Alwis, Flournoy + Irges)

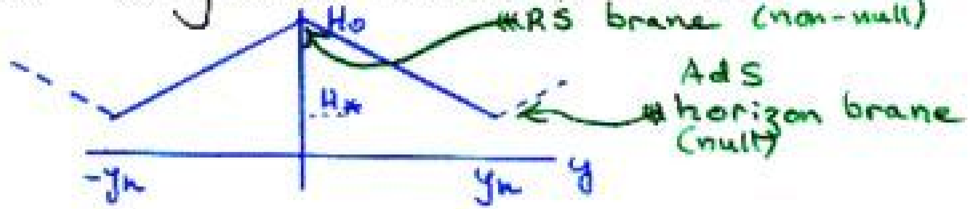
- we shall pick $\tilde{b}_2 > 0$, $\tilde{b}_1 < 0$ to ensure reality of metric and permit a $k \to 0$ limit to be taken

Finding Randall - Sundrum

DLS

- If one picks the harmonic function slope k to be negative, $k<0$, then the "kink-down" $\wedge^{H}$ brane at $y=0$ is of <u>positive tension</u>, the basic requirement for "binding gravity" in RS geometry

- Choose the integration constant φ_0 such that $H(0) = e^{-\frac{7}{\sqrt{15}}\varphi_0} > H_* = e^{-\frac{7}{\sqrt{15}}\varphi_*}$.
Then for $k<0$, $H(y)$ decreases as $|y|$ increases, reaching $H_* = e^{-\frac{7}{\sqrt{15}}\varphi_*}$ at $y = \pm y_h$.
Now <u>identify</u> $y_h \leftrightarrow -y_h$, thus including a second (negative tension) brane:



- this solution is a "semi-interpolating soliton": it evolves to a stationary point φ_* at $y_h \leftrightarrow -y_h$ (from above: breathing mode crucial here Cvetic, Lü, Pope) from a non-null RS brane generated by the delta-function source at $y=0$

- <u>RS limit</u>: let $H_0 = e^{-\frac{7}{\sqrt{15}}\varphi_0} = H_* + \beta|k|$ $\quad y_h = \beta, \ \beta > 0$
take $k \to 0_-$ limit (joint $\varphi_0 \to \varphi_*$, $k \to 0$ limit)
obtain $ds^2 = \frac{2}{\sqrt{L}}(\beta - |y|)^{1/2} dx^\mu dx^\nu \eta_{\mu\nu} + \frac{L^2}{16}\frac{dy^2}{(\beta-|y|)^2}$; $L = m^{-1}\left(\frac{10m^2}{R_5}\right)^{\frac{1}{2}}$
— this is AdS: recoordinatize $\beta - |y| = \beta e^{-4|\tilde{y}|/L}$
to obtain Poincaré coord. form $ds^2 = e^{-2|\tilde{y}|/L} dx^\mu dx^\nu \eta_{\mu\nu} + d\tilde{y}^2$

D=10 view of all this

- D=5 double 3-brane solution can be oxidized back to D=10. Spherical oxidation: codimension 1 transverse coord. becomes a <u>radius</u>.
  - make two coord. transformations

$$\rho = \sqrt{\frac{20}{R_5}}\, H^{3/28} \quad \text{then} \quad r^4 = \rho^4 - \tilde{k} \qquad x^{\mu'} = \tilde{b}_2^{-1/4} x^\mu$$
$$\tilde{k} = m^4 \left(\frac{20 m^2}{R_5}\right)^{5/2}$$

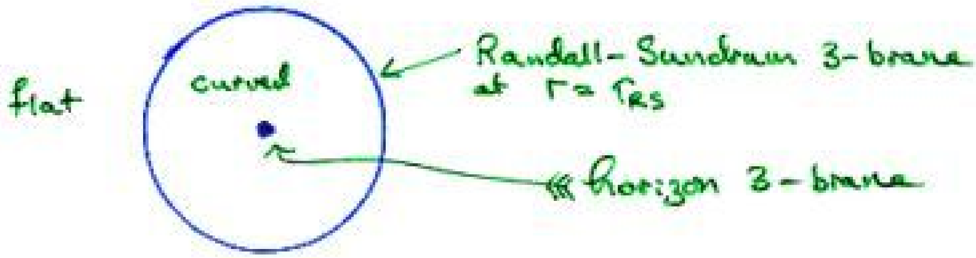
to obtain $ds_{10}^2 = \left(1+\frac{\tilde{k}}{r^4}\right)^{-\frac{1}{2}} dx^\mu dx^\nu \eta_{\mu\nu} + \left(1+\frac{\tilde{k}}{r^4}\right)^{1/2}\left(dr^2 + r^2 d\Omega^2\right)$

$\Rightarrow$ Just the usual D3 brane of IIB theory

- Z_2 symmetric solution in D=5 oxidizes to two Z_2 reflected ($\pm m$) copies of the D3 brane
  - patchings are at the D3 brane horizon $r=0$ and at $r_{RS} = \sqrt{\frac{20}{R_5}}\left[e^{-\sqrt{\frac{3}{8}}\varphi_0} - e^{-\sqrt{\frac{3}{8}}\varphi_*}\right]^{1/4}$

$\to 0$ as $k \to 0$

D=10 (half) picture



- In the RS limit, $r_{RS} \to 0$:

To orbifold or not to orbifold?

If one expands the theory about the above background, excluding Z_2 odd modes, the result is equivalent to compactifying on an S^1/Z_2 orbifold.

- What happens if one just expands without making this truncation?
  - Z_2 odd modes are under suspicion of negative energies, owing to negative tension brane
- However, the theory is more robust than this: continuity conditions for φ and metric at the two 3-branes imply that Z_2 odd modes distort $|m|$, $|k|$ and R_5: unequal in $y \gtrless 0$ zones
  - but m and R_5 are curvature values, subject to Bianchi identities.
    - so the attempt to let m, R_5 fluctuate becomes entangled with Kaluza-Klein massive modes

e.g. let $m \to m(x^\mu)$ in

KK modes

$$H_{[5]} = 4m\, e^{8\alpha\varphi} d^5x + 4m\, \epsilon_{[5]}(S^5) + h_{[5]}$$

then $dH_{[5]} = 0$ implies

$$4\partial_\mu m\, dx^\mu \wedge \epsilon_{[5]}(S^5) + dh_{[5]} = 0$$

so if $\partial_\mu m \neq 0$, then KK massive modes are excited

- Hence the projection into Z_2 even modes happens dynamically, by ordinary Kaluza-Klein processes.