ANTI DE SITTER D-BRANES

C. Bachas (ENS)

Mumbai, Strings '01

based on:

C.B, M. Petropoulos th/0012234

C.B, M. Douglas, C. Schweigert th/0003037

Will consider D-branes in SL(2, R) (and SU(2)) WZW models.

Why ?

- D-branes $\begin{cases}\text{openned window} \\ \text{gave glimpses}\end{cases}$ of microscopic struct. of spacetime, but still poorly understood in curved geometry

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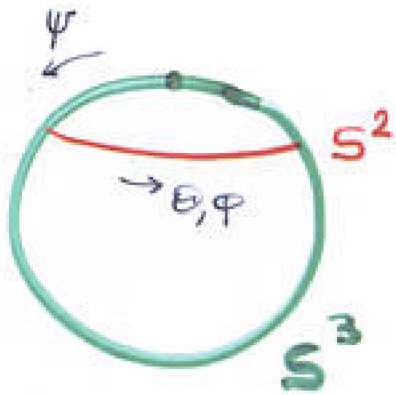
- get exact info on important CFT:
  SL(2,R) WZW + related Liouville cf Giveon ......
  Maldacena,
  Ooguri
  ⋮
  (non-critical strings
  mh of black holes)

- get insights into
  $\begin{cases}\text{warped compactificnts} \\ \text{cosmology in brane worlds ....}\end{cases}$

Q: Why are these branes stable despite $\pi_2(S^3)=0$?



↳ quantized magnetic flux

$$\int F = 2\pi n$$

does not, a priori, suffice because of strong field (relativistic) effects. In flat space

$$\mathcal{E}_{DBI} \sim \left\{ (\text{Area})^2 + n^2 \right\}^{1/2}$$

minimum for Area → 0

↳ stability due to interaction with background B-field

"T-dual" to "dielectric blow-up" of n D-particle bound state

Myers, ...

↳ Surprisingly, analysis based on (abelian) DBI action gives exact results

CB, Douglas, Schweigert, Pawelczyk

simple access to (some) exact CFT data.

.... hidden susy ?

Using $S_i{}^j = \sqrt{\frac{2}{k+2}} \sin\left(\frac{(2i+1)(2j+1)\pi}{k+2}\right)$

& standard manipulations
(eg. cylinder amplitude) can compute:

mass $\quad M_j = \frac{(2\pi)^{3/2}(2k+4)^{1/4}}{K_{(7)}} \sin\frac{(2j+1)\pi}{k+2}$

RR charge $\quad Q_j = \frac{(2\pi)^{3/2}(2k+4)^{1/4}}{2K_{(7)}} \sin\frac{(2j+1)2\pi}{k+2}$

open spectrum $\quad \psi^{\alpha}_{-1/2}|s\rangle$ in $(s+1)\oplus(s-1)$ rep

$m^2 = \frac{s(s+1)}{k+2} \qquad 0 \leq s \leq j$

integer

For $k \to \infty$, j fixed: decoupling of n.c. f. t. on "fuzzy sphere"

Alekseev, Recknagel, Schomerus

Geometric interpretnt:

$\bar{J}^{\alpha} = \omega(J^{\alpha}) \rightarrow$ world volume is (twined) conjugacy class

$$W = \{\, \omega(h)\, g\, h^{-1} \mid \forall h \in G \,\}$$

Alekseev, Schomerus
Felder, Fröhlich, Fuchs, Schweigert

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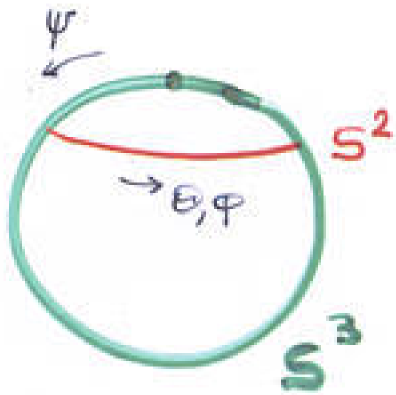

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$$\begin{cases} ds^2 = L^2\left\{d\psi^2 + \sin^2\psi\,(d\theta^2 + \sin^2\theta\, d\varphi^2)\right\} \\ B = L^2\left\{\psi - \frac{\sin 2\psi}{2}\right\}\sin\theta\, d\theta\wedge d\varphi \Rightarrow H = dB \propto \text{volume form} \\ F = -\frac{n}{2}\sin\theta\, d\theta\wedge d\varphi \quad (\text{quantizn robust under large gauge transfos.}) \end{cases}$$

$$\hookrightarrow \quad \mathcal{E}_{DBI} = 4\pi L^2 T\left\{\sin^4\psi + \left(\psi - \frac{\sin 2\psi}{2} - \frac{\pi n\alpha'}{L^2}\right)^2\right\}^{1/2}$$

- Unique minimum at $\psi_n = \frac{\pi n\alpha'}{L^2}$ for $0 < n < L^2/\alpha'$
- mass $M_n = 4\pi L^2 T_{(2)} \sin\frac{\pi n\alpha'}{L^2}$
- 'RR charge' $Q_n = T_{(2)}\int B + 2\pi\alpha' F = 2\pi L^2 T_{(2)} \sin\frac{2\pi n\alpha'}{L^2}$

  ↳ gauge inv. not quantized
  cf. W. Taylor; Marolf; .....

- spectrum of small fluctuations

$$\frac{d^2}{dt^2} = -\frac{1}{L^2}\begin{pmatrix} \square+2 & 2 \\ 2\square & \square \end{pmatrix} \Rightarrow \frac{s(s+1)}{L^2} \text{ in } (s-1)\oplus(s+1)$$

triplet of zero modes $\sim S^3$ translations of S^2

All these results in exact agreement with CFT if we set

$$\begin{cases} L^2 = (k+2)\alpha' \\ n = 2j+1 \end{cases}$$

Comments:

↳ cutoff $s \leq j$ invisible from DBI (?)

in decoupling limit $L^2/\alpha' \to \infty$, n fixed:

$$B + 2\pi\alpha' F \sim \sin\frac{2\pi n\alpha'}{L^2} \gg \hat{g} \sim \sin^2\frac{\pi n\alpha'}{L^2}$$

dipoles in strong magnetic field

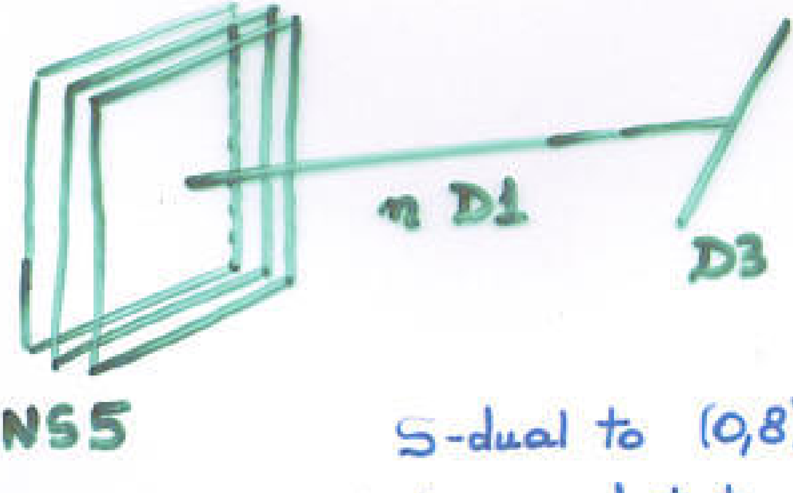
$\Rightarrow \sim n^2$ states

↳ 'Stringy exclusion principle' $0 < n < k+2$

can be related to Pauli-exclusion

n.h.g. of $N_5 = k+2$ NS5 branes

$= SU(2)_k \oplus FF \oplus \mathbb{R}^{1,5}$



S-dual to (0,8) system in which ground state of stretched F1 is fermion

C.B, Douglas, Green

:

cf. also Pelc

D-BRANES OF SL(2,R) WZW

$$g = \frac{1}{L}\begin{pmatrix} X^0+X^1 & X^2+X^3 \\ X^2-X^3 & X^0-X^1 \end{pmatrix}$$

$$\det g = 1 \quad \Rightarrow \quad (X^0)^2+(X^3)^2-(X^1)^2-(X^2)^2 = L^2$$

(part of) AdS_3

Regular conj. classes: Stanciu (+Figueroa O'Farrill)

$$\frac{1}{2}\operatorname{tr} g = \frac{X^0}{L} = \tilde{C} \quad \begin{array}{ll} \nearrow \ \tilde{C} < 1 & H_2 \\ \searrow \ \tilde{C} > 1 & dS_2 \end{array}$$

Twined conj. class: C.B, Petropoulos

$\omega = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ outer automorphism

$$\frac{1}{2}\operatorname{tr} g\omega = \frac{X^2}{L} = C \qquad AdS_2$$

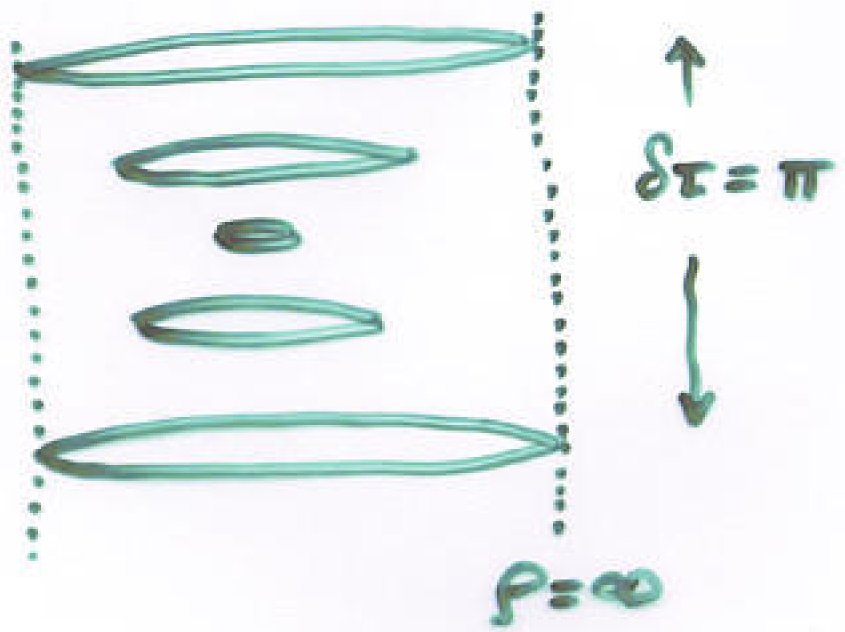
these are only 'physically allowed' D-branes

dS_2 : draw them in cylindrical coords :

$$X^0 + iX^3 = L\cosh\rho\, e^{i\tau}\,,\quad X^1 + iX^2 = L\sinh\rho\, e^{i\phi}$$

$$\begin{cases} ds^2 = L^2(-\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\phi^2) \\ B = L^2 \sinh^2\rho\, d\varphi \wedge d\tau \end{cases}$$

$\hookrightarrow \quad \frac{X^0}{L} = \cosh\rho\cos\tau = \tilde{C} > 1$



(1,q) string reaching AdS boundary in finite time. Long string ?

Seiberg, Witten
Maldacena, Ooguri

Impossible because tension > NS charge

For circular string:

$$\mathcal{E}_{DBI} = \underbrace{\frac{M(\rho)\cosh^2\rho}{\sqrt{\cosh^2\rho - \dot{\rho}^2}}}_{\text{blue-shifted mass + K.E.}} - \underbrace{2\pi L\, q T_F \sinh^2\rho}_{\text{B-coupling}}$$

$$M(\rho) = 2\pi L \sinh\rho \, T_{(1,q)}$$

$$\mathcal{E}_{DBI} \xrightarrow[\rho\to\infty]{} \infty \qquad \therefore \text{ forbidden region}$$

'Physical motions'

$$\rho_{max} \sim \log \frac{E/L}{T_{(1,q)} - qT_F}$$

Examples of non-sym. WZW branes

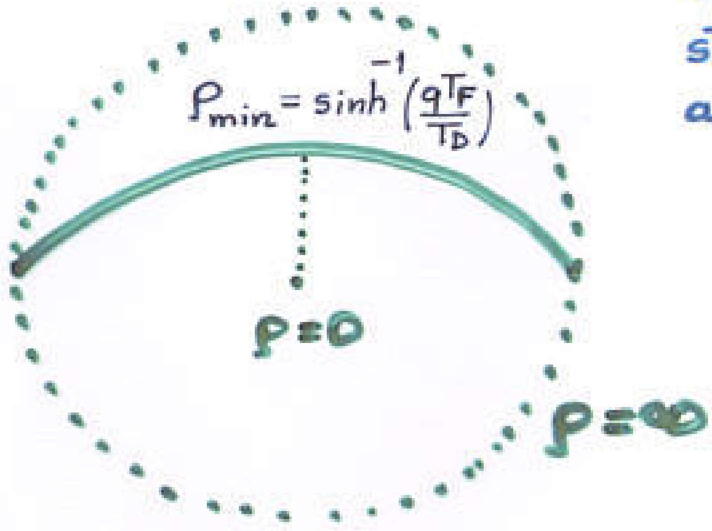
What about dS_2 ?

soln. in unphysical region of supercritical Electric field $\Rightarrow$ imaginary action

(Interpretnt ?)

AdS_2

$$\frac{X^2}{L} = \sinh\rho \sin\phi = C$$

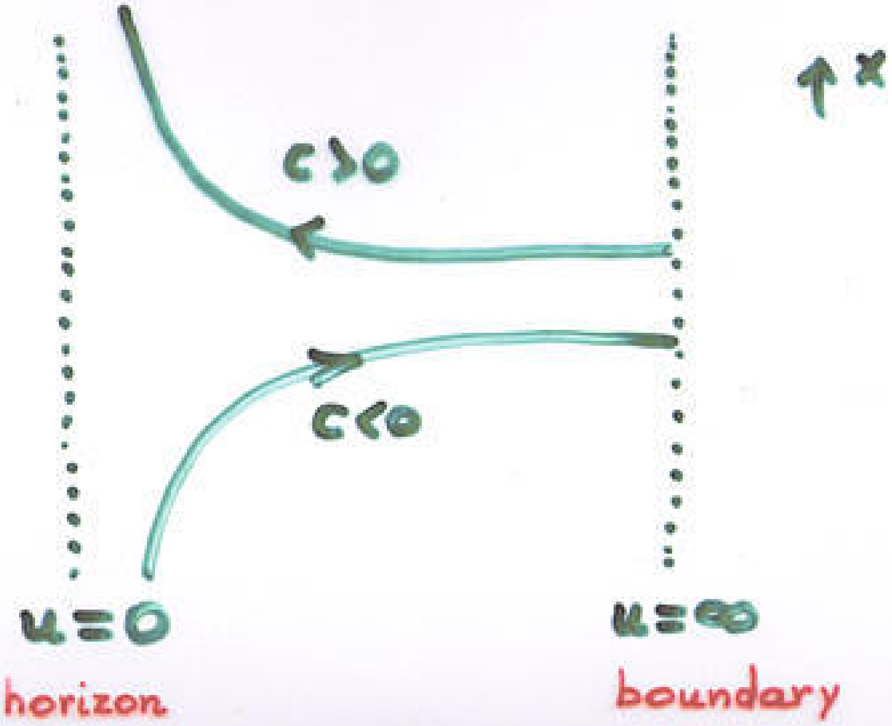


static $(1,q)$ string stretching across antipodal points on boundary

* can 'Lorentz boost' to oscillating string

↳ draw also in Poincaré coords:

$$x \pm t = \frac{1}{u}\left(\sinh\rho \sin\phi \pm \cosh\rho \sin\tau\right)$$
$$u = \cosh\rho \cos\tau + \sinh\rho \cos\phi$$
$$\Longrightarrow \quad u = \frac{C}{x}$$



Check DBI eqns:

$$ds^2 = L^2\left(\frac{du^2}{u^2} + u^2(dx^2 - dt^2)\right)$$

$$B = L^2 u^2\, dx \wedge dt$$

For static D-string: $u = u(x)$ with F_{xt} worldvol. field, one finds:

↳ (integrated) Gauss constr. : $\displaystyle \frac{2\pi\alpha' T_D \mathcal{F}_{xt}}{\sqrt{-\det \hat{g} - \mathcal{F}_{xt}^2}} = -q$

↑ integer # of bound F-strings

$\mathcal{F} = B + 2\pi\alpha' F$

↳ cons. of wv em tensor

$$\Rightarrow \Theta^x{}_x = L^2\left\{ \frac{T_{(1,q)}\, u^4}{\sqrt{u^4 + u'^2}} - q T_F u^2 \right\} = \text{const.}$$

$u' = \frac{\partial u}{\partial x}$

$\Theta^x{}_x = 0 \quad \Leftarrow$ free b.c. in x-direction

$$\Downarrow \quad u = \frac{C}{x - x_0}, \quad \text{with } C = \pm \frac{q T_F}{T_D} \quad (q > 0)$$

NB Relaxing above constraint leads to more general solns ~ strings in $q\bar{q}$-potential calculnt.

SUSY, open/closed metrics & speculation

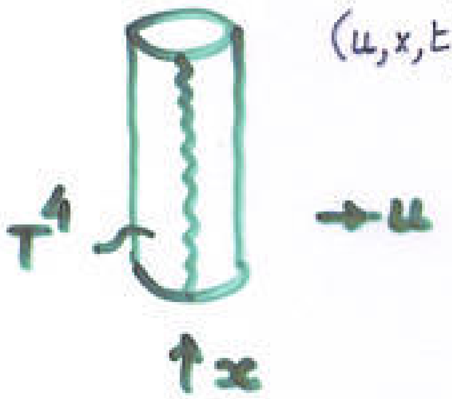
Simplest susy background:

nhg of NS5/F1 black string

$$SL(2,R)_k \oplus SU(2)_k \oplus U(1)^4$$

$$AdS_3 \times S_3 \times R^4$$

(u, x, t)



$$L^2 = N_5 \alpha' = (k+2)\alpha' \leftarrow$$

Has $AdS_2 \times S_2$ D3-branes

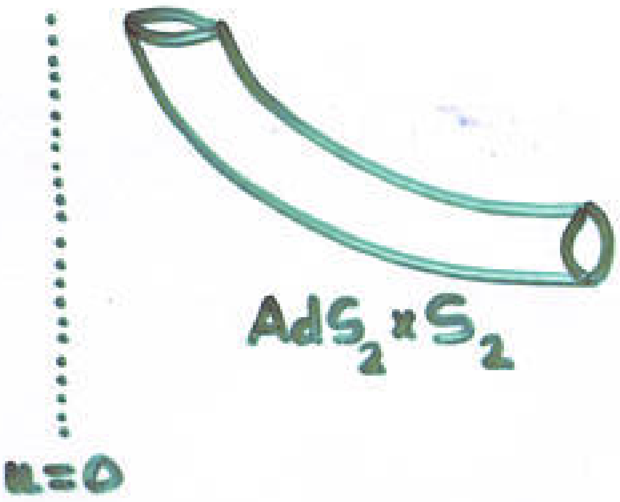
∵ ~ tensoring boundary states

or factorization of DBI eqns

one subtlety:

quantizn. cond. modified

$$n \frac{2\pi\alpha' T_D \mathcal{F}_{xt}}{\sqrt{-\det\hat{g} - \mathcal{F}_{xt}^2}} = -q \Longrightarrow C = \pm \frac{q\, T_F}{n\, T_D}$$



$$\int_{S_2} F = n$$

$$\int_{S_2} * \frac{\partial \mathcal{L}}{\partial F} = q$$

D3 carries quantum numbers of (n, q) string

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susy? $\psi_0^{A=0,\dots,9} = \Gamma^A$ ← flat tangent index

background ½ susy because the superVirasoro constraint imposes chiral projection:

$$G = \psi^A J_A + f_{ABC}\,\psi^A \psi^B \psi^C \Rightarrow$$

$$\Big(\prod_{A \notin M^4} \Gamma^A\Big) Q = Q \quad \text{\& likewise for } \bar{Q}$$

↑ unbroken susys

Antoniadis, CB, Sagnotti '89

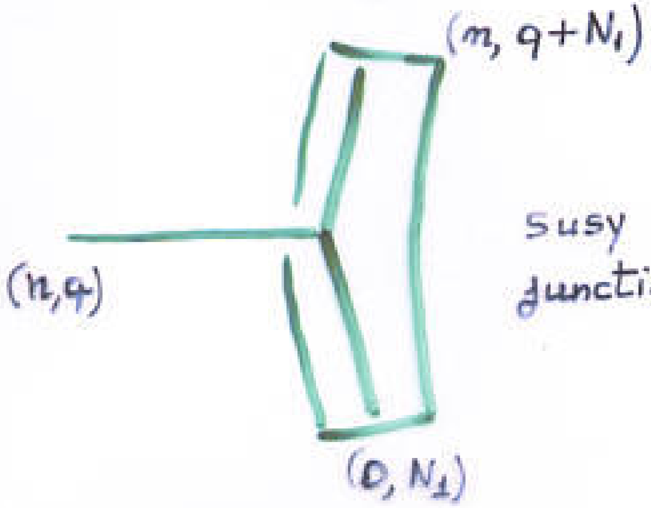
ω induces automorphism in spinor space:

$$\tfrac{i}{4}\left[\omega(\Gamma^A),\omega(\Gamma^B)\right] = \tfrac{i}{4}\,\Omega\left[\Gamma^A,\Gamma^B\right]\Omega^{-1}$$

$\therefore$ 1/4 unbroken susys: $Q+\Omega\bar{Q}$

Do these survive in asympt. flat region?

most likely, but 'back-reaction' must be accounted for



Do these survive in (more general) 5D BHs ?

↳ momentum modes
angular momentum

Strominger, Vafa
Cvetic, Larsen
Maldacena, Maoz
Balasubramanian et al
⋮

nhg: $AdS_3 \times S_3/_{\Gamma} \times T^4$

↳ discrete identifications
(+ remove regions where Killing isometry timelike)

↳ BTZ identificnt:

$$g = g_L \, g \, g_R^{-1}$$

with $g_L = \exp[\pi(r_+ - r_-)\sigma^3]$

$g_R = \exp[\pi(r_+ + r_-)\sigma^3]$

No inv. spinor under Lorentz boost ⇒ backgr. breaks susy, unless $g_L = 1 \Leftrightarrow r_+ = r_-$

↳ extremal Strominger-Vafa BH

But remaining susys chiral ⇒ D-brane breaks them all.

↳ ∃ however different possibility:

Γ acts as rotation in both AdS_3 and S_3.

Equal-angle combined rotnts preserve both left & right susys (1/4 of total)

⇒ D-brane can be 1/8 susy.

So ∃ susy D3-branes, ~~ending~~ in nhg of 6D black string, & 5D rotating BHs.

How is this realized on the brane worldvolume ?

↳ radii of S_2 and AdS_2

$$\underbrace{\frac{T_{(p,q)}}{p\,T_D}}_{\sqrt{1+C^2}} L = \ell_{AdS} \geq L \geq \ell_{Sph} = L \sin\left(\frac{\pi n \alpha'}{L^2}\right)$$

↳ can be arbitrarily flat

↳ can be arbitrarily small

worldvolume susy requires that they be equal. Puzzle ?

↳ but relevant radii are those of effective open-string metric, not induced closed string metric :

$$G_{\alpha\beta} = \hat{g}_{\alpha\beta} + \tilde{\mathcal{F}}_{\alpha\gamma}\, \hat{g}^{\gamma\delta}\, \tilde{\mathcal{F}}_{\beta\delta}$$

↳ cov. constant on worldvolume

for AdS_2:

$$\left.\begin{aligned} \text{closed } d\hat{s}^2 &= L^2(1+C^2)\frac{du^2}{u^2} - L^2u^2dt^2 \\ \mathcal{F} &= L^2C\, dt\wedge du \end{aligned}\right\} \Rightarrow$$

$$\text{open } dS^2 = L^2\left[\frac{du^2}{u^2} - u^2d\tilde{t}^2\right] \qquad \text{with } \tilde{t} = \frac{t}{\sqrt{1+C^2}}$$

Similarly for S_2. Therefore:

$$L_{AdS} = L = L_{Sph}$$

i.e. effective open-metrics have radii frozen to the radius of bulk geometry.

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* Consistent with susy, eff. metric as in near-horizon of 4D RN black hole.

* Obvious from CFT: spectrum of open vertex ops $h = \frac{j(j+1)}{L^2 = (k+2)\alpha'}$ same as for closed vertex ops.

Could have more general validity ??

Summary & open problems

↳ Interesting new class of (susy) D-branes with AdS_2 geometry

↳ Worldvol. theory is curved-space version of NCOS (analogous to NCYM on fuzzy sphere)

↳ branes cannot be 'flatenned out' in effective metric. Valid more generally?

↳ Extract info on SL(2,R) CFT
  ↳ Cardy states ?
  S-matrix ?

↳ Holographic interpretn ?