SUPERSTRINGS ON AdS_3

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hep-th/0009242

AMIT GIVEON

STRINGS 2001

⋮

A.G., KUTASOV, SEIBERG	9806194
KUTASOV, LARSEN, LEIGH	9812027
A.G., ROCEK	9904024
BERENSTEIN, LEIGH	9904040
A.G., KUTASOV, PELC	9907178
MALDACENA, OOGURI	0001053
HIKIDA, HOSOMICHI, SUGAWARA	0005065

⋮

MOTIVATION

- TIME DEPENDENT BACKGROUNDS
- AdS/CFT
- STRING THEORY IN GENERAL
- BLACK HOLES
-
-

AdS_3 = ∞ COVER OF SL(2)

LORENTZIAN:

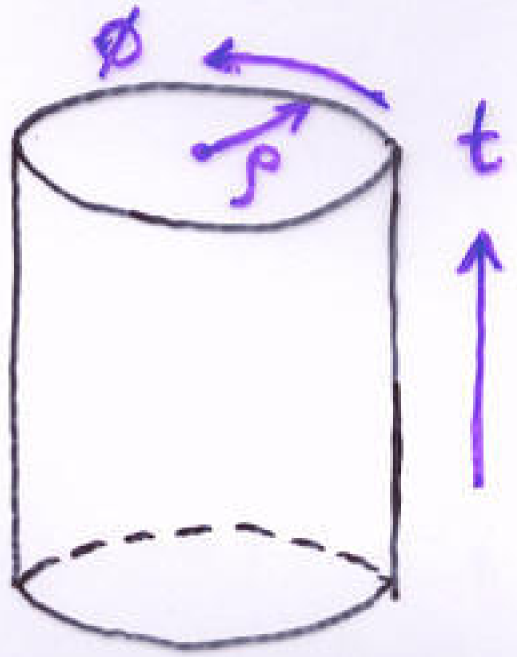
$$-X_{-1}^2 - X_0^2 + X_1^2 + X_2^2 = -\ell^2$$

GLOBAL COOR.

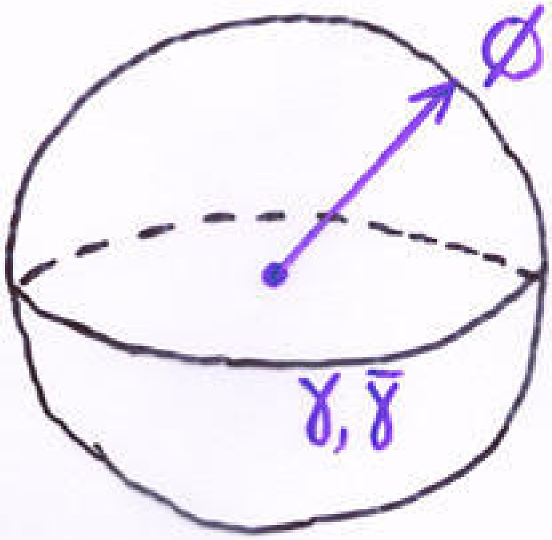
$$g = e^{iu\sigma_2} e^{\rho\sigma_3} e^{iv\sigma_2} \in SL(2)$$

$$ds^2 = \ell^2\left[-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\phi^2\right]$$

$$u = \tfrac{1}{2}(t+\phi) \qquad v = \tfrac{1}{2}(t-\phi)$$



$$ds^2 = \ell^2\left(d\phi^2 + e^{2\phi}\, d\gamma\, d\bar{\gamma}\right)$$



$$ds^2 = -\ell^2\left(1+\frac{r^2}{\ell^2}\right)dt^2 + \left(1+\frac{r^2}{\ell^2}\right)^{-1}dr^2 + r^2 d\theta^2$$

$$\ell^2 = \ell_s^2\, k$$

SUMMARY

PERTURBATIVE WORLDSHEET STUDY

+

NON-PERTURBATIVE CONSEQUENCES

FROM AdS_3/CFT_2

$\Downarrow$

STRING THEORY

ON

$AdS_3 \times \mathcal{N}$ $\longleftrightarrow$ $\mathcal{M}(\mathcal{N})^p / S_p$

WORLDSHEET

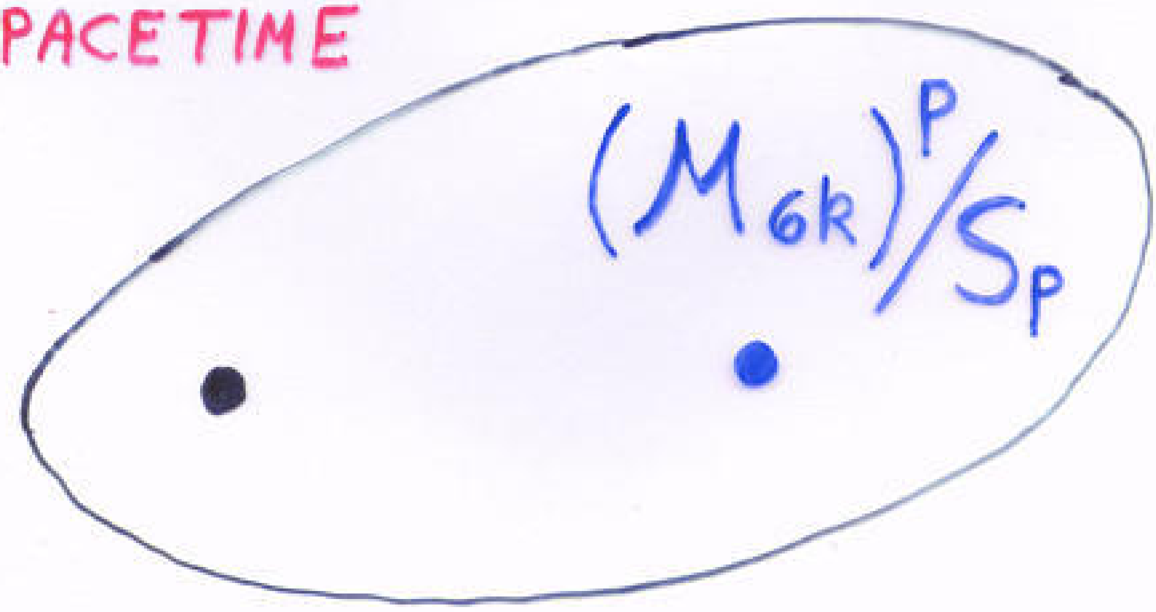
- $(AdS_3)_k \times \mathcal{N}$ $\quad g_s^2 \sim \frac{1}{p}$

$$p = \begin{cases} \bullet\ \#\ \text{OF F1} \\ \bullet\ \text{MAX. \# OF "LONG STRINGS"} \\ \bullet\ \text{"EXCLUSION PRINCIPLE": MAX. } p \text{ STRINGS STATE} \end{cases}$$

A.G., Kutasov, Seiberg
Maldacena, Michelson, Strominger
Seiberg, Witten

Maldacena, Strominger

SPACETIME



$$\mathcal{M}_{6k} = \text{2-d CFT w/ } c = 6k$$

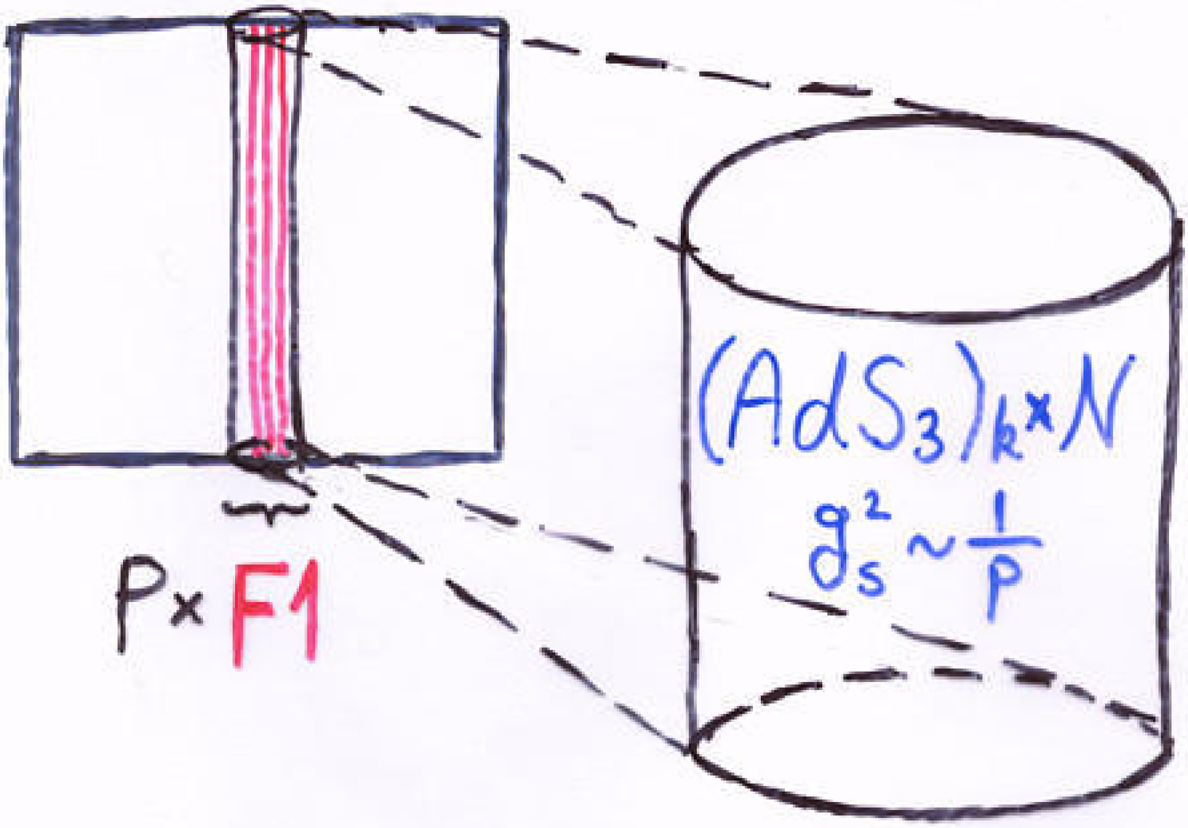
$$\updownarrow$$

W.S. "SHORT STRINGS" + SINGLE "LONG STRING"

A.G., Kutasov, Pelc

$\mathbb{R}^{1,1} \times \mathbb{R}_\phi \times N$

$\Phi = -\sqrt{\frac{2}{k}}\,\phi$



$p \times$ F1

Callan, Harvey, Strominger

$k \times$ NS5

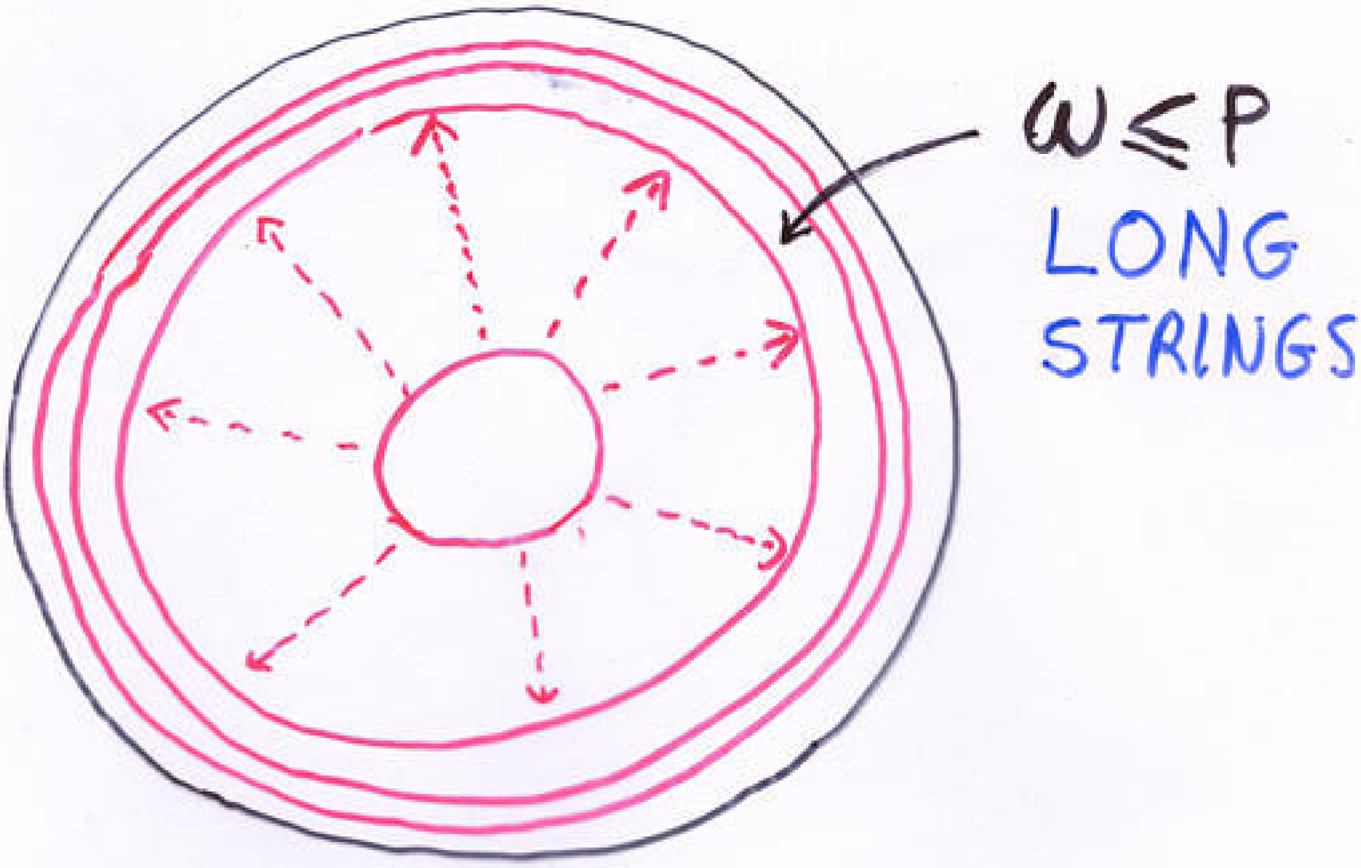
$\mathbb{R}^{1,1} \times \mathbb{R}_\phi \times SU(2) \times T^4$

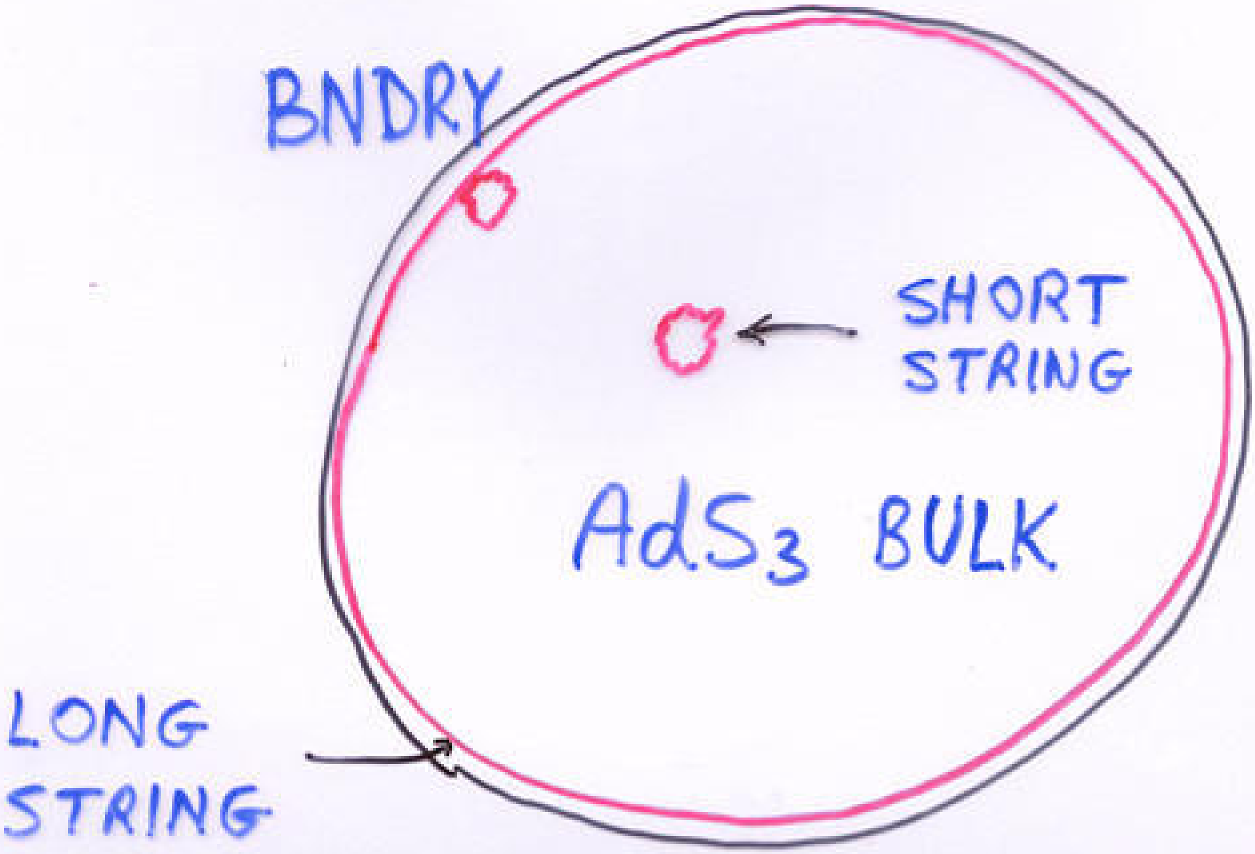
$\downarrow$

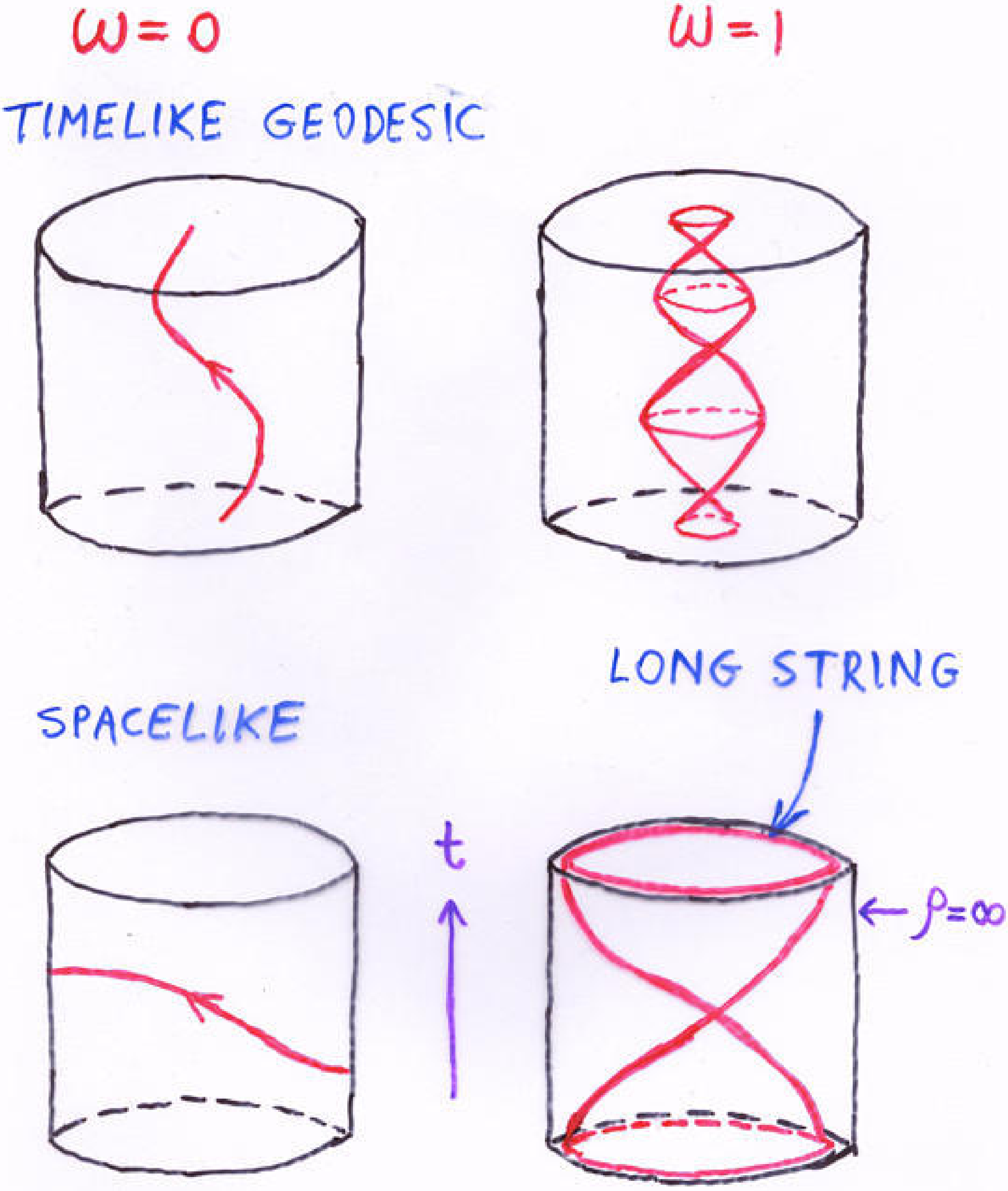
$p \times$ F1

$AdS_3 \times S^3 \times T^4$

$$g_s^2 = \frac{k}{p} V_{T^4}$$


$\omega \leq p$
LONG
STRINGS


BNDRY
SHORT
STRING
AdS_3 BULK
LONG
STRING



Maldacena-Ooguri

W.S. $AdS_3 \times N$

VERTEX OPERATORS $V^{\omega} = t^{\omega}(V)$

$\downarrow$

"SINGLE PARTICLE" STRING STATES

S.T. $\mathcal{M}^p / S_p$

$t^n(O_{\mathcal{M}})$

t^n = TWIST FIELD IN THE $\mathbb{Z}_n$ TWISTED SECTOR OF $\mathcal{M}^n / \mathbb{Z}_n$ (SYMM. BY S_p)

CHECKS

- W.S. AND S.T. SPECTRUM HAVE THE SAME PATTERN
- CHIRAL SPECTRUM

BOSONIC STRING - REVIEW & "TWIST"

$AdS_3 = \infty$ COVER OF $SL(2)_k$

W.S. $\widehat{SL}(2)_L \times \widehat{SL}(2)_R$

$$J^a(z) \qquad \bar{J}^a(\bar{z})$$

$$\downarrow$$

S.T. $SL(2) \times SL(2)$

$$L_0 = -\oint J^3 \qquad L_{\pm 1} = -\oint J^{\pm}$$

$$[L_n, L_m] = (n-m) L_{n+m} \qquad n, m = 0, \pm 1$$

$\downarrow$ A.G., Kutasov, Seiberg

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{c_{ST}}{12}(n^3 - n)\,\delta_{n+m,0}$$

$$L_n = \oint \left[(n^2-1) J^3 \gamma^n - \frac{n(n-1)}{2} J^- \gamma^{n+1} - \frac{n(n+1)}{2} J^+ \gamma^{n-1} \right]$$

$$c_{ST} = 6kP$$

$$\updownarrow$$

$$\oint \gamma^{-1} \partial \gamma$$

WORLDSHEET $\leftrightarrow$ SPACETIME CORRESPONDENCE

GKS

$$\widehat{SL}(2)_k \times \widehat{SL}(2)_k \leftrightarrow \text{VIR.} \times \text{VIR.} \quad c = 6kp$$

$$\hat{G}_k \leftrightarrow \hat{G}_{kp}$$

PRIMARY j $\leftrightarrow$ PRIMARY $h = j + 1$

R OF G $\leftrightarrow$ R OF G

LEFT/RIGHT MOVING $\leftrightarrow$ L/R CHIRALITY

$\vdots$ $\qquad\qquad$ $\vdots$

$\langle V \cdots V \rangle$ SATISFY WARD IDENTITIES OF 2-d CFT

AdS_3 / 2-d CFT

$$J^3 = -\sqrt{\tfrac{k}{2}}\,\partial X$$

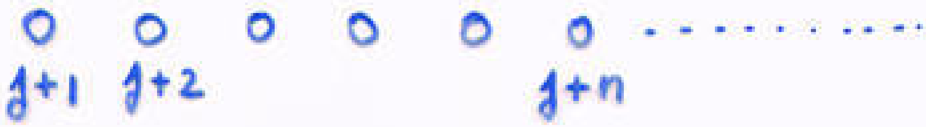
$$SL(2) \longrightarrow \frac{SL(2)}{U(1)} \times U(1)_X$$

$$\Phi_{jm\bar{m}} = \Psi_{jm\bar{m}}\, e^{\sqrt{\frac{2}{k}}(mX + \bar{m}\bar{X})}$$

$$\Delta(\Phi_j) = -\frac{j(j+1)}{k-2} \qquad \Delta(\Psi_{jm}) = -\frac{j(j+1)}{k-2} + \frac{m^2}{k}$$

UNITARITY: DLP, ..., EGP, GK, MO

(1) $D_j^{\pm}$: $m = \pm(j+n)$ $\quad n = 1, 2, \ldots$



$$j \in \mathbb{R} \qquad -\tfrac{1}{2} < j < \frac{k-3}{2}$$

$$\Delta(\Psi_{jm}) \geq 0$$

(2) $C_{j,\mu}$: $m = \mu, \mu \pm n$ $\quad 0 \leq \mu < 1$

$$j \in -\tfrac{1}{2} + i\mathbb{R}$$

W.S.

$$V(j; m, \bar{m}) \subset AdS_3 \times N$$

IN $D_j^{\pm}$

$\updownarrow$ GKS

S.T.

MODES $A^h_{m\bar{m}}$ OF PRIMARY W/

$$h = j + 1$$

$$[L_n, V(j, m)] = (nj - m) V(j, m+n)$$

IN 2-d CFT

$$[L_n, A^h_m] = (n(h-1) - m) A^h_{m+n}$$

THE "TWIST"

- SINGLE VALUEDNESS ON AdS_3

$$\Downarrow$$

(1) $m - \bar{m} \in \mathbb{Z}$

- TO OBTAIN (1) "INTRINSICALLY" IN AdS_3 W.S.: INCLUDE "TWIST FIELDS"

$$t^{\omega}_{SL(2)}$$

WHICH IMPOSE (1) VIA MUTUAL LOCALITY

$$\Downarrow$$

$$t^{\omega}_{SL(2)} = e^{\omega\sqrt{\frac{k}{2}}(X+\bar{X})} \equiv e^{-\omega\left(\int J^3 + \int \bar{J}^3\right)}$$

$$t^{\omega}_{SL(2)}(z,\bar{z})\,\phi_{jm\bar{m}}(0) \sim z^{(m-\bar{m})\omega}|z|^{-2\bar{m}\omega}\,\phi^{\omega}_{jm\bar{m}}$$

$$\phi^{\omega}_{jm\bar{m}} = \Psi_{jm\bar{m}}\, e^{\sqrt{\frac{2}{k}}\left[(m+\frac{k}{2}\omega)X + (\bar{m}+\frac{k}{2}\omega)\bar{X}\right]}$$

$$\Delta(\phi^{\omega}_{jm}) = -\frac{j(j+1)}{k-2} - \frac{k}{4}\omega^2 - m\omega$$

Maldacena-Ooguri (USED SPECTRAL FLOW)

ω "TWISTED SECTOR"

$\updownarrow$

ω "LONG STRINGS" IN AdS_3

$\mathcal{N} \times AdS_3$

$$V^{\omega} = V_{\Delta}\, \emptyset^{\omega}_{jm} \qquad (\omega < 0)$$

$$[L_n, V^{\omega}] = \begin{cases} -\left(m + \frac{k}{2}\omega\right) V^{\omega} & n = 0 \\ 0 & n \geq 1 \end{cases}$$

$$\Downarrow$$

W. S.

$$V^{\omega} \leftrightarrow \underbrace{A^{h}_{m_{ST} = -h}}_{\substack{\text{THE MODE} \\ m_{ST} = -h \quad h = -\left(m + \frac{k}{2}\omega\right) \\ \text{OF A PRIMARY } h}} + \left\{ \begin{array}{c} \text{SOMETHING THAT} \\ \text{ANNIHILATES} \\ |0\rangle_{ST} \end{array} \right\}$$

ON-SHELL CONDITION $\Rightarrow$

$$h = \frac{k|\omega|}{4} + \frac{1}{|\omega|}\left(- \frac{j(j+1)}{k-2} + \Delta - 1 \right) \qquad \text{MO}$$

SUPERSTRINGS ON AdS_3 - REVIEW & "TWIST"

$SL(2)$ SUPERCURRENTS

$$\psi^a + \theta\sqrt{\tfrac{2}{k}}\, J^a$$

$$\underbrace{J^a}_{\substack{\text{TOTAL}\\ \text{CURRENT}}} = \underbrace{j^a}_{\substack{\text{BOSONIC}\\ \text{CURRENT}}} - \underbrace{\tfrac{i}{2}\,\varepsilon^a{}_{bc}\,\psi^b\psi^c}_{\substack{\text{FERMIONIC}\\ \text{CURRENT}}}$$

$k \qquad k_B = k+2 \qquad k_F = -2$

$N=2$ SUPERSTRING ON $AdS_3 \times \mathcal{N}$

A.G., Rocek
Berenstein, Leigh

(1) $\mathcal{N} \supset U(1)_Y \qquad \mathcal{N} \simeq \frac{\mathcal{N}}{U(1)} \times U(1)_Y$

$$\chi^Y + \theta J^Y \qquad J^Y = i\,\partial Y$$

(2) $\mathcal{N}/U(1)$ HAS $N=2$

$$J_R^{\mathcal{N}/U(1)} = i\sqrt{\frac{c_{\mathcal{N}/U(1)}}{3}}\,\partial Z = i a\,\partial Z$$

$$a^2 = 3 - \frac{2}{k}$$

SPACETIME SUPERCHARGES

$$G_r^{\pm} \sim \oint e^{-\frac{\varphi}{2}} S_r^{\pm} \qquad r = \pm\tfrac{1}{2}$$

$$S_r^{\pm} = e^{-ir(H_1 \mp H_2) \mp \frac{i}{2}\left(aZ + \sqrt{\frac{2}{k}}Y\right)}$$

$$\partial H_1 = \psi^1\psi^2 \qquad i\partial H_2 = \psi^3\chi^Y$$

$$N=2 \begin{cases} \{G_r^+, G_s^-\} = 2L_{r+s} + (r-s)J_0 & r,s = \pm\tfrac{1}{2} \\ [L_m, L_n] = (m-n)L_{m+n} & m,n = 0,\pm 1 \\ [L_m, G_r^{\pm}] = \left(\frac{m}{2}-r\right)G_{m+r}^{\pm} \\ [J_0, G_r^{\pm}] = \pm G_r^{\pm} \end{cases}$$

$$J_0 = \oint J^R = \sqrt{2k}\oint J^Y$$

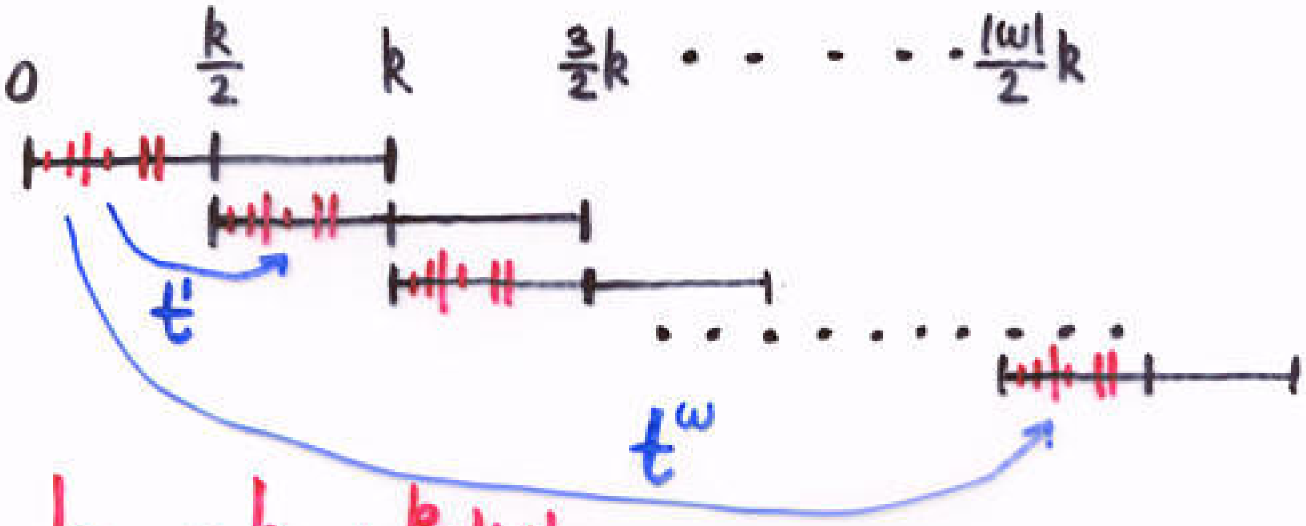
$$\mathcal{N} \simeq \frac{\mathcal{N}}{U(1)} \times U(1)_Y$$

THE "TWIST"

$$AdS_3 \times \mathcal{N}$$

$$\underbrace{\frac{SL(2)}{U(1)} \times U(1)_X}_{AdS_3} \times \underbrace{\frac{\mathcal{N}}{U(1)} \times U(1)_Y}_{\mathcal{N}}$$

$$t^{\omega}_{\pm} = e^{\omega \sqrt{\frac{k}{2}}(X \pm iY)} \cong e^{-\omega \int (J^3 \mp \frac{1}{2} J^R)}$$



$$h_{\omega} = h_0 + \frac{k}{2}|\omega|$$

THE SAME AS CHIRAL SPECTRUM OF

$$(\mathcal{M}_{6k})^p / S_p$$

IF $\quad MAX(\omega^{total}) = p$

CHIRAL SPECTRUM

THE UNTWISTED SECTOR:

(1) $\mathcal{X}_{-k(1-r_v)} = e^{-\varphi} e^{-i\sqrt{\frac{2}{k}}(j+1)Y} V \phi_{j=\frac{k}{2}(1-r_v)-1,m}$

$r_v + \frac{1}{k} < 1 \qquad \frac{1}{2} < h \leq \frac{k}{2}$

(2) $\mathcal{W}_{kr_v} = e^{-\varphi} e^{i\sqrt{\frac{2}{k}}jY} V(\psi\phi_{j=\frac{k}{2}r_v})_{j-1,m}$

$r_v + \frac{1}{k} < 1 \qquad 0 \leq h < \frac{k-1}{2}$

(3) $\mathcal{Y}_{1+k(r_v-1)} = e^{-\frac{\varphi}{2}} (S e^{i\sqrt{\frac{2}{k}}jY} V \phi_{j=\frac{k}{2}(r_v-1)})_{j-\frac{1}{2},m}$

$1 < r_v + \frac{1}{k} < 2 \qquad 0 < h < \frac{k}{2}$

V = CHIRAL OF $N/U(1)$: $\Delta_V = \frac{r_v}{2}$

THE TWISTED SECTOR:

(1) $t_+^{\omega}(\mathcal{X}) = \mathcal{X}^{\omega}$

$$h_\omega = \underbrace{\frac{k}{2}(1-r_v)}_{h_0} + \frac{k|\omega|}{2} = -\frac{R}{2}$$

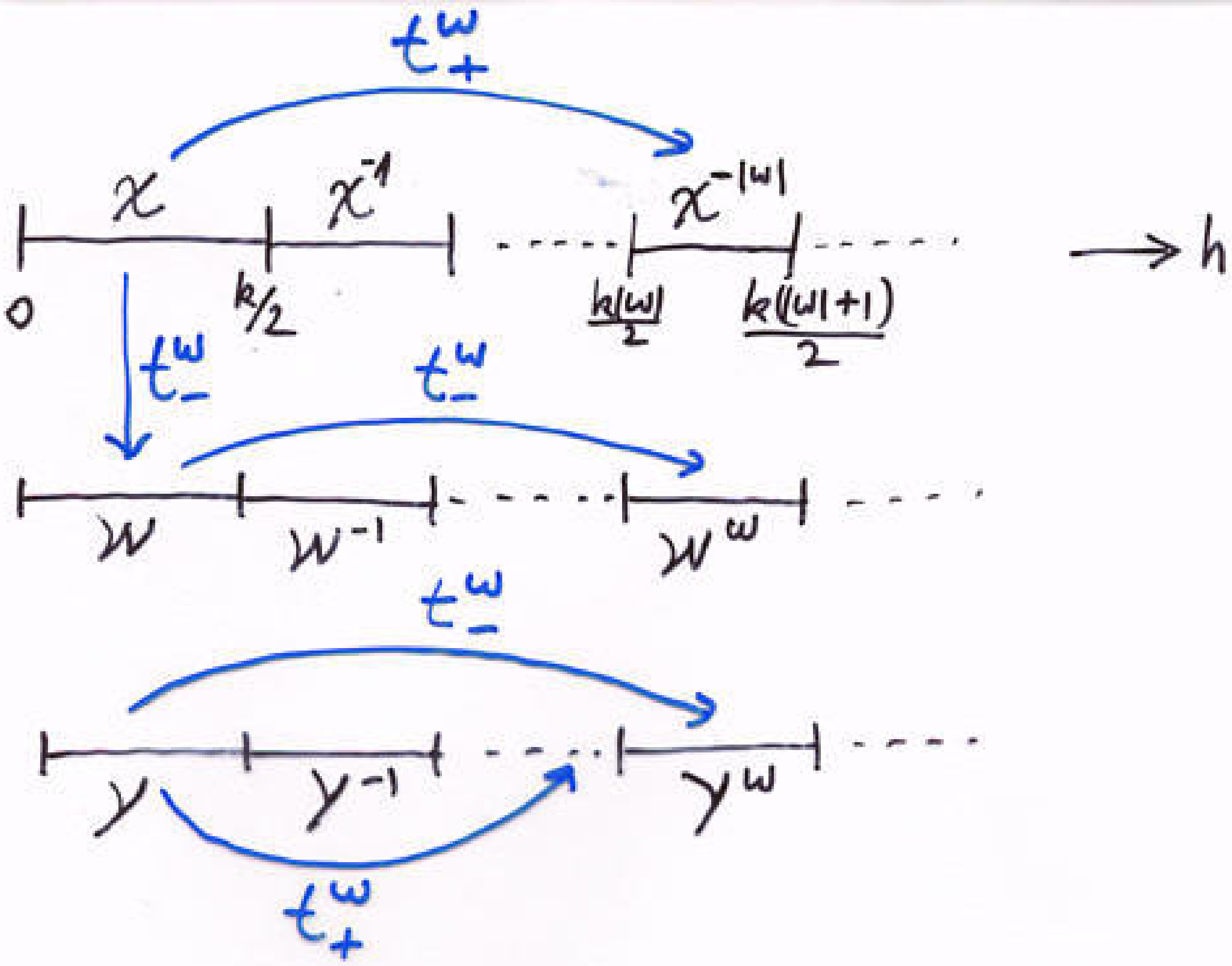
(2) $t_-^{\omega}(\mathcal{W}) = \mathcal{W}^{\omega}$

$$h_\omega = \underbrace{\frac{k}{2}r_v}_{h_0} + \frac{k|\omega|}{2} = \frac{R}{2}$$

(3) $t_-^{\omega}(\mathcal{Y}) = \mathcal{Y}^{\omega}$

$$h_\omega = \underbrace{\frac{1}{2} + \frac{k}{2}(r_v-1)}_{h_0} + \frac{k|\omega|}{2} = \frac{R}{2}$$

$$h_\omega = h_0 + \frac{k}{2}|\omega|$$



$\updownarrow$

THE SAME AS CHIRAL SPECTRUM

OF $(M_{6k})^P / S_P$

IF

$$|\omega| = 0, 1, 2, \ldots, P-1$$

AND

$$\mathrm{MAX}(\omega^{total}) = P$$

$C_{j,m}$

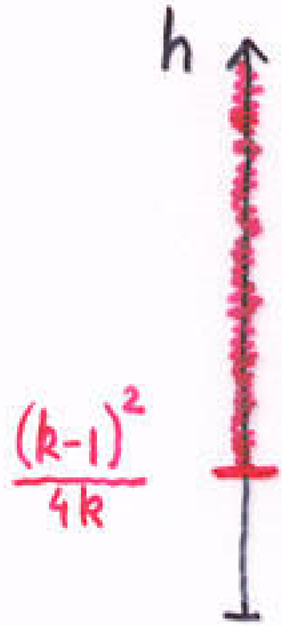
$$e^{-\varphi}\,\Phi^{\omega=-1}_{j,m}$$

$$j = -\frac{1}{2} + is \qquad s \in \mathbb{R}$$

$$m = \frac{k}{2} - \frac{(k-1)^2}{4k} - \frac{s^2}{k}$$

$$\Downarrow$$

$$h = \frac{(k-1)^2}{4k} + \frac{s^2}{k}$$



SINGLE LONG STRING

Seiberg, Witten
Maldacena, Ooguri