Strings 2001

N. DOREY
UW SWANSEA

From $\mathcal{N}=4$

To $\mathcal{N}=1$

with

O. Aharony
(Weizmann)

P. Kumar
(Swansea)

A. Parnachev
(Chicago)

work in progress with

T. Hollowood
(Swansea)

AdS/CFT for confining gauge theory:

Asymptotic Freedom $\Rightarrow \Lambda \ll M$ (I)

Λ: dynamical scale; M: UV cut off

SUGRA $\Rightarrow \Lambda \sim M$ (II)

Aim

Exact QFT results which interpolate between regimes I and II

$\mathcal{N}=4$ SYM $\xrightarrow[\text{deformation}]{\text{mass}}$ $\mathcal{N}=1$

SUGRA

Girardello et al
Pilch + Warner
Polchinski + Strassler

QFT

Vafa + Witten
Donagi + Witten

$\mathcal{N}=4$ SUSY Yang-Mills $G=SU(N)$

$$\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi} \qquad q = \exp(2\pi i \tau)$$

$$\lambda = g^2 N / 4\pi$$

$\mathcal{N}=1$ content: V vector

Φ_i chiral $i=1,2,3$

$\mathcal{N}=1^*$: masses M_i for Φ_i

$M_1 \sim M_2 \sim M_3 \sim M$

$\mathcal{N}=4$ SYM $\xrightarrow{\text{RG flow}}$ $\mathcal{N}=1$ SYM

matching:

$$\Lambda_{\mathcal{N}=1} = M q^{1/3N} \sim M e^{-\frac{2\pi}{3\lambda}}$$

't Hooft coupling

$$\Rightarrow \mu = \frac{\Lambda_{\mathcal{N}=1}}{M} \sim \exp[-2\pi/3\lambda]$$

usual problem:

SUGRA valid for $\lambda \gg 1$

asymptotic freedom requires $\lambda \ll 1$

$\mathcal{N}=1 \Rightarrow$ holomorphy

calculate exact λ-dependence of holomorphic quantities:

chiral condensates

BPS domain walls

- Quantitative tests of AdS/CFT Polchinski + Strassler
- Predictions for stringy corrections to SUGRA

scalar + fermion masses ~ M ⟷ sources on boundary

AdS/CFT

IIB geometry asymptotic to $AdS_5 \times S^5$

D = 5 approach Girardello et al Pilch + Warner

RG flow → Naked singularity

Singularity resolved by brane polarization Polchinski + Strassler

Myers effect

N × D3 ⟶ 5-brane on $\mathbb{R}^4 \times S^2$ ← non-com.

quantitative predictions for condensates, domain wall tensions....

$\lambda \ll 1 \Rightarrow \Lambda_{\mathcal{N}=1} \ll M \Rightarrow$ Pure $\mathcal{N}=1$ SYM

Gluino condensate: $S = \lambda_\alpha \lambda^\alpha$

$$U(1)_R \longrightarrow \mathbb{Z}_{2N} \xrightarrow{\langle S \rangle \neq 0} \mathbb{Z}_2$$

anomaly

spontaneous breaking

N vacua $|p\rangle$

$p = 0, \ldots, N-1$

$$\langle p|S|p\rangle = M^3 q^{1/N} \exp\left(\frac{2\pi i p}{N}\right)$$

Q: What happens as we increase λ ???

1) $\mathbb{Z}_{2N}$ broken

2) Extra vacua with $G = SU(N) \longrightarrow H$

Vafa + Witten

't Hooft classification

Donagi + Witten

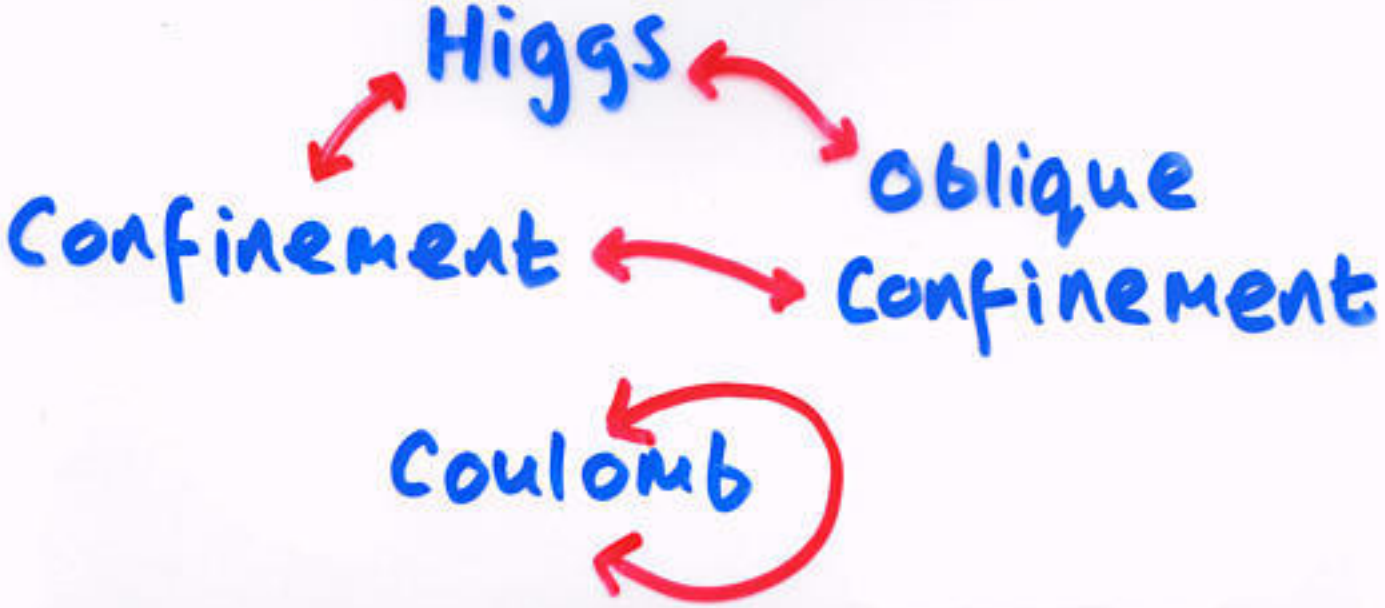
massive phases $\longleftrightarrow$ subgroups $K \subset \mathbb{Z}_N \times \mathbb{Z}_N$

E and M charges $\rightarrow$ $|K| = N$

$$\text{total } \# = \sum_{d|N} d$$

One vacuum in each phase

$SL(2,\mathbb{Z})$ action on vacua:



Compactification on $\mathbb{R}^3 \times S^1$

- $G = SU(2)$

New scalars: ← large gauge transformations

Wilson line $\omega \in [0, 2\pi]$

Dual photon $\sigma \in [0, 2\pi]$ ↑ quantization of magnetic charge

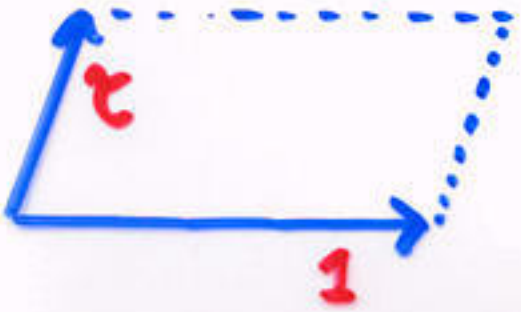
$\mathcal{N}=1$ SUSY:

$X = -i(\tau\omega + \sigma) \in E/\mathbb{Z}_2$ ← Weyl group

↑ chiral superfield

$E = \mathbb{C}/\Gamma \qquad \Gamma = \mathbb{Z} \oplus \tau\mathbb{Z}$

↑ complex torus



Manifest $SL(2,\mathbb{Z})$

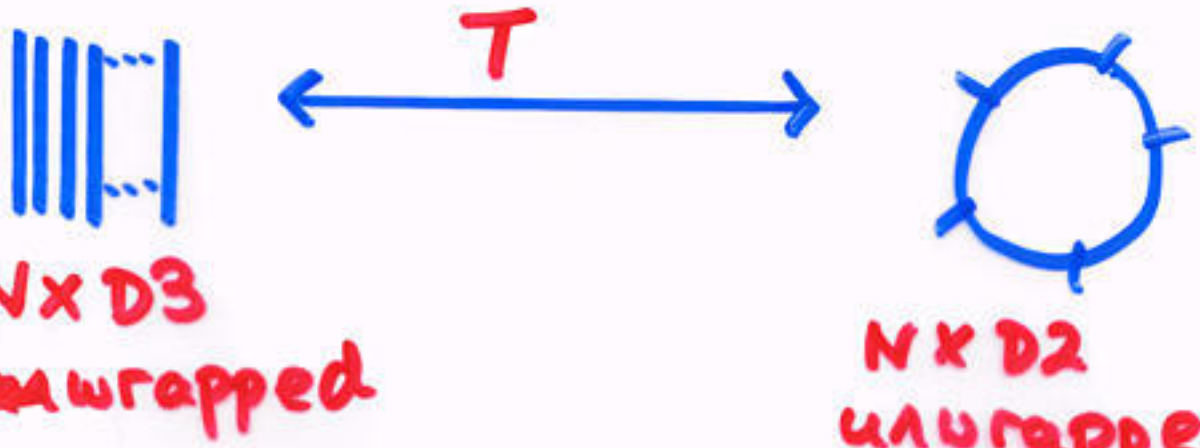
Seiberg

Geometrization of S-duality

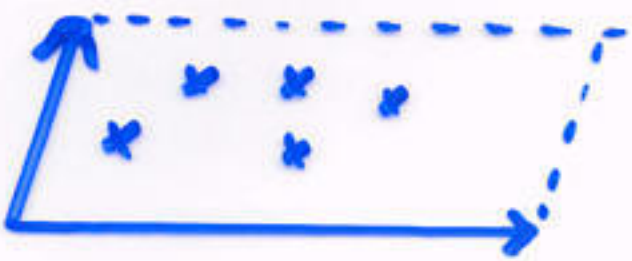
Schwarz

- $G = SU(N)$

IIB on $\mathbb{R}^9 \times S^1$ ⟷ IIA on $\mathbb{R}^9 \times \tilde{S}^1$



IIA/M

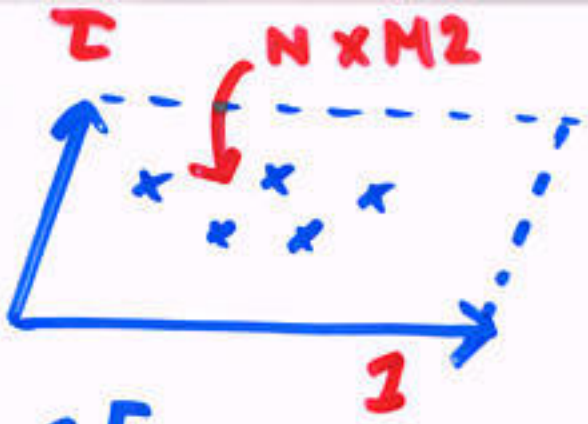


M theory on $\mathbb{R}^9 \times E$

Classical Coulomb Branch:

$$\mathcal{M}_{cl} = Sym^N(E)$$ ← config space of M2 branes

$\mathcal{N}=1^*$ $G=U(N)$



N complex scalars:

$$X_i = -i(\tau\omega_i + \sigma_i) \in E \qquad i=1,\dots,N$$

BPS monopoles ≡ instantons on $\mathbb{R}^3 \times S^1$

Superpotential $W \sim e^{X_i - X_j}$

Affleck, Harvey, Witten

Exact superpotential ND

$$W = M_1 M_2 M_3 \sum_{i>j} \wp(X_i - X_j)$$

↑ Weierstrass

cf Calogero-Moser

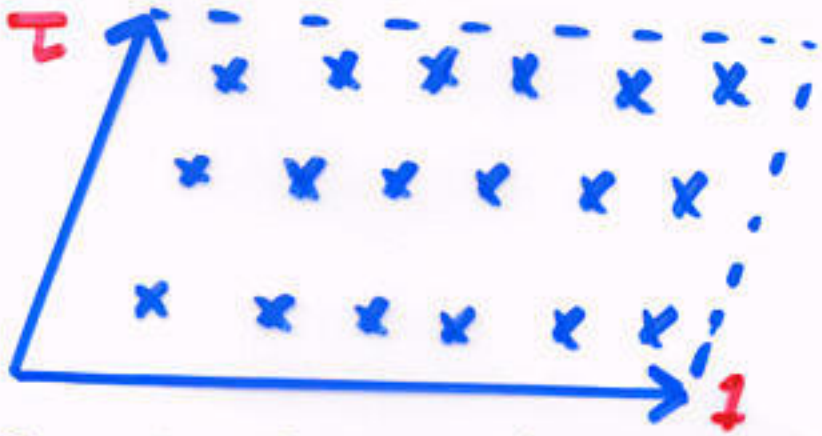
derivation:

1) holomorphy + double periodicity
   semiclassical analysis
2) $\mathcal{N}=2^* \rightarrow \mathcal{N}=1^*$
   integrable systems

$$\wp(x_i - x_j) \sim \sum_{w \in \Gamma} \frac{1}{(x_i - x_j - w)^2}$$

= Σ over pair-wise interactions between M2s cf Polchinski + Pouliot

SUSY vacua



$\{x_i\}$ form lattice Γ' with $\Gamma \subset \Gamma'$ order N

SUSY vacua $\longleftrightarrow$ subgroups $k \subset \mathbb{Z}_N \times \mathbb{Z}_N$ $|k| = N$

reproduces Donagi-Witten

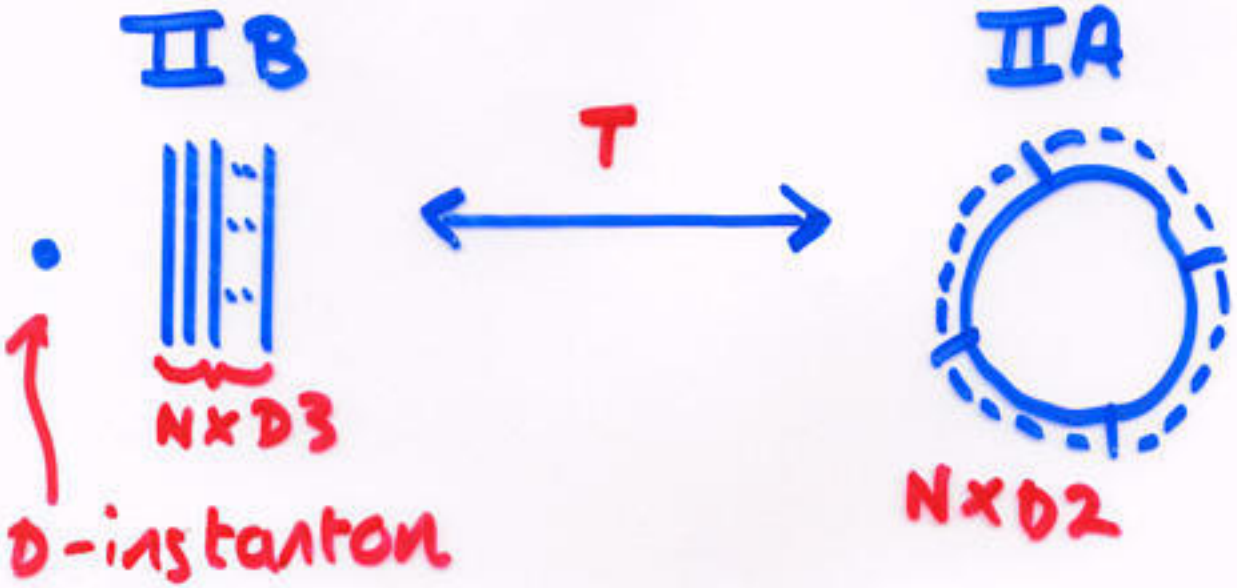
magnetic monopoles on $\mathbb{R}^3 \times S^1$

- finite action $= \frac{4\pi}{g^2}\omega$ ← Wilson line $G = SU(2)$
- fractional Pontryagin # $= \frac{\omega}{2\pi}$

$\equiv$ fractional instantons

Brodie
Lee + Yi

$G = SU(N)$



- - - - - $\equiv$ Euclidian D0 worldline

segments:

D2 —— D0 —— D2 $\equiv$ magnetic monopoles

Exact results for confining vacua $|p\rangle$ $p = 0, \ldots, N-1$

$$\langle p|W|p\rangle \sim M^3 \left[E_2(\tau) - E_2\left(\frac{\tau+p}{N}\right)/N \right]$$

E_2 ← 2nd Eisenstein series

$$\sim \sum_{k=0}^{\infty} c_k q^{k/N}$$

← "fractional instanton" expansion

- **$P=0$ vacuum**
  't Hooft limit $g^2 \to 0$, $N \to \infty$, $\lambda = g^2 N/4\pi$ fixed

$$\langle 0|W|0\rangle \sim E_2\left(i/\lambda\right)$$

- **$\tilde{S}$-duality** ≡ IR duality of SW theory

$$\lambda \longleftrightarrow 1/\lambda$$

Asymptotic freedom regime ⟷ SUGRA valid regime

Comparison with Polchinski-Strassler SUGRA dual:

$N \times D3 \xrightarrow[\text{effect}]{\text{Myers}} (p,q)$ 5-brane on $\mathbb{R}^4 \times S^2$

Detailed agreement for:

- BPS domain walls $\equiv$ 5-brane junctions
- chiral condensates

modulo operator mixing ambiguity (Akhmedov, ND, Kumar)

stringy corrections:

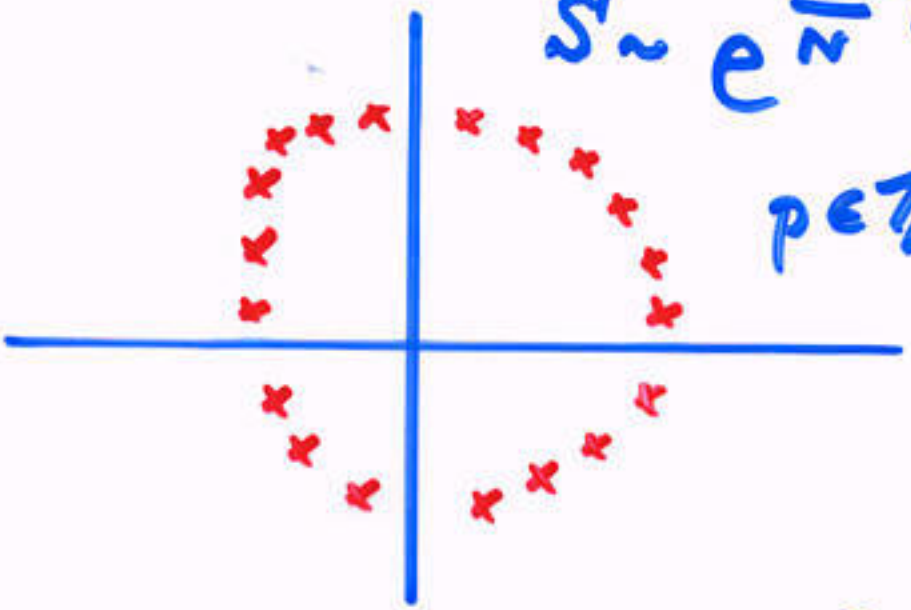
$$\langle O|W|O\rangle \overset{\lambda \gg 1}{\sim} \sum_{k=0}^{\infty} \tilde{c}_k \exp(-2\pi k \lambda) + \frac{A}{\lambda}$$

$\exp(-2\pi k\lambda)$: IIB worldsheet instantons wrapped on S^2

$\frac{A}{\lambda}$: anomalous term

$\lambda << 1$

$S = \partial_z W$



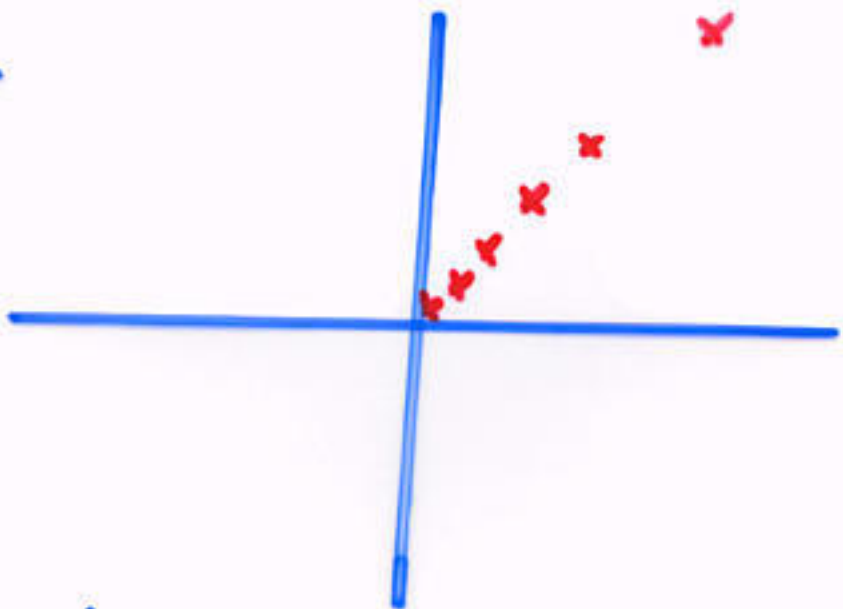
$S \sim e^{\frac{2\pi i}{N} p} \Lambda^3$

$p \in \mathbb{Z}$

$S \sim \frac{M^3}{j^2}$

$\lambda >> 1$

$j \in \mathbb{Z}$



Outlook

Similar results for other $\mathcal{N}=1$ theories?

Klebanov + Strassler

Leigh + Strassler