STRINGS '00

"OPEN/CLOSED STRING DUALITY"

OR

"HOW GRAVITY COUPLES TO MATTER"

BASED ON WORK w/

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C. S. CHAN . P. PAUL

+ WORK IN PROGRESS:

w/ ARKANI-HAMED, DIMOPOULOS, KALOPER

+ DISCUSSIONS w/

BERKOOZ, LARSEN, KACHRU, E. SILVERSTEIN

INGREDIENTS

① STRING THEORY:

TYPE I

MATTER

OPEN

GRAVITY

CLOSED

② GAUGE HIERARCHY: M_W : M_{PL}

1 : 10^{16}

③ HOLOGRAPHY: RG - SCALE = EXTRA DIMENSION

① + ②

$\Downarrow$

"BRANE WORLD"

BACK REACTION → $\Downarrow$

WARPED COMPACTIFICATION

AdS / CFT → $\Downarrow$

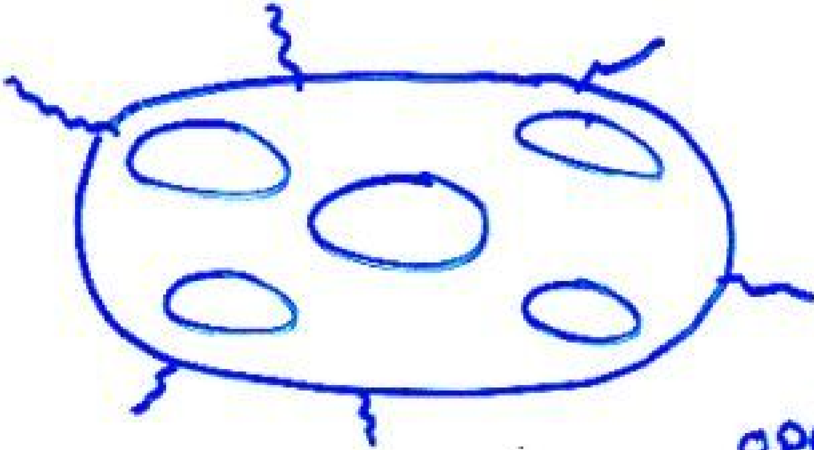
③

"GEOMETRIZATION OF THE RENORMALIZATION GROUP"

ADD

RS

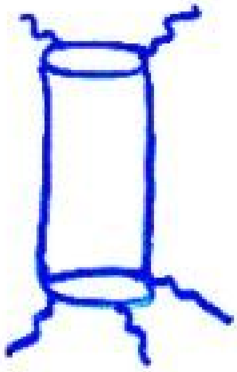
H.V.



OPEN STRING
AMPLITUDES

OPEN STRING LOOP

$\Leftrightarrow$

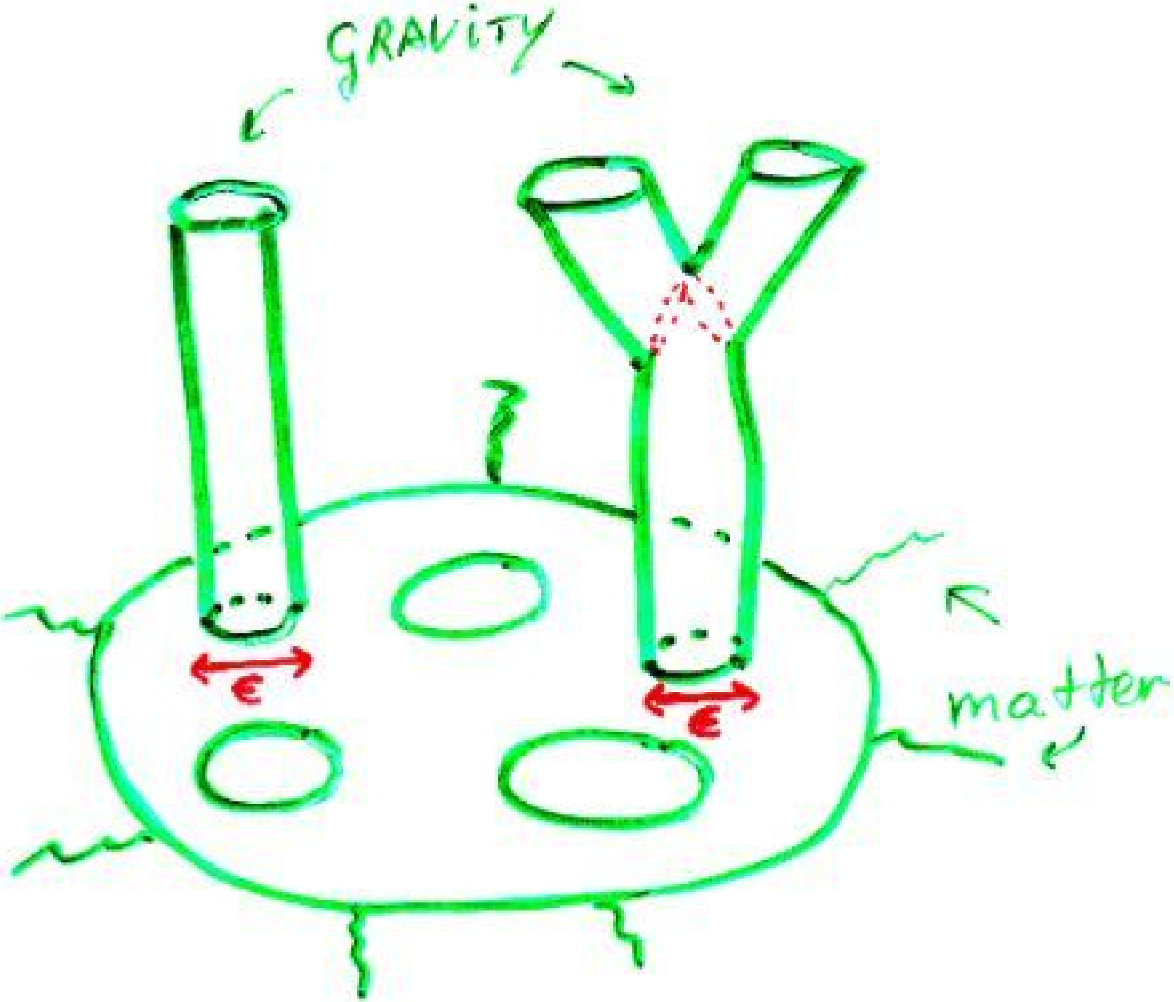


CLOSED STRING
PROPAGATOR

UV $\longleftrightarrow$ IR

IN TYPE I, OPEN + CLOSED
STRINGS ARE IN PEACEFUL ...

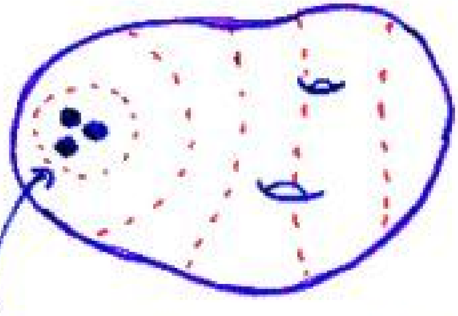
....COEXISTENCE!



OPEN + CLOSED STRING DIAGRAM

RG RELATION
$\epsilon \to \epsilon + \delta\epsilon$

OPEN STRING DIVERGENCES $\Longleftrightarrow$ SOURCE TERMS FOR CLOSED STRING BACKGD



$K_6 \times \mathbb{R}^4$

$\#$ D3 - BRANES

$\parallel$

$\chi(K_6)$

SOME TOPOLOGICAL EULER NUMBER

F- THEORY

WORLD BRANES D3

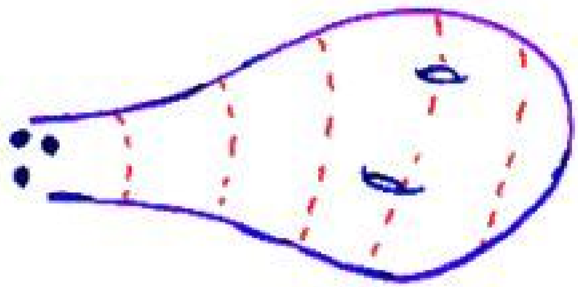
3+1 DIM^{L}

$\Downarrow$

BACK REACTION

$\Downarrow$

WORLD BRANES $\Rightarrow$



$K_6 \times \mathbb{R}^4$

$\uparrow$

WARPED PRODUCT

$\longrightarrow r$

$$ds^2 = \underbrace{a^2(r)\, dx_\mu^2 + dr^2} + \underbrace{h_{mn}(r,y)\, dy^m dy^n}$$

DEFORMATION OF COMPACT SLICE OF AdS_5 $\times$ K_5

$$\Delta_\perp \, a^2(x_\perp) = \sum_i^N \delta(x_\perp - y_i) - \underbrace{R^3\text{-term}}_{\text{EULER DENSITY}}$$

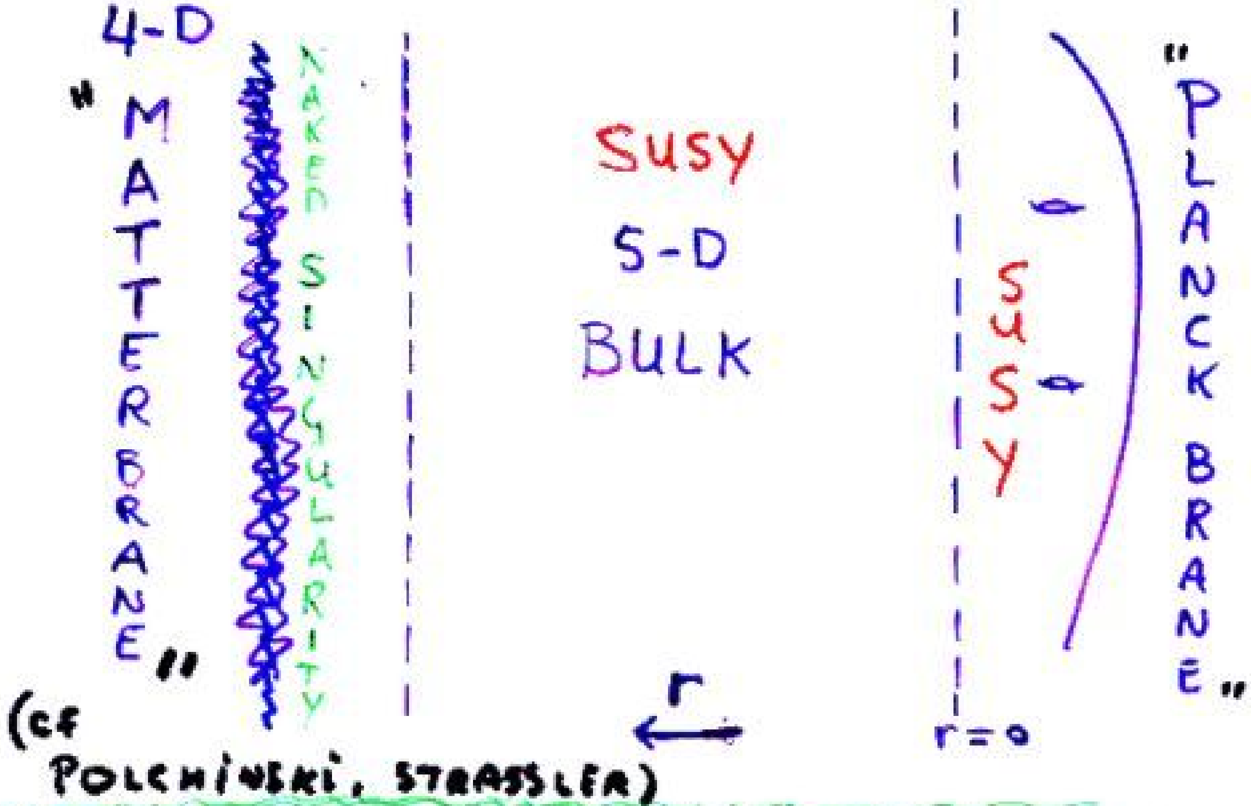
"PLANCK BRANE"

= REGION OF K_6 OUTSIDE

OF AdS - THROAT

AFTER DIMENSIONAL REDUCTION OVER K_5

$$\boxed{10\text{-D} \rightarrow 5\text{-D}}$$



$$S_{TOTAL} = S^{4\text{-D}}_{MATTER} + S^{5\text{-D}}_{BULK} + S^{4\text{-D}}_{PLANCK}$$

$$S_{BULK} = \int \sqrt{-g} \left[R_5 + (\nabla\phi)^2 + V(\phi) \right]$$

+ FERMIONS

ϕ = MODULI OF K_5, ETC.

MATTER BRANE & HOLOGRAPHY

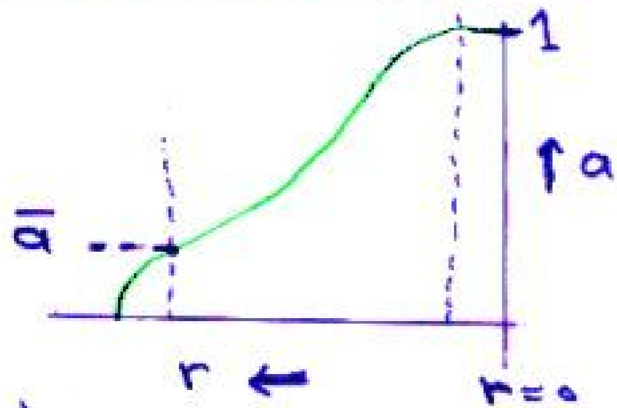
$$e^{\frac{i}{\hbar} S_{\text{MATTER}}(\bar{a}, \phi)} = \int_{\text{Energy} \leq \bar{a}} d[\text{MATTER}]\, e^{\frac{i}{\hbar} S(\text{MATTER}, \phi)}$$

ϕ = COUPLING CONSTANTS

$\bar{a}$ = UV CUT-OFF ENERGY SCALE

$$ds_5^2 = a^2(r)\, ds_4^2 + dr^2$$

$$\phi = \phi(r)$$



$$a \frac{\partial}{\partial a} \phi(a) = \beta_\phi(\phi, a)$$

$\Rightarrow$ RADIAL EVOLUTION OF a & ϕ REPRESENTS RG-FLOW OF THE QFT COUPLINGS. $\blacksquare$

CLAIM:

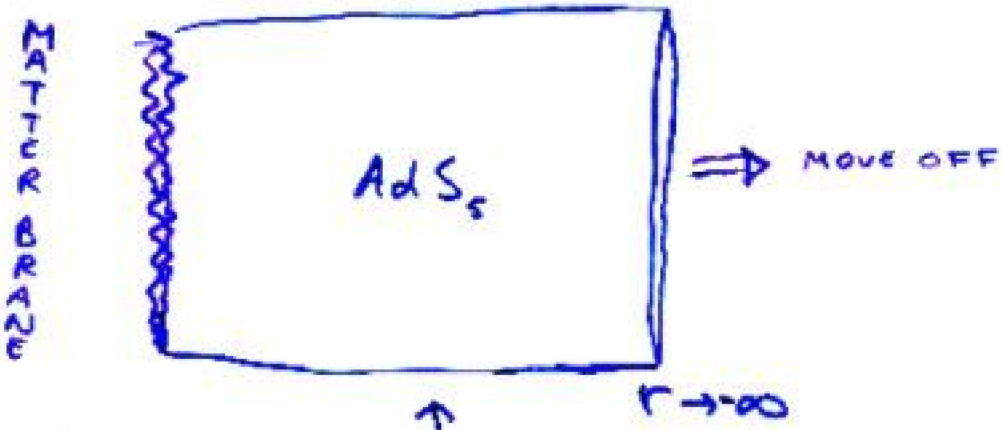
4-D COSMOLOGICAL CONSTANT Λ_4 IS **NOT** **FIXED** BY THE MATTER BRANE EQN. OF MOTION. INSTEAD, IT'S AN UNDETERMINED INTEGRATION CNST.

REASON / MECHANISM

1. AdS / CFT
2. ADJUSTMENT OF WARP FACTOR
3. OPEN STRING PERTURBATION THEORY
4. RG-SCALE DEPENDENCE OF $S_{MATTER}(\phi, a)$

ALL 1 - 4 ARE IN FACT THE SAME.

(1) SINCE 5-D EQNS ARE LOCAL, WE CAN SEND THE "PLANCK BRANE" OFF TO ∞, AND JUST LOOK AT THE MATTER BRANE SEPARATELY, EMBEDDED INSIDE A NON-COMPACT AdS-LIKE 5-D SPACE.



AdS/CFT: $\updownarrow$ EQUIVALENCE

4-D QFT WITHOUT 4-D GRAVITY !

$M_{PL} \to \infty$

CAN LIVE ON ANY 4-D GEOMETRY

$\Rightarrow R^{(4)}$ IS DETERMINED BY BOUNDARY COND.S

$r \to -\infty$

② 5-D EINSTEIN EQUATION:

$$\boxed{(\partial_r g_{\mu\nu})^2 - (\partial_r \phi)^2 + V(\phi) + R_4 = 0}$$

$$\downarrow \quad ds_5^2 = a^2(r)\, ds_4^2 + dr^2$$

$$\boxed{\left(\frac{\partial_r a}{a}\right)^2 - (\partial_r \phi)^2 + V(\phi) + \frac{k}{a^2} = 0}$$

cf. FRW

$k = \Lambda_4$, THE 4-D COSMOLOGICAL CONSTANT

$\Rightarrow$ $\boxed{\Lambda_4 = \text{INTEGRATION CONSTANT OF THE } r \text{ EVOLUTION.}}$

$\Rightarrow$ Λ_4 IS DETERMINED BY INITIAL CONDITIONS IN THE PLANCK REGION!

$\Rightarrow$ CLOSED + OPEN STRING E.O.M.

$$L_0 \, | V_{\text{closed}} \rangle = \sum_{\substack{\text{closed} \\ \text{string} \\ \text{states}}} | \phi_i \rangle \langle \phi_i \rangle_{\text{annulus}}$$

$$\Downarrow$$

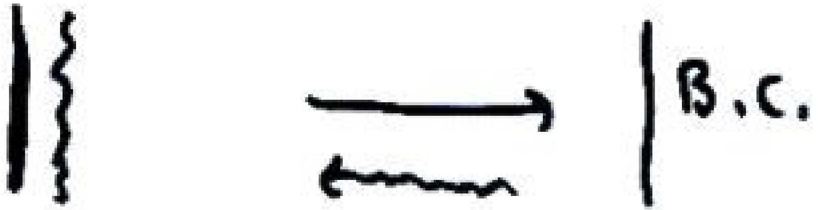
(CALLAN e.a.)

$$a \, \partial_r g_{\mu\nu} + b R_{\mu\nu} = T_{\mu\nu}$$

$$- \lambda^{\text{closed}} g_{\mu\nu} + K_4 R_{\mu\nu} = \lambda^{\text{open}} g_{\mu\nu}$$

$\Rightarrow$ K_4 & $\lambda^{\text{open + closed}}$

DETERMINED BY HIGH ENERGY BOUNDARY CONDITIONS.



SCHRÖDINGER = RG FLOW EQN:

$$H \, e^{\frac{i}{\hbar} S_{\text{MATTER}}[\phi, \bar{a}]} = 0 \qquad (*)$$

WITH

$$H = \left(\bar{a}\frac{\partial}{\partial \bar{a}}\right)^2 - \left(\frac{\partial}{\partial \phi}\right)^2 + a^4 V(\phi) + a^2 R^{(4)}$$

THIS EQN FOLLOWS FROM THE INVARIANCE UNDER SHIFTING THE DIVIDING LINE BETWEEN THE BULK & THE MATTER BRANE.

$(*) \Rightarrow$ SUPPOSE THAT:

$$S_{\text{MATTER}} = \int \sqrt{-g}\left[U(\phi, \bar{a}) + \Phi(\phi, \bar{a}) R^{(4)} + \ldots\right]$$

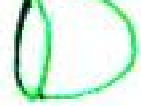
THEN:

$$(**) \qquad (\partial_\phi U)^2 - \left(U + \frac{\bar{a}}{4}\frac{\partial}{\partial \bar{a}} U\right)^2 = V(\phi)$$

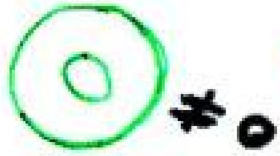
$$\hbar \to 0$$

$$\left[\bar{a}\frac{\partial}{\partial \bar{a}} - \beta_\phi \partial_\phi\right]\Phi = V_{5}/\tilde{U}$$

(3) CONSIDER OPEN STRING PERT. SERIES:

Λ_4^{OPEN} =  DISK $\frac{1}{g_s}\lambda^0$

\+ +

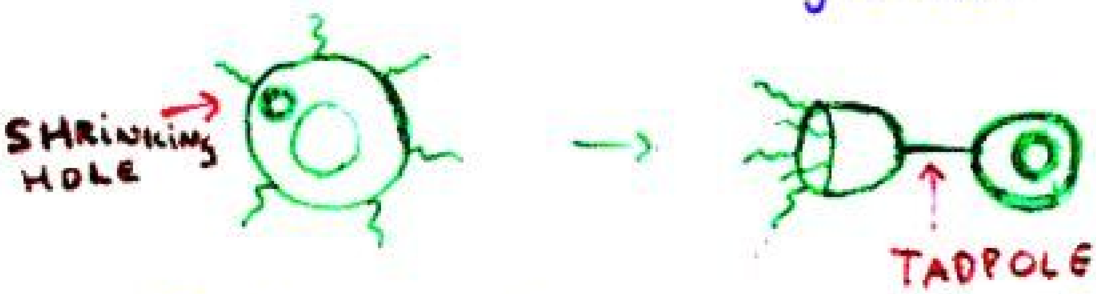
 ANNULUS $(g_s)^0 \lambda^1$

HOW IS IT CANCELED?

\+ +

ETC ...

$\Rightarrow$ ~~ONE~~ TWO LOOP DIVERGENCE:

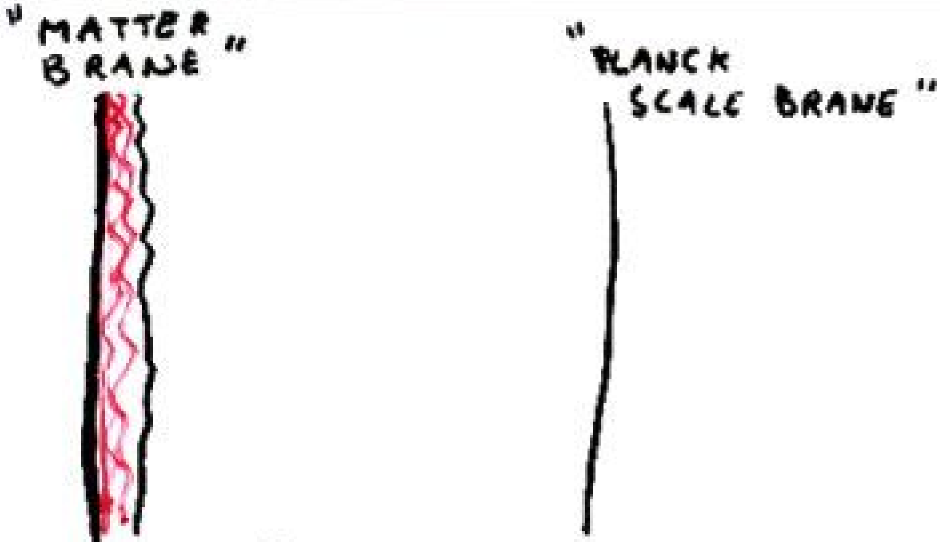


$\Rightarrow$ MUST ADD COUNTERTERM:

 ← VERTEX OPERATOR THAT MODIFIES CLOSED STRING BG

RENORMALIZES CLOSED STRING BACKGROUND SUCH THAT:

Λ_4^{CLOSED} =  $= -\Lambda^{OPEN}$



$ds_4^2 = \bar{a}^2 dx_\mu^2$ $\qquad\qquad$ $a = 1$

$$\frac{1}{2}\int d^4x \left[\bar{a}^2(\partial h)^2 - m_0^2 h^2 \bar{a}^4\right]$$

h = scalar field on matter brane.

$$\Big\downarrow \text{ 1-loop determinant}$$

$$\frac{\mu^\epsilon}{2}\int \frac{d^{3-\epsilon}k}{(2\pi)^{3-\epsilon}}\sqrt{k^2 + m_0^2\bar{a}^2}$$

$$\downarrow$$

$$\frac{\bar{a}^4 m_0^4}{32\pi^2}\left[\frac{1}{\epsilon} - \frac{1}{2}\log\left(\frac{m_0^2\bar{a}^2}{4\pi\mu^2}\right) - \frac{\gamma}{2} + \frac{3}{2}\right]$$

Goldberger, Rothstein

AFTER A FEW STANDARD STEPS ONE FINDS THAT BOTH PLANCK + MATTER BRANE ARE STABILIZED, PROVIDED ONE SATISFIES THE MATCHING RELATIONS:

$$\frac{\dot{a}}{a} = \boxed{\left(1+\frac{\bar{a}}{4}\frac{\partial}{\partial \bar{a}}\right) U(\bar{\phi},\bar{a}) = -W(\bar{\phi})}$$

$$\dot{\phi} = \boxed{\partial_\phi U(\bar{\phi},\bar{a}) = -\partial_\phi W(\bar{\phi})}$$

FOR GENERAL $U(\bar{\phi},\bar{a})$ (SOLVING EQN (**)) THERE EXIST SOLUTIONS TO THESE CONDITIONS. IN FACT, THERE'S A WHOLE "RG-TRAJECTORY" OF SOLUTIONS, GENERATED BY

$$\bar{a}\frac{\partial}{\partial \bar{a}} \bar{\phi}(\bar{a}) = \beta_\phi(\bar{\phi})$$

WITH $\beta_\phi = 6\,\frac{\partial_\phi W}{W}$

SUPPOSE WE CHOOSE: (cf. TALKS BY R. KALLOSH) K. STELLE

$$S_{PLANCK} = \int_{4-D} \sqrt{-g}\, W(\phi)$$

$\Rightarrow$ BOUNDARY CONDITION AT $r=0$ BECOMES

$$\partial_r g_{\mu\nu} = W(\phi)\, g_{\mu\nu}$$
$$\partial_r \phi = \partial_\phi W(\phi)$$

SINCE PLANCK REGION $r \simeq 0$ IS SUPER-SYMMETRIC, IT'S NOT UNREASONABLE TO ASSUME A FINE-TUNED RELATION BETWEEN $V(\phi)$ AND $W(\phi)$ SUCH THAT:

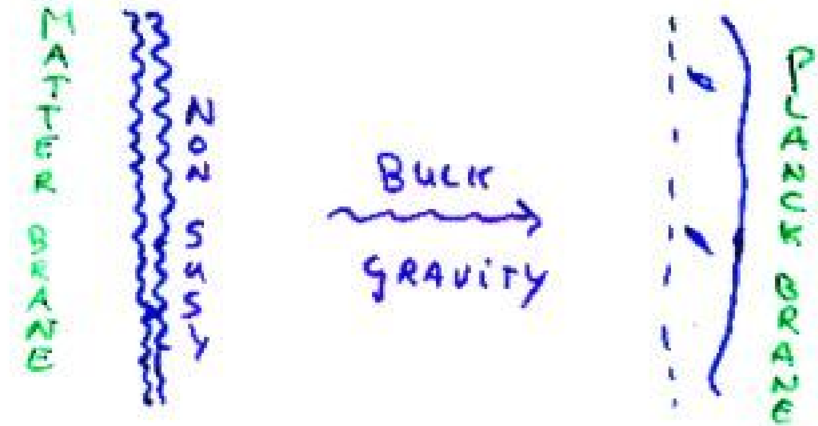
$$\Lambda_4 = V(\phi) + (W(\phi))^2 - (\partial_\phi W)^2 \equiv 0$$

$\Rightarrow$ $W(\phi)$ = SUPERPOTENTIAL OF 5-D GAUGED SUGRA

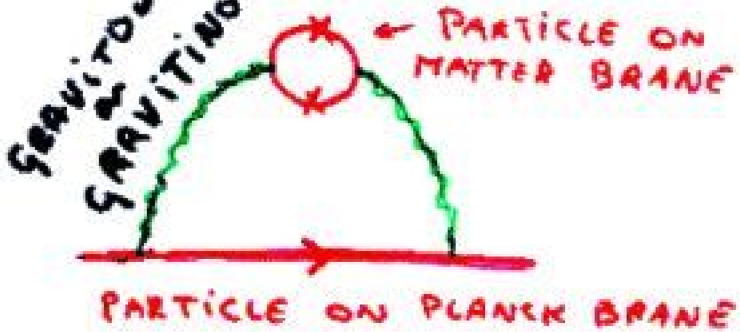
$\Rightarrow$ "PLANCK BRANE" IS FLAT !

FINALLY: HOW LARGE/SMALL IS Λ_4?

NATURAL SCALE IS SET BY THE AMOUNT OF SUSY VIOLATION IN PLANCK REGION.



ONE LOOP CALCULATION GIVES:



$$m_{3/2} \simeq \frac{m_{susy}^2}{m_{pl}}$$

$$\Lambda_4 \simeq \frac{(m_{3/2})^4}{m_{pl}^2} \simeq \frac{(m_{susy})^8}{(m_{pl})^6}$$

(Cf. T. BANKS, C. SCHMIDHUBER.)

CONCLUSIONS

① WARPED GEOMETRY NATURALLY ARISES IN STRING THEORY. LARGE HIERARCHY STILL REQUIRES FINE-TUNING, HOWEVER.

$\Updownarrow$

SUSY

② HOLOGRAPHIC INTERPRETATION OF EXTRA DIMENSION AS RG SCALE $\Rightarrow$ KK PARADIGM MODIFIED.

③ Λ_4 IS DETERMINED LOCALLY IN THE PLANCK REGION. CAN BE PROTECTED BY SUPERSYMMETRY $\Rightarrow$ $\Lambda_4 \simeq (m_{susy})^8$?

QUESTIONS

① EXPLICIT STRING REALIZATIONS.

(MAYR)
H.V.

② STABILITY, FLUCTUATIONS

③ GRAVITY LOOPS, $1/N$?

④ PHASE TRANSITIONS ?

⑤ 4-D EFFECTIVE FIELD THY ?