Space-Time Uncertainties and Noncommutativity in String Theory

Tamiaki Yoneya
University of Tokyo – Komaba

A brief review on the idea of a "space-time uncertainty principle" in string theory

1. *Motivations*

2. *Space-time uncertainty relation as a characterization of string theory*

3. *Space-time uncertainties and noncommutativity*

hep-th/0004074
and references therein

1 Motivation

What is string theory?

$$\ell_s \sim \sqrt{\alpha'}$$

as a natural cut-off parameter

$$\Downarrow$$

nonlocality or space-time fuzziness

- keeps (all) axioms for analytic S-matrix,

  (causality and unitarity)

  at least perturbatively.

Most of the past attempts with nonlocality (and/or space-time noncommutativity) toward quantum gravity were not successful.

- internal inconsistency and/or impossibility of including interactions consistently

How to characterize the nonlocality in string theory?

Importance of the question :

- uncover the underlying principles of string theory
- recall an analogous situation in the early quantum theory

world-sheet conformal invariance
$\updownarrow$
adiabatic invariance of the Bohr-Sommerfeld quantization condition

$\Downarrow$

What is the universal principle behind the world-sheet conformal invariance requirement?

(must also be related to the characterization of l_s)

2 Space-time uncertainty relation as a characterization of string theory

What is the origin for the elimination of UV divergence in string theory ?

Modular Invariance

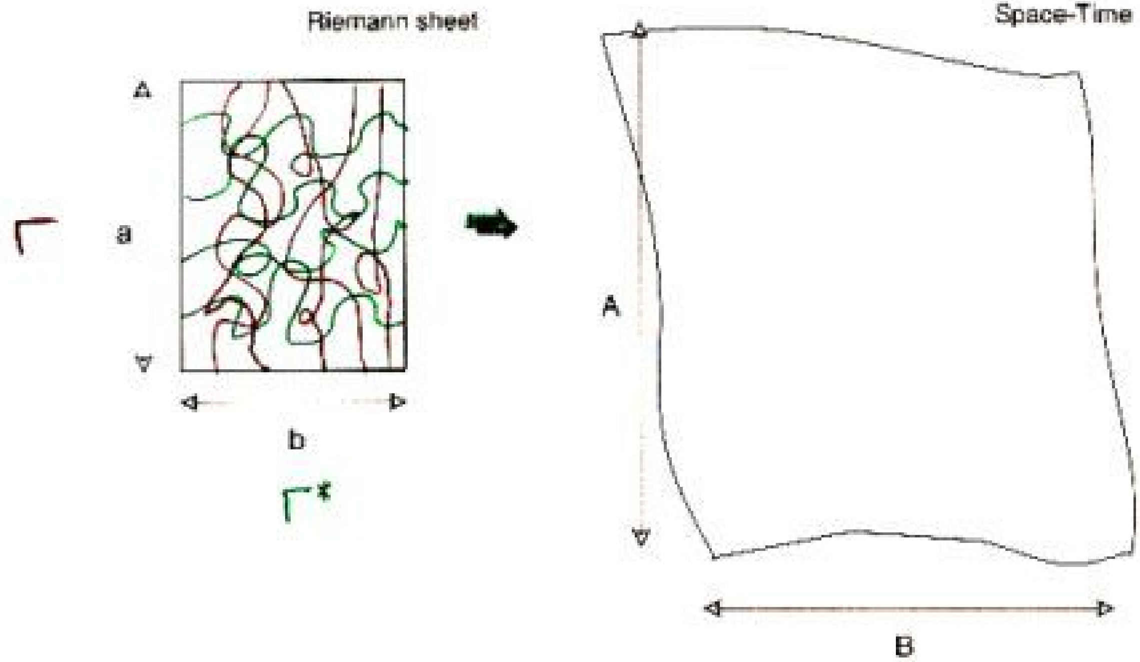
$\Downarrow$

What is the most fundamental expression for the modular invariance?

> reciprocity of
>
> "extremal lengths"

extremal length $\Lambda(\gamma)$ = a conformally invariant notion of length for a family $\{\gamma\}$ of curves on general Riemann surfaces

Example : Consider the amplitude for the mapping



$$\Lambda(\gamma \in direction\ a) = \frac{a}{b} \equiv \Gamma$$

$$\Lambda(\gamma \in direction\ b) = \frac{b}{a} \equiv \Gamma^*$$

The reciprocity relation

$$\Gamma\Gamma^* = 1$$

is a general relation that is valid for any quadrilateral region on the Riemann surface.

Choose the boundary condition of Dirichlet type :

$$X^\mu(\tau,0) = X^\mu(\tau,b) = \delta^{\mu 0}\frac{A\tau}{a},$$

$$X^\mu(0,\sigma) = X^\mu(a,\sigma) = \delta^{\mu 1}\frac{B\sigma}{b}.$$

Then, due to conformal invariance, the amplitude depends on the parameters a, b only through the extremal lengths

$$\exp[-\frac{1}{\ell_s^2}(\frac{A^2}{\Gamma}+\frac{B^2}{\Gamma^*})]$$

where

$$\Gamma \equiv \frac{a}{b},\ \Gamma^* \equiv \frac{b}{a}, \quad (\Gamma\cdot\Gamma^* = 1).$$

$$\Downarrow$$

$$\Delta T \Delta X \sim \langle A\rangle\langle B\rangle \sim \ell_s^2$$

In Minkowski metric, one of the directions must be of dominantly time-like, and the other be of dominantly space-like.

Taking into account the random nature of the boundaries in actual scattering amplitudes, we should expect, in general, an *inequality* for any string amplitude to all orders in string perturbation theory.

$$\Delta T \Delta X \gtrsim \ell_s^2$$

where

ΔT - uncertainty of the lengths measurable along the time-like direction

ΔX - uncertainty of the lengths measurable along the longitudinal spatial direction

" Space-Time Uncertainty Relation "

(T.Y. '87)

Remarks

1. Consistent with the energy-time uncertainty relation

$$\Delta t \Delta E \gtrsim 1, \quad \Delta E \sim \frac{\Delta X}{\ell_s^2}$$

2. More general than the familiar notion of "minimal distance"

Gross, Amati et al, ... '87

3. Lorentz invariance?

In principle, the inequality is compatible with kinematical Lorentz invariance ;

For example,

$$\frac{1}{2}[X^\mu, X^\nu]^2 \sim \ell_s^4,$$

$$\Downarrow$$

$$\sqrt{\langle -[X^0, X^i]^2 \rangle} = \sqrt{\frac{1}{2}\langle -[X^i, X^j]^2 \rangle + \ell_s^4} \gtrsim \ell_s^2.$$

4. High-energy string scattering

Wave packet analyses show that

(up to possible logarithmic corrections)

- high-energy fixed angle scattering (tree and fixed loops) Gross-Mende

$$\Delta T \Delta X \gg \ell_s^2$$

- high-energy fixed angle scattering, after Borel summation over all string loops Mende-Ooguri

$$\Delta T \Delta X \sim \ell_s^2$$

- Regge (tree) behavior for nearly forward scattering by graviton exchange

$$\Delta T \Delta X \gtrsim \ell_s^2$$

- high-energy resonance (tree) scattering Seiberg-Susskind-Toumbas

$$\Delta T \Delta X \gg \ell_s^2$$

5. Compatible with S- and T- dualities ;

   → use the lengths in the string-frame metric

6. Closely related to the ideas of "Black-hole complementarity", "UV/IR correspondence", and "Holography"

7. Effective for D-branes

   Example : D-particle scattering Douglas et al.

   Probe the spatial region of order ΔX.

$$\Delta T \Delta X \sim \frac{(\Delta X)^2}{v} \gtrsim \ell_s^2$$

Given the D-particle mass $m \sim 1/g_s\ell_s$ and taking into account the ordinary quantum-mechanical spread of wave functions, this leads to

$$\Delta X \gtrsim g_s^{1/3}\ell_s,$$

and

$$\Delta T \gtrsim g_s^{-1/3}\ell_s$$

Further implication

The coexistence of gravity and stringy space-time uncertainties leads, in general, to the characteristic scale of 10D string physics which is related to the M-theory scale :

Suppose we first ignore the string higher-modes.

Then, the only relevant scale is the gravitational length scale

$$\ell_P^{(D)} \sim g_s^{2/(D-2)} \ell_s = G_{(D)}^{1/(D-2)}$$

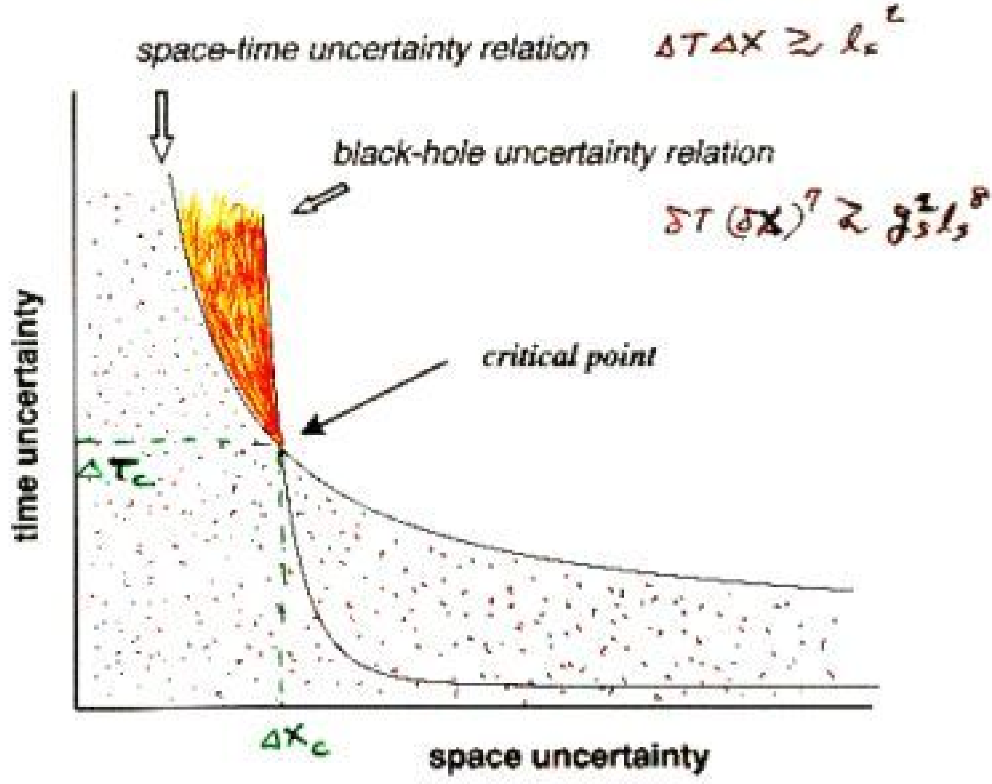
For a given time resolution δT, the validity of usual flat space-time background leads to the restriction of the spatial scale of order

$$\delta X \gtrsim (G_{(D)}/\delta T)^{1/(D-3)}$$

namely, 'black-hole uncertainty relation'

$$\delta T (\delta X)^{D-3} \gtrsim \ell_P^{(D)}$$

Truly stringy gravitational effects are characterized by the crossover point between the stringy space-time uncertainty relation and the 'black-hole uncertainty relation'



This occurs at $\Delta T \sim \delta T, \Delta X \sim \delta X$ leading to the critical scale as (D=10)

$$\Delta X_c \sim g_s^{1/3}\ell_s, \quad \Delta T_c \sim g_s^{-1/3}\ell_s.$$

How to "observe" this ?

3 Uncertainties and noncommutativity

The relation $\Delta T \Delta X \gtrsim \ell_s^2$ suggests the existence of certain noncommutative space-time structure that underlies string theory.

↓

Any alternative formulations of string quantum mechanics which explicitly exhibit the noncommutativity?

[*even with* $B_{\mu\nu} = 0$]

Usually, the world-sheet conformal invariance is expressed as the Virasoro condition

$$\mathcal{P}^2 + \frac{1}{4\pi\alpha'}\acute{X}^2 + \cdots = 0, \quad \mathcal{P}\cdot\acute{X} + \cdots = 0.$$

→ Polyakov approach

Let us consider a less familiar but equally possible formulation based on the Nambu-Goto-Schild action ($\lambda \sim \ell_s^2$, only bosonic part).

$$S_{ngs} = -\frac{1}{2}\int d^2\xi\,\{-\frac{1}{e}\frac{1}{2\lambda^2}(\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu)^2 + e\}$$

The auxiliary field e ensures the Virasoro constraint ($\leftrightarrow \delta e$)

$$-\frac{\mathcal{P}^2}{\acute{X}^2} \sim \frac{1}{e}\sqrt{-\frac{1}{2}(\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu)^2} = \lambda$$

Introducing another auxiliary field $b_{\mu\nu}(\xi)$ (world-sheet scalar, space-time antisymmetric tensor),

$$S_b = -\int d^2\xi \,\{\frac{1}{2\lambda}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu b_{\mu\nu} + \frac{1}{2}e(\frac{1}{2}b^2_{\mu\nu} + 1)\}.$$

If the lagrange multiplier field e is integrated over first, we can interpret this form as defining a 'random Poisson structure' for space and time;

$$\frac{1}{2}b^2_{\mu\nu} = -1 \quad \rightarrow \quad \sqrt{b^2_{0i}} \gtrsim 1$$

Dirac-bracket relation:

at each instant of world-sheet time, and for each random ("annealed") b field configuration,

$$\{X^\mu(\sigma_1), \partial_\sigma b^\mu_\nu(\sigma_2) X^\nu(\sigma_2)\}_D = \lambda\delta(\sigma_1 - \sigma_2).$$

$$\Downarrow$$

$$[X^0, \int d\sigma\, b_{0\mu}(\sigma)\partial_\sigma X^\mu] = i\lambda$$

between the center-of-mass target time operator

$$X^0 = \int d\sigma X^0(\sigma)$$

and the spatial extension $\int d\sigma\, b_{0\mu}(\sigma)\partial_\sigma X^\mu$.

$$\sqrt{\langle(\int d\sigma\, b_{0\mu}(\sigma)\partial_\sigma X^\mu)^2\rangle} \sim \Delta X$$

Semiclassically,

$$b_{\mu\nu} \sim 0(9,1)\text{“}spin\text{”}$$

tangential to strings

- *Space-time noncommutativity* : much milder than those in the 'ordinary' noncommutative field theory with constant external B field.

– b field $\leftrightarrow$ random 'spin' field
– $\partial X(\xi)$ $\leftrightarrow$ continuous field representing the string extension
– (macro) causality $\leftrightarrow$ the dominance of time-like area element $\partial_\tau x^\mu \wedge \partial_\sigma x^\nu$

- In contrast to this, the Moyal product $* = e^{i\frac{\theta}{2}(\overleftarrow{\partial}_x\overrightarrow{\partial}_t - \overleftarrow{\partial}_t\overrightarrow{\partial}_x)}$ in noncommutative field theory with space-time noncommutativity, $[x,t] = i\theta$, leads to

$$(\pi\theta)^2 \int dx dt \, (\phi_1 * \phi_2 * \phi_3)(x,t)$$

$$= (\prod_{i=1}^{3} \int dx_i dt_i) \exp i[\frac{2}{\theta} \sum_{cyclic} (x_i t_j - t_i x_j)] \prod_{i=1}^{3} \phi_i(x_i, t_i)$$

$$\Downarrow$$

$$\Delta(t_1 - t_2) \sim \theta p_3, \quad etc$$

The center-of-mass momenta are directly related to the nonlocal shifts of time.
$\Rightarrow$ acausal (and also nonunitary) behavior

- In the case of open strings, the addition of constant external 'electric field' B_{0i} induces acausal behavior when $|B_{0i}| \gtrsim \lambda^{-1}$.

  → large (space-like) fluctuations in the area element at $\lambda^{-1} b_{0i} \pm B_{0i} \sim 0$

  → well-known instability

- *Quantum equivalence with the Polyakov formulation ?*

No rigorous proof, unfortunately.

However, the application of a similar procedure to particle quantum mechanics leads simply to the familiar momentum-space representation of the amplitude

$$L = -\int d\tau \left(-\frac{1}{e}\left(\frac{dx^\mu}{d\tau}\right)^2 + em^2 \right)$$

$$\downarrow$$

$$L_p = \int d\tau \left(p_\mu \frac{dx^\mu}{d\tau} - \frac{1}{2} e (p^2 + m^2) \right)$$

(particle)

on-shell condition

$\downarrow$

(string, conformal gauge)

Virasoro condition

$\Updownarrow$

(string, 'b'-gauge)

'random space-time noncommutativity'

suggests

an entirely new representation of the string amplitudes, in which the auxiliary 'b' field is used, while the ordinary moduli parameters are integrated over.

If possible, might lead to a new formulation of string theory,
both perturbatively and nonperturbatively.

Other important questions :

- Symmetry structure associated with the space-time uncertainty relation from the viewpoint of target space geometry

  - general coordinate transformation at large distances,

  and so on .....

- *M theory viewpoint ?*

  'tripod relation' ?

$$\Delta T \Delta X \gtrsim \ell_s^2 \sim \ell_M^3 / R_{11}$$

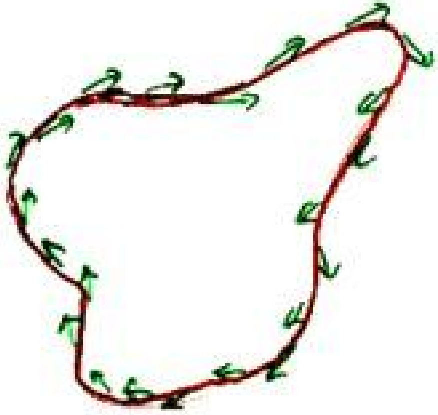
$$\Downarrow$$

$$\Delta T \Delta X_t \Delta X_{11} \gtrsim \ell_M^3 \sim G_{11}$$

in conformity with the membrane picture.

→ *What is the underlying symmetry?*

- Closed string



- Open string

 : $O(9,1)$ spin