NORMALISATION OF BACKGROUND INDEPENDENT SFT ACTION

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Plan of the talk:

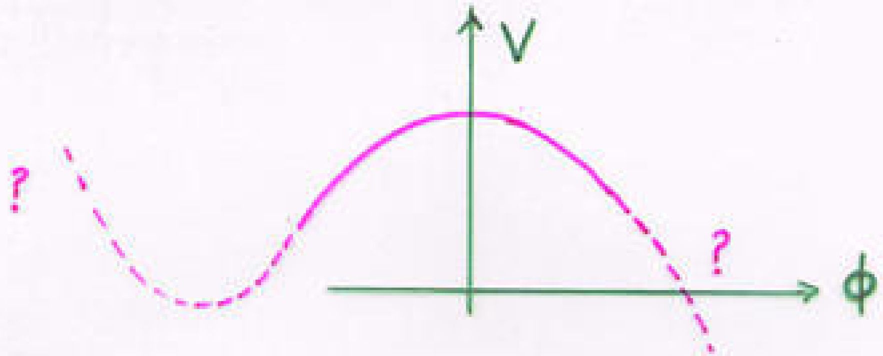
1. Introduction
2. Background Independent OSFT:
    - (a) Far off-shell
    - (b) Near on-shell
4. Comparison with Cubic OSFT
5. Conclusion

- The $(m+1)$-dimensional worldvolume field theory of a D-m-brane of bosonic string theory contains a scalar whose mass-square appears to be negative. This is called the *tachyon*.

- Perturbative vacuum described by the first quantised theory is *unstable*. The D-m-brane is at a *maximum* of the "string potential".

  Effective potential of the tachyon field.

  What is the shape of this potential?



String Field Theory (SFT) appears to be a promising framework to answer this (and other) non-perturbative issues.

Based on his pioneering work on non-susy excitations of supertsrings, Sen proposed the following conjectures:

- The potential of the tachyonic scalar field has a local minimum.

  At the minimum the potential energy of the tachyon *exactly* cancels the energy of the D-brane.

- The minimum describes the (closed string) *vacuum*.

  There are no perturbative (open string) excitations around this configuration.

- There are solitonic lump solutions (of all codimensions) of the tachyon equation of motion.

  These represent D-n-brane for $n = 0, 1, \cdots, 24$

The conjectures have been checked in

- World-sheet CFT [Sen] [Harvey-Kutasov-Martinec] [Zehra-Husain-Zabzine]
- Cubic Open String Field Theory (COS-FiT) [Witten]

  Numerically using the level truncation scheme. ([Kostelecy-Samuel] [Kostelecy-Potting]) [Sen-Zwiebach], [Moeller-Taylor], ⋯
- Effective Field Theory of Tachyon in the large B-field background. [Gopakumar-Minwalla-Strominger]

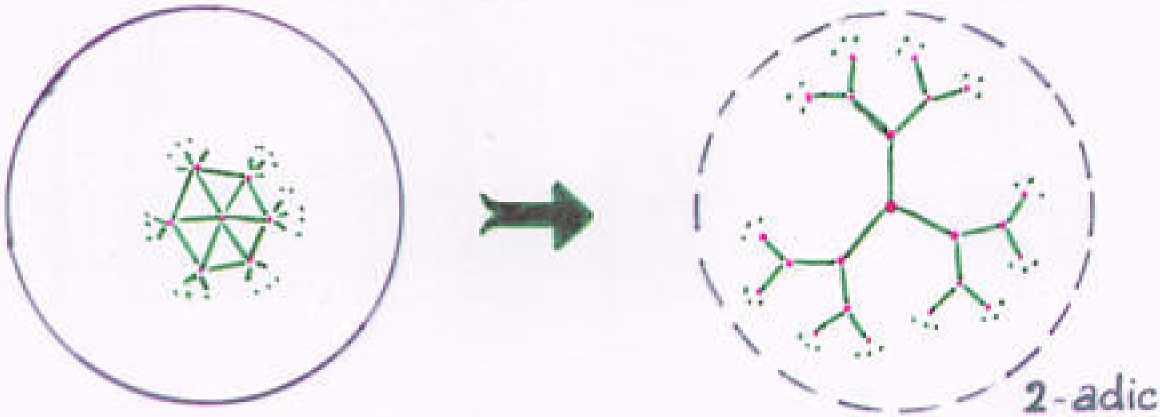
  Non-commutative tachyons.

  [Dasgupta-Mukhi-Rajesh], [Harvey et al], [Witten], ⋯

- Toy Models [DG-Sen]

  - *p*-adic String Theory: [Freund-Olson] [Brekke et al]

    A discretisation of the string world-sheet.
    [Zabrodin], [Chekhov-Mironov-Zabrodin]



    Conjectures verified in *exact* effective field theory of the tachyon.

  - Zwiebach Models

  - Minahan-Zwiebach model

- Background Independent Open String Field Theory (BIOSFiT) [Gerasimov-Shatashvili], [Kutasov-Marino-Moore], [DG-Sen]

Background Independent OSFT: [Witten], [Li-Witten], [Shatashvili]

- 2d field theory on a disc D (string world-sheet) with standard flat metric described by the action

$$\mathcal{S}_{Bulk} = \frac{1}{4\pi\alpha'}\int_D \left[\partial_\alpha X^M \partial^\alpha X^N \eta_{MN} + \text{ghost}\right]$$

(Trivial closed string background)

- String field $|\Phi\rangle$ is a ghost number 1 state in the ghost + matter Hilbert space $\mathcal{H}^{(1)}$.

  Also need $|\mathcal{V}\rangle = b_{-1}|\Phi\rangle$.
  $\Phi(z), \mathcal{V}(z)$ are the corresponding vertex operators.

  In terms of a set of basis vectors $\{\phi_{1,i}\}$ in $\mathcal{H}^{(1)}$

$$|\Phi\rangle = \sum_i \psi_i |\phi_{1,i}\rangle$$

  ψ_i: Dynamical variables of SFT,
  Couplings of the world-sheet FT.

- Deform the world-sheet theory by the *boundary* perturbation

$$\mathcal{S}_{2d} = \mathcal{S}_{Bulk} + \frac{1}{\alpha'} \int d\theta \, \mathcal{V}(e^{i\theta})$$

- Correlation functions:

$$\langle \cdots \rangle_{\mathcal{V}} = \int \mathcal{D}X \, \mathcal{D}b \, \mathcal{D}c \, e^{-\mathcal{S}_{2d}} (\cdots)$$

SFT action determined from the relation

$$dS_{BI} = \frac{K}{2} d\psi_i \int d\theta \int d\theta' \left\langle \phi_{1,i}(e^{i\theta}) \{ Q_B, \Phi(e^{i\theta'}) \} \right\rangle_{\mathcal{V}}$$

- K is a normalisation constant to be determined.

- $Q_B = \oint dz j_B(z)$ is the BRS operator.

- S_{BI} is formally background independent.

- Additive ambiguity in defining the action.

To determine the tachyon dependent part of the SFT, deform by the tachyon field

$$
\begin{aligned}
\Phi(z) &= cT(X)(z) \\
\mathcal{V}(z) &= T(X(z))
\end{aligned}
$$

Behaviour far off-shell is given by Taylor expansion

$$
T(X) = \frac{1}{2\pi}a + \frac{1}{4\pi}\sum_{r=n+1}^{m} u_r (X^r)^2 + \cdots
$$

Quadratic perturbation – can be solved exactly.

[Witten]

$$
\begin{aligned}
S_{BI} &= K\left[(1+a)Z + \sum\left(u_r Z - u_r \frac{\partial Z}{\partial u_r}\right)\right] \\
Z &= e^{-a}\prod\left(2\pi\sqrt{u_r}e^{\gamma u_r}\Gamma(u_r)\right)
\end{aligned}
$$

[Kutasov-Marino-Moore]

- Perturbative vacuum ($\sim$ D-m-brane) at $a = 0, u = 0$.

- S_{BI} has an extremum at $a = \infty, u = 0$.

$$S_{BI}(a = \infty) - S_{BI}(a = 0) = K$$

- Other extrema given by

$$u_r = \infty$$

and a determined in terms of u.

This describes a solitonic n-brane. Energy of this configuration is

$$(2\pi)^{m-n} K$$

Expected behaviour *if* $K = T_m$.

Near on-shell tachyon field

$$T(X) = \frac{1}{2\pi}\int \frac{d^{m+1}k}{(2\pi)^{m+1}}\phi(k)e^{ik.X}$$

with $\phi(k)$ supported over *near on-shell* momenta $k^2 \simeq 1$.

Quadratic term requires evaluation of correlators only in the undeformed theory:

$$S^{(2)}_{BI} = \frac{K}{4}\int \frac{d^{m+1}k}{(2\pi)^{m+1}}(k^2-1)\phi(k)\phi(-k)$$

Cubic term requires evaluation of correlators in presence of the deformation.

Contribution from change in Q_B, as well as change in correlation function. (Conf. Pert. Th. [Sen])

For near on-shell fields these are plagued by UV divergences.

Another way to evaluate the cubic term:

Whenever the world-sheet action describes a *CFT*, the corresponding *string field* configuration is a solution of the equations of motion.

[Sen], [Callan et al.], [Das-Sathiapalan], ...

SFT Eqns of motion are proportional to the β-functions of the deformed 2d world-sheet FT.

[Affleck-Ludwig]
If in some basis $\mathcal{V} = \sum \lambda^I \mathcal{V}_I$, where $\mathcal{V}_I$ are primaries of dimension $h_I \simeq 1$, with OPE

$$\mathcal{V}_I(x)\mathcal{V}_J(y) = \frac{C_{IJ}^K}{2\pi|x-y|^{h_I+h_J-h_K}} \mathcal{V}_K(y)$$

the β-function is

$$\beta^I(\lambda) \propto (h_I - 1)\lambda^I + C_{JK}^I \lambda^J \lambda^K + \cdots$$

We have

$$\mathcal{V}_I \sim e^{ik.X} \quad \text{with } h(k) = k^2 \simeq 1$$

and OPE

$$e^{ik.X}(x)e^{ik'.X}(y) = \frac{(2\pi)^{-1}}{|x-y|^{k^2+k'^2-(k+k')^2}} e^{i(k+k').X}$$

Eqn of motion (near on-shell)

$$(k^2-1)\phi(k) + \frac{1}{2\pi}\int \frac{d^{m+1}k'}{(2\pi)^{m+1}}\phi(k')\phi(k-k') + \cdots = 0$$

Background indept SFT action for near on-shell tachyon:

$$\begin{aligned} S_{BI} &= \frac{K}{4}\int \frac{d^{m+1}k}{(2\pi)^{m+1}}\Big[(k^2-1)\phi(k)\phi(-k) \\ &+ \frac{1}{3\pi}\int \frac{d^{m+1}k'}{(2\pi)^{m+1}}\phi(k)\phi(k')\phi(-k-k') + \cdots\Big] \end{aligned}$$

Cubic Open String Field Theory action: [Witten], [Leclair-Peskin-Preitschof], [Sen], ...

$$
\begin{aligned}
S_{Cu} &= 2\pi^2 T_m \Big[\frac{1}{2}\langle \Phi | Q_B | \rangle \Phi \rangle \\
&\quad + \frac{1}{3}\langle f_1 \circ \Phi(0)\, f_2 \circ \Phi(0)\, f_3 \circ \Phi(0) \,\rangle \Big]
\end{aligned}
$$

- T_m: tension of the D-m-brane.
- $\Phi(z) = \int \frac{d^{m+1}k}{(2\pi)^{m+1}} \chi(k) e^{ik.X}(z)$ with $k^2 \simeq 1$.
- f_i are specific conformal maps.

Near on-shell tachyon dependent part of the action [Sen], [Sen-Zwiebach]

$$
\begin{aligned}
S_{Cu} &= 2\pi^2 T_m \int \frac{d^{m+1}k}{(2\pi)^{m+1}} \Big[\frac{1}{2}(k^2 - 1)\chi(k)\chi(-k) \\
&\quad + \frac{1}{3}\int \frac{d^{m+1}k'}{(2\pi)^{m+1}} \chi(k)\chi(k')\chi(-k-k') + \cdots \Big]
\end{aligned}
$$

Compare the two action:

- The relation between the tachyon fields of the two SFT

$$\chi(k) = \frac{1}{2\pi}\sqrt{\frac{K}{T_m}}\,\phi(k) + \cdots$$

  Corrections from terms containing derivatives (momenta), and terms higher order in ϕ.

- Now identifying the cubic terms give

$$\boxed{K = T_m}$$

This is *exactly* what is needed to prove the conjectures in the framework of background independent open SFT.

- Straightforward to incorporate constant NS B-field background. [Li - Witten]

  - This can also be used to fix normalisation. (Comparison with Born-Infeld action.)

  - Expected result: ordinary product $\rightarrow$ star product. [Cornalba], [Okuyama],

- UV problem prohibits a proper understanding of the formalism of Background Independent OSFT.

- Exact effective action for the tachyon field in Background Independent OSFT, (as in p-adic string).

- Technology and exact results for integrable (boundary) perturbations may shed light on the BIOSFiT for at least a class of string field configurations. [Fujii - Itoyama]

- superstring