NONCOMMUTATIVE GAUGE THEORY

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Mumbai
STRINGS 2001

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Solitons in NCGT

FLUXONS
U(1)
U(2) MONOPOLES

N. NEKRASOV & D.G.

hepth 0005204, 0007204, 0010090

Observables in NCGT

GAUGE INVARIANT
'LOCAL' OPERATORS

DG, A. HASHIMOTO, N. ITZHAKI

hepth 0008075.

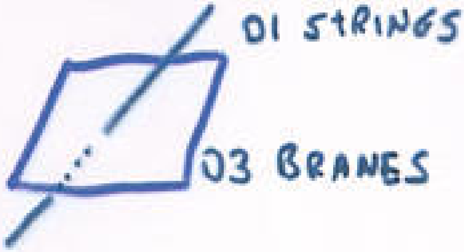
OUTLINE

BUT

- INTRODUCTION TO NCGT
- COMPLETE ANALYSIS OF CODIMENSION 2 SOLITONS

} Gopakumar, Minwalla

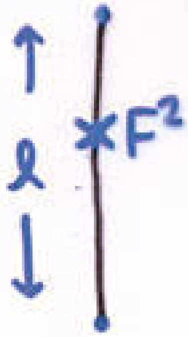
FLUXONS, VORTICES



Harvey

- GAUGE INVARIANT OBSERVABLES
  Straight Wilson lines

Ooguri



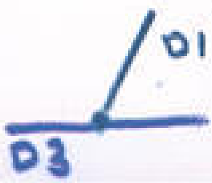
~ DUAL TO BULK FIELDS IN AdS

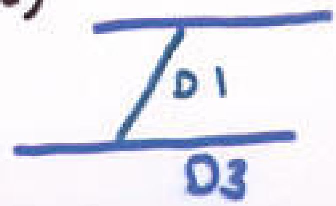
Das

CARRY MOMENTUM

$\theta \ell$

- NON COMMUTATIVE $U(1) + U(2)$ MONOPOLES





AN EXACT PREDICTION OF $\mathcal{N}=4$ SUSYM THEORY & COMPARISON WITH STRING THEORY

STRINGS 2001
MUMBAI

NADAV DRUKKER + D.G.

hepth 0010274

AN EXACT PREDICTION OF $\mathcal{N}=4$ SUSYM Theory

+ Comparison with II_B STRING THEORY in $AdS_5 \times S_5$

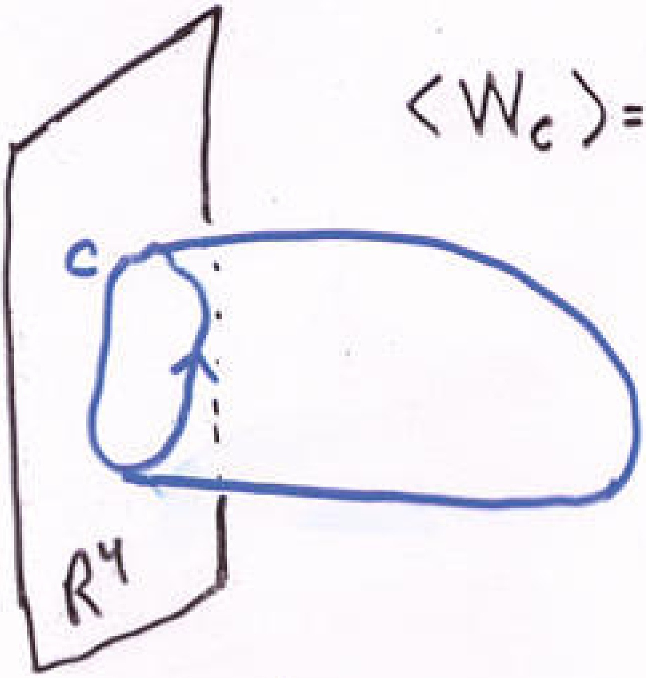
—

CIRCULAR BPS WILSON LOOP IN $\mathcal{N}=4$ SUSYM CAN BE CALCULATED EXACTLY

$$W_0(\lambda = g^2 N, N) = \frac{1}{N} L^1_{N-1}\left(\frac{-\lambda}{4N}\right) e^{\lambda/8N}$$

AGREES WITH STRING THEORY IN $AdS_5 \times S_5$ FOR $\lambda >> 1$ (SMALL α') TO ALL ORDERS IN $\frac{1}{N^2}$.

WILSON LOOPS + AdS/CFT.



$$\langle W_C \rangle = \frac{1}{N} \left\langle \mathrm{Tr}\, P\, e^{i\oint_C (A\cdot\dot{x} + i\phi_i \theta^i |\dot{x}|)\,dt} \right\rangle$$

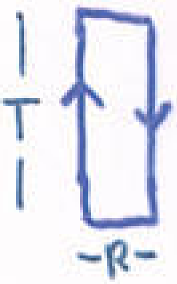
SUSY (BPS) LOOP

MALDACENA
Rey
DRUKKER, D.G., OOGURI

$$W_C \sim e^{-\sqrt{\lambda}\left(A^{\text{MIN}}_{\text{rea}} - \text{BOUNDARY TERM}\right)}$$

$\lambda = g_s N \gg 1$
$N \gg 1$

$$\equiv e^{-\sqrt{\lambda}\,\tilde{A}}$$



$$\tilde{A} = -\frac{4\pi\sqrt{2}}{\Gamma^4(\frac{1}{4})}\,\frac{T}{R}$$

$$\tilde{A} = -1$$

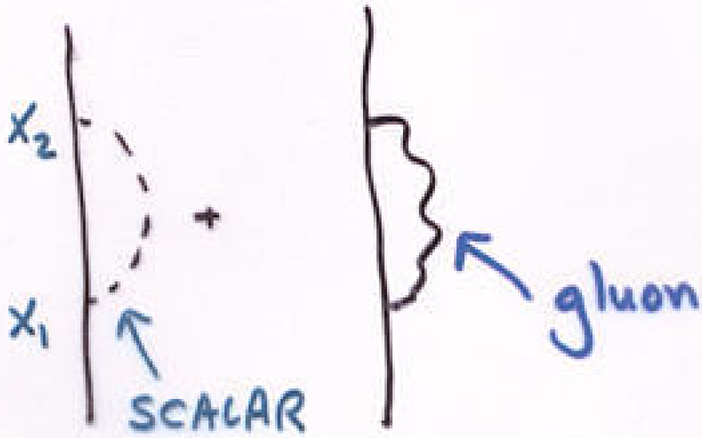
$$W_O \sim e^{\sqrt{\lambda}}$$

$\lambda \gg 1$
$N = \infty$

STRAIGHT LINE $\qquad W(\uparrow) = 1$

Since: $W_{\uparrow \updownarrow T} \simeq e^{-T\delta m_{BPS}=0} = 1$

Indeed:



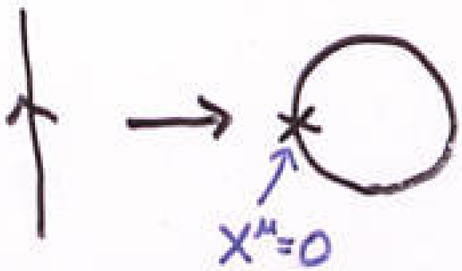
$$i^2 \int dt_1\, dt_2 \, \frac{|\dot{x}_1||\dot{x}_2|}{(x_1-x_2)^2} \qquad \int dt_1\, dt_2 \, \frac{\dot{x}_1 \cdot \dot{x}_2}{(x_1-x_2)^2}$$

CANCEL

SINCE $\quad -|\dot{x}_1||\dot{x}_2| + \dot{x}_1 \cdot \dot{x}_2 = 0$

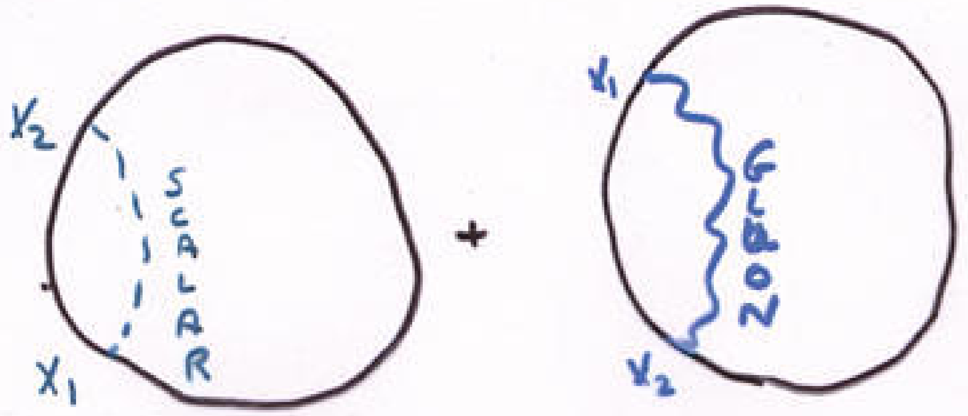
BUT $\mathcal{N}=4$ SUSYM CONF. INVARIANT.

INVERSION: $X^\mu \rightarrow \dfrac{X^\mu}{X^2}$



$\therefore\ W_{\bigcirc} = 1 \qquad$ WRONG!

CIRCLE



$$\frac{\dot{x}_1\cdot\dot{x}_2 - |\dot{x}_1||\dot{x}_2|}{(x_1-x_2)^2} = -\frac{1}{2}$$

$$\therefore \quad \langle W_0 \rangle = 1 + \frac{\lambda}{16\pi^2}(2\pi)^2\cdot\frac{1}{2} + \cdots = 1 + \frac{\lambda}{8} + \cdots$$

$\sum$  $= \sum \frac{(\lambda/4)^n}{n!(n+1)!} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$

RAINBOW GRAPHS

$$\sim e^{\sqrt{\lambda}} \checkmark$$

Erikson, Semenoff, Zarembo.

why is circle non trivial?

why do rainbow graphs agree for $\lambda \gg 1$, $N = \infty$?

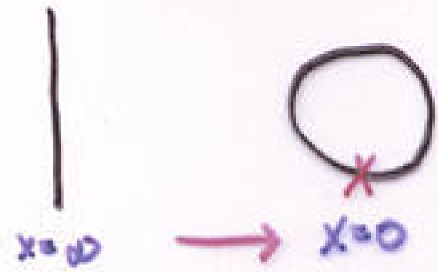
CONFORMAL ANOMALY

$\mathcal{N}=4$ SUSYM is CONFORMALLY INVARIANT

OKAY for 'small' conformal transformations $\infty \to \infty$

OKAY on S^4

PROBLEMATIC FOR 'large' TRANSFORMATIONS ON R^4 that take $\infty \to 0$, LIKE $x^\mu \to \frac{x^\mu}{x^2}$

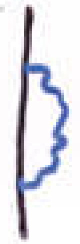


SUGGESTS THAT THE DIFFERENCE BETWEEN CIRCLE AND LINE GIVEN BY FIELDS AT 1 POINT

$\Rightarrow$ MATRIX MODEL

PERTURBATIVE ANALYSIS

$$X^\mu \to \frac{X^\mu}{X^2} = \tilde{X}^\mu$$

COMPARE  AND  SCALAR + GLUON PROPAGATORS

SCALAR PROPAGATOR: $\tilde{\phi}(\tilde{x}) = x^2 \phi(x)$

$$G(x,y) = \langle \phi(x)\phi(y) \rangle = \frac{1}{(x-y)^2} \to \tilde{G}(\tilde{x},\tilde{y}) = \frac{x^2 \cdot y^2}{(x-y)^2}$$

$$|\dot{x}| \to \frac{|\dot{x}|}{x^2}$$

$$\therefore \int dt \int ds \, |\dot{x}(s)| \, |\dot{y}(t)| \langle \phi(x)\phi(y) \rangle$$

INVARIANT UNDER $x \to \frac{1}{x}$

GLUON PROPAGATOR

$$\tilde{V}_\mu(x) = x^2 I_{\mu\nu}(x) V_\nu(x)$$

$$I_{\mu\nu} = g_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}$$

$$G_{\mu\nu}(x,y) = \frac{g_{\mu\nu}}{(x-y)^2} \to x^2 y^2 I_{\mu\rho}(x) I_{\nu\sigma}(y) G_{\rho\sigma}(x,y) = \tilde{G}_{\mu\nu}(\tilde{x},\tilde{y})$$

$$\tilde{G}_{\mu\nu}(\tilde{x},\tilde{y}) = x^2 y^2 \left\{ \frac{g_{\mu\nu}}{(x-y)^2} + \frac{1}{2}\left[\partial^x_\mu \left(\ln\frac{(x-y)^2}{|x|} \partial^y_\nu \ln|y|^2 \right) \right] + \begin{smallmatrix} \mu \leftrightarrow \nu \\ x \leftrightarrow y \end{smallmatrix} \right\}$$

FEYNMAN GAUGE

1 LOOP

$$W_{1}^{(1)} - W_{0}^{(1)} = -\frac{1}{16\pi^2}\oint dx^{\mu}\oint dy^{\nu}\,\partial_x^{\mu}\left[\ln\frac{(x-y)^2}{|x|}\,\partial_y^{\nu}\ln|y|^2\right]$$

$$\overset{\shortparallel}{0}$$

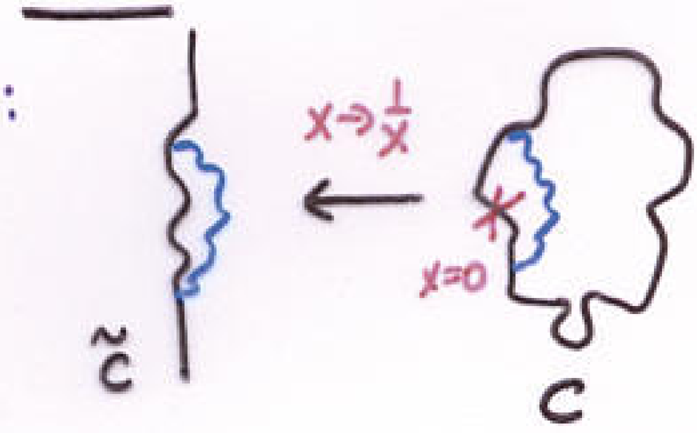
TOTAL DERIVITIVE

$\approx$ GAUGE TERM

WOULD $=0$ EXCEPT FOR SINGULARITIES AT $x=y=0$

$$W_1^{(1)} - W_0^{(1)} = -\frac{1}{8} \Rightarrow W_0^{(1)} = \frac{1}{8}$$

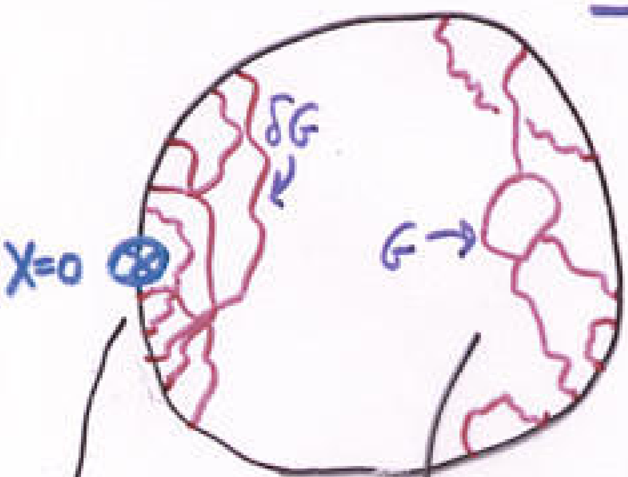
MORE GENERALLY:



$$W_{\tilde{C}}^{(1)} - W_C^{(1)} = -\frac{1}{8}$$

SINCE THE ONLY CONTRIBUTION COMES WHEN ALL POINTS ARE AT $X^{\mu}=0$ ← INVERSION POINT.

CIRCLE TO ALL ORDERS



- δG's CONTRIBUTE ONLY when all points $\to 0$

$\equiv$ EQUIVALENT TO 0-DIM FIELD THEORY

- MATRIX MODEL.

CANCEL

Appears that ONLY NON-INTERACTING GRAPHS SURVIVE

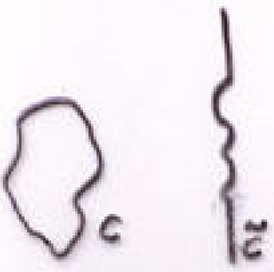
- g^4 terms explicity cancel

- Result agrees with string  in A&S.

IF SO: $$\langle W_0 \rangle \equiv F(\lambda, N) = \frac{1}{Z} \int \mathcal{D}M \, \frac{1}{N} \mathrm{Tr}\, e^{M} e^{-\frac{2N}{\lambda} \mathrm{tr} M^2}$$

NxN Hermitean matrix

C=-2



$$\langle W_C \rangle = F(\lambda, N) \langle W_{\tilde{C}} \rangle$$

CIRCLE

$$W_0 = \frac{1}{Z}\int \mathcal{D}M \, \frac{1}{N} \mathrm{tr}\, e^{M} \, e^{-\frac{2N}{\lambda}\mathrm{tr} M^2}$$

CALCULABLE (othogonal poly).

$$W_0 = \frac{1}{N} L^{1}_{N-1}\left(-\frac{\lambda}{4N}\right) e^{\frac{\lambda}{8N}}$$

$\frac{1}{N^2}$ Expansion $$= \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{\lambda}{48N^2} I_2(\sqrt{\lambda})$$

$$+ \frac{\lambda^2}{1280 N^4} I_4(\sqrt{\lambda}) + \frac{\lambda^{5/2}}{9216 N^4} I_5(\sqrt{\lambda}) + \cdots$$

large λ $$= \sum_p \frac{1}{N^{2p}} e^{\sqrt{\lambda}} \frac{\lambda^{\frac{6p-3}{4}}}{96^p \, p!} \left[1 - \frac{3(12p^2+8p+5)}{40\sqrt{\lambda}} + \cdots\right]$$

Solution of Matrix Model

$$\left\langle \frac{1}{N}\operatorname{tr} e^{M} \right\rangle = \frac{1}{Z}\int \prod dm_i \, \Delta^2(m_i)\, \frac{1}{N}\sum e^{m_i}\, e^{-\frac{2N}{\lambda}\sum_i m_i^2}$$

Eigenvalues

$\Delta = \prod(m_i - m_j) = \det[m_i^{\,j-i}]$

TRICK: REPLACE $m_i^{\,j-1} \rightarrow P_{j-1}(m_i) = m_i^{\,j-1} + \dots$

Choose polynomials to be orthonormal w.r.t. measure $\int dm\, e^{-\frac{2N}{\lambda}m^2}$.

$\Rightarrow$ HERMITE POLYNOMIALS

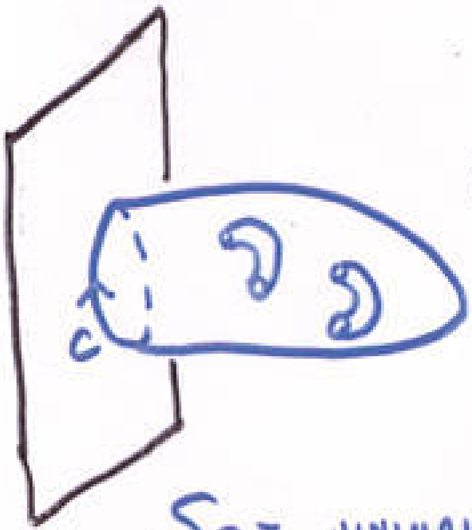
$$P_N(x) = \frac{H_N(x)}{\sqrt{2^N N! \sqrt{\pi}}}$$

$$\left\langle \frac{1}{N}\operatorname{tr} e^{M} \right\rangle = \frac{1}{N}\int dm \sum_{j=0}^{N-1} (P_j(m))^2 \exp\left[-m^2 + \sqrt{\frac{\lambda}{2N}}\, m\right]$$

$$= \frac{1}{N}\sum_{j=0}^{N-1} L_j\left(-\frac{\lambda}{4N}\right) e^{\lambda/8N}$$

$$= \frac{1}{N} L^{1}_{N-1}\left(-\frac{\lambda}{4N}\right) e^{\lambda/8N}$$

COMPARISON WITH STRING THEORY.



$$\langle W_c \rangle = \sum \frac{1}{N^{2p}} e^{-S_p'} f_p(\lambda)$$

p = # of handles

$\frac{1}{\sqrt{\lambda}} \sim \alpha'$ expansion of fluctuations about minimal surface

S_p = MINIMAL AREA OF SURFACE ENDING ON CIRCLE with p handles – bd. term.

$$S_p' \sim S_{p=0}' = -\sqrt{\lambda}$$

Degenerate handles. ✓

↓

$2p$ (+3) constraints on moduli space $\rightarrow \left(\frac{1}{\lambda^{1/4}}\right)^{2p+3}$

$$g_s^2 = \frac{\lambda^2}{N^2}$$

$$\left(\frac{\lambda^2}{N^2}\right)^p \left(\frac{1}{\lambda^{1/4}}\right)^{2p+3} = \frac{\lambda^{\frac{6p-3}{4}}}{N^{2p}}$$

$$\langle W \rangle^{\text{STRING}} = \sum_p \frac{1}{N^{2p}} \frac{1}{p!} \lambda^{\frac{6p-3}{4}} e^{\sqrt{\lambda}} \,\text{Const.} \left(1 + O\left(\frac{1}{\sqrt{\lambda}}\right)\right)$$

$$\langle W \rangle^{\text{YM}} = \sum_p \frac{1}{N^{2p}} \frac{1}{p!} \lambda^{\frac{6p-3}{4}} e^{\sqrt{\lambda}} \sqrt{\frac{2}{\pi}} \frac{1}{96^p} \left(1 + O\left(\frac{1}{\sqrt{\lambda}}\right)\right).$$

S DUALITY

$$\langle W \rangle^{YM} \sim e^{\sqrt{\lambda} + \frac{\lambda^{3/2}}{96 N^2}} \sim e^{\sqrt{\lambda}}$$

for $N^2 \gg \lambda \gg 1$

Agrees with String in AdS

FOR $N \gg \lambda \gg 1$ $\quad g_s = \frac{\lambda}{N} \ll 1$

WHAT ABOUT: $\quad N^2 \gg \lambda \gg N \gg 1$

PERFORM S-DUALITY TRANSFORMATION ← $\quad g_s \gg 1 \quad g_{YM} \gg 1$

matrix model $\sim \langle W_{\text{t'HOOFT LOOP}} \rangle_{\tilde{g} = \frac{1}{g}} = e^{\sqrt{\lambda}}$

$\tilde{g}_s = \frac{1}{g_s} \quad \tilde{\lambda} = \frac{\lambda}{g_s^2}$

prediction

AdS check:

$$\langle W^{\text{DUAL STRING}} \rangle = e^{\sqrt{\tilde{\lambda}}/\tilde{g}_s} = e^{\sqrt{\lambda}} \checkmark$$

Test of

- AdS/CFT (AND OF QUADRATIC MATRIX MODEL)
- Test of S-duality in $\mathcal{N}=4$ SUSYM

REMARKS

- ANALYSIS CAN BE EXTENDED TO MUTIPLY WOUND LOOPS.

$$= \langle \mathrm{tr}\, e^{n_1 M}\, \mathrm{tr}\, e^{n_2 M} \cdots \mathrm{tr}\, e^{n_k M} \rangle$$

- NEED RIGOROUS PROOF OF QUADRATIC MATRIX MODEL for $\mathcal{N}=4$ SUSYM
- For $\mathcal{N}=2,1$ conformal YM we might expect more complicated matrix models?
- CAN ONE DERIVE the MATRIX MODEL ($c=-2$) directly from II_B String theory on $AdS_5 \times S_5$?