Instantons at large N

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($N=4$ SUSY)

- Systematics of large N amplitudes
  cf. Topological expansion in powers of $\frac{1}{N^2}$
- 'Test' AdS/CFT

Investigate correlation fns. of short multiplets

Role of fermionic partners of gauge orientations
$SU(2) \subset SU(N)$

→ Kaluza-Klein modes
Supercovariant completion
Higher order α' effects

M.B.G + Kovacs hep-th/010????

[c.f. Bianchi, MBG, Kovacs + Rossi hep-th/9807033

esp. Dorey, Hollowood, Khoze, Mattis, Vandoren hep-th/9901128]

Recall 'minimal' correlation fns:
(semiclassical $\lambda = g_{YM}^2 N \to 0$)

– Saturated by 16 superconformal fermion modes η^A_α $\bar{\xi}^{A\dot{\alpha}}$

Broken SUSY + Conformal SUSY

$$\sim N^{\frac{1}{2}} c_k e^{2\pi i k \tau} \left(1 + O(g_{YM}^2)\right)$$

$\frac{1}{\alpha'}$ $\qquad$ $\sum_{m|k} \frac{1}{m^2}$ $\qquad$ $\theta + \frac{4\pi i}{g_{YM}^2}$

– Matches D-instanton in $AdS_5 \times S^5$

WHY? – leading term indep of λ.

en. mom.

e.g. $\langle T(x_1) \ldots T(x_4) \rangle_k \longleftrightarrow \frac{c_k}{\alpha'} \int \sqrt{g}\, e^{2\pi i k \tau} R^4$

$\mathrm{Tr}(F^- \lambda)$ $\qquad$ $\langle \hat{\Lambda}(x_1) \ldots \hat{\Lambda}(x_4) \rangle_k \longleftrightarrow \frac{c_k}{\alpha'} \int \sqrt{g}\, e^{2\pi i k \tau} \Lambda^{16}$

dilatino

$\vdots \qquad\qquad \vdots$

Let $k=1$

Bosonic moduli $\underline{SU(2) \subset SU(N)}$

Position x_0^μ

Gauge orientations $w_u^{\dot\alpha}$ $\bar w_u^{\dot\alpha}$ $u=1,2,..N$

$$4N = 4 + 2N + 2N - 4$$

constraints

$$\bar w_u^{\dot\alpha} w_u = \rho_0^2 \quad \text{scale}$$

Fermionic moduli

16 Broken SUSY's η_α^A $\bar\xi^{A\dot\alpha}$

$\underline{\nu}_u^A$ $\underline{\bar\nu}_u^A$ $A=1,2,3,4$ $SU(4)$

$$8N = 8 + 8 + 4(N-2) + 4(N-2)$$

NOTE: $\bar\nu_u^A \nu_u^B \equiv \left(\bar\nu_u^{[A} \nu_u^{B]}\right)_{\underline{6}} + \left(\bar\nu_u^{(A} \nu_u^{B)}\right)_{\underline{10}}$

- Integrate out $\nu, \bar\nu$ $\quad i=1,\dots,6$

$$\left\langle \left(\bar\nu \nu\right)_{\underline{6}} \right\rangle \underset{N\to\infty}{\sim} \left(g_{YM}^2 N\right)^{\frac{1}{2}} \rho_0 \, \underline{\Omega}^i$$

$\Omega^2 = 1$ $\quad$ 5-Sphere

$$\Rightarrow \int \frac{d^4x_0 \, d\rho_0}{\rho_0^5} \, d\underline{\Omega} \, d^8\eta \, d^8\bar\xi$$

Measure $\quad AdS_5 \times S^5$ $\quad$ (Dorey et al)

Correlation fns.

$$\langle O_1(x_1)\cdots O_M(x_M)\rangle_1 \underset{g\to 0}{=} \int d\mu\, \hat{O}_1(x_1)\cdots\hat{O}_M(x_M)\, e^{-S_N}$$

gauge-inv. composite op.

Use classical instanton soln. $F^- = \hat{F}^-(x-x_0;\rho_0)$

and solns. of $\not{D}\lambda = 0$ and $D^2\phi^{AB} = i[\lambda^A,\lambda^B]$

in terms of $\eta, \bar{\xi}, \nu, \bar{\nu}$

→ Classical profiles

Superconformal current multiplet

$\mathrm{Tr}(F^{-2})$ $\quad C = (\rho_0 f)^4 \equiv \dfrac{\rho_0^4}{[(x-x_0)^2+\rho_0^2]^4} \equiv K_4$

$\mathrm{Tr}(F^-\lambda)$ $\quad \hat{\Lambda} = (\rho_0 f)^4 \zeta$

$\mathrm{Tr}(\lambda^A\lambda^B)$ $\quad \mathcal{E} = (\rho_0 f)^4 \zeta\zeta + \rho_0^2 f^3 (\bar{\nu}\nu)_{\underline{10}}$

$\vdots$

$\mathrm{Tr}(\lambda\phi)$ $\quad \chi = (\rho_0 f)^4 \zeta\zeta\zeta + \rho_0^2 f^3 (\bar{\nu}\nu)_{\underline{10}}\zeta$

$\mathrm{Tr}(\phi^i\phi^j)$ $\quad O_2 = (\rho_0 f)^4 \zeta\zeta\zeta\zeta + \rho_0^2 f^3 (\bar{\nu}\nu)_{\underline{10}}\zeta\zeta + f^2 (\bar{\nu}\nu)_{\underline{10}}(\bar{\nu}\nu)_{\underline{10}}$

$\vdots$

$$\zeta^A_\alpha = \eta^A_\alpha + \sigma_{\alpha\dot{\beta}}\cdot(x-x_0)\bar{\xi}^{A\dot{\beta}}$$

BROKEN SUSY

Note structure of $(\bar{\nu}\nu)_{10}$ factors

Substitute profiles into corr. fns and integrate over moduli

- Must soak up 16 ζ's e.g. $\langle \Lambda(x_1) \ldots \Lambda(x_{16}) \rangle$

- Attaching powers of $\phi^i \sim \left(\bar{\nu}^{[A}\nu^{B]}\right)_6$

  → Kaluza-Klein excitations

  $(\bar{\nu}\nu)_6 \sim (g_{YM}^2 N)^{1/2} \rho_0 \Omega^i$ KK vector S^5

- $\int (\bar{\nu}\nu)_{\underline{10}} (\bar{\nu}\nu)_{\underline{10}} (\bar{\nu}\nu)_{\underline{10}} (\bar{\nu}\nu)_{\underline{10}} (- - - - -)$

  $10 \times 10 \times 10 \times 10 = 1 + \ldots$

  $\sim g_{YM}^4 \left(a + \frac{b}{N} + \ldots \right)$

Produces exact agreement with a variety of AdS/CFT expectations

- Kaluza-Klein excitations in AdS/CFT

e.g. $\langle O_4(x_1)\, O_4(x_2)\, O_2(x_3)\, O_2(x_4)\rangle \longleftrightarrow \frac{1}{\alpha'} R^4$

- Supercovariant completion

e.g. $\langle \Lambda(x_1) \cdots \Lambda(x_{14})\, \chi(x_{15})\, \chi(x_{16})\rangle \longleftrightarrow \frac{1}{\alpha'} \Psi \gamma \bar{\Psi}\, \Lambda^{14}$

Couples to gravitino

- Higher derivative terms ($\frac{1}{N}$ corrections)

e.g. $\langle T(x_1) \cdots T(x_4)\, \mathcal{E}(x_5) \cdots \mathcal{E}(x_8)\rangle \longleftrightarrow \alpha' G^4 R^4$

couples to $G = dB$

Requires use of several technical tricks.

- Systematic expansion in $\frac{1}{N}$ (in progress!) arbitrary k