Phase separation

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Part I: black hole instabilities in AdS

- Gregory-Laflamme transition revisited
- Spinning branes can be thermodynamically unstable
- Numerical demonstrate ∃ an unstable mode leading to clumping of R-charge — "phase separation"

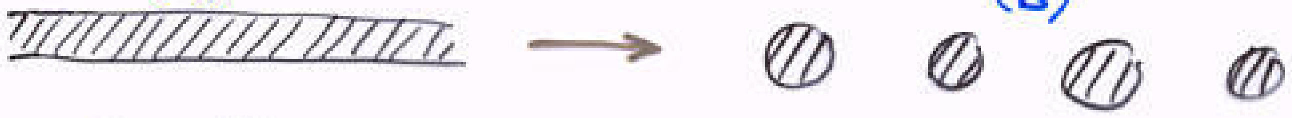
Part II: quantum Hall soliton

- Briefly recapitulate Bernevig-Brodie-Susskind-Toumbas
- inter-"electron" repulsion
- Try to qualitatively describe dynamics: ∃ local stability against clumping, but just barely!

Part I: work with I. Mitra

Gregory & Laflamme (93-94) demonstrated that non-extremal black branes are often unstable:

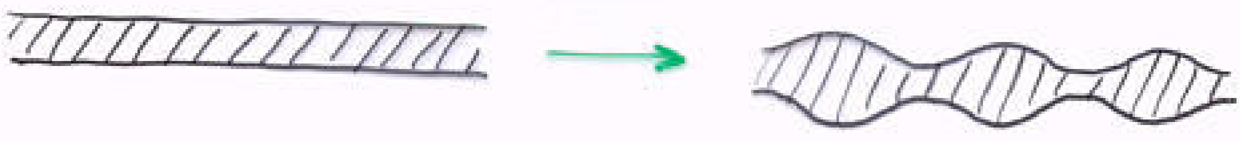
(A) (B)



uniform black string array of black holes

① The simple explanation was a scaling argument demonstrating that (B) has more entropy.

② The actual (semi-numerical) computation was to prove (A) is unstable toward small fluctuations:



We may ask, how closely are ① and ② related?

① indicates a global instability which would generically be expected to proceed through a tunneling mechanism, (A) → (B).

② indicates a local instability proceeding in Lorentzian time through the classical dynamics.

Conjecture: a dynamical instability like ② arises precisely when there is a local thermodynamic instability.

What I mean by thermodynamic instability is that given $S = S(E, Q_1, \ldots, Q_n)$ where Q_A are the conserved, extensive charges,

$$H^{S}_{E,Q_A} \equiv \begin{pmatrix} \frac{\partial^2 S}{\partial E^2} & \frac{\partial^2 S}{\partial Q_A \partial E} \\ \frac{\partial^2 S}{\partial E \partial Q_B} & \frac{\partial^2 S}{\partial Q_A \partial Q_B} \end{pmatrix} \leq 0$$

is the condition for thermodynamic <u>stability</u>

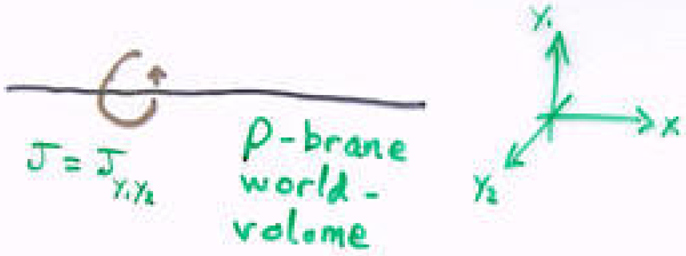
If H^{S}_{E,Q_A} has a positive eigenvector, $\vec{v}$, then a small variation

$$\left(\frac{E}{V}, \frac{Q_A}{V}\right) \longrightarrow \left(\frac{E}{V}, \frac{Q_A}{V}\right) + \lambda(x)\,\vec{v}$$

where $\int d^px\, \lambda(x) = 0$, will raise S.

Gregory-Laflamme's explicit examples happen to be always thermodynamically unstable in the local sense explained here.

To test the conjecture, examine near-extremal "spinning" D3, M2, M5-branes.



J is a globally conserved R-charge in p-brane world-volume theory.

"spin" is rotation that leaves world-volume stationary.

Classical solns are known [Cvetic-Youm & others],
thermodynamics is well-explored [SSG, SSG-Cvetic, Cai-Soh, SSG-Mitra, Chamblin-Myers-Johnson-Emparan, & others]

In particular, $H^S_{E,Q_A} \lesssim 0$ only if

$$\star \quad |\Omega| < c_1 T \quad \text{for some } c_1 \sim O(1)$$

angular velocity, conjugate to J

temperature

Suggestion: transverse scalars $X^1 \pm i X^2$ carry the $U(1)$ R-charge under consideration. If X^1, X^2 pick up $m \sim T$, then $\star$ becomes condition to avoid Bose-condensing.

Now, is the thermodynamic tendency toward Bose condensation / phase separation / clumping visible in the classical black brane dynamics?

→ Yes for the following particular case:

Spinning M2's with
$J_{12} = J_{34} = J_{56} = J_{78}$

angular momentum $\xrightarrow{\text{KK}}$ electric charge in gSUGRA

$\downarrow$ near-horizon

$$(AdS_4 - RN) \times S^7 \xrightarrow{\text{KK}} AdS_4 - RN \text{ with scalars + gauge fields}$$

Thermodynamics suggests a mode with
$\delta J_{12} = \delta J_{34} = -\delta J_{56} = -\delta J_{78}$ is unstable.

Relevant part of $\mathcal{N}=8$ gSUGRA Lagrangian is

$$\mathcal{L} = R - \frac{1}{2}(\partial\varphi)^2 + \frac{2}{L^2}(2 + \cosh\varphi) - 4e^{\varphi}(F^{(1)}_{\mu\nu})^2 - 4e^{-\varphi}(F^{(2)}_{\mu\nu})^2$$

φ controls shape of S^7; $J_{12} = J_{34} \rightarrow F^{(1)}_{\mu\nu}$
$J_{56} = J_{78} \rightarrow F^{(2)}_{\mu\nu}$

AdS$_4$-RN: $ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$

$$F^{(1)}_{0r} = F^{(2)}_{0r} = \frac{Q}{\sqrt{8}\,r^2} \qquad f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{L^2}$$

Perturbation: $F^{(A)} = F^{(A)}_{\text{background}} \pm \delta F$

$$d(\delta F) = 0 \qquad d * \delta F + d(\delta\varphi) \wedge * F = 0$$

$$\left[\Box + \frac{2}{L^2} - \delta F^2\right]\delta\varphi - 16\, F^{\mu\nu}\delta F_{\mu\nu} = 0$$

Spinorial / dyadic indices facilitate computation, which comes down to numerics on a 4th order ODE... $\exists$ an instability!

Smooth horizon, no ergosphere, even works in global AdS$_4$ provided horizon size $\gg L$.

Thus no-hair theorems are violated in an interesting way in AdS.

Compare two notions of stability:

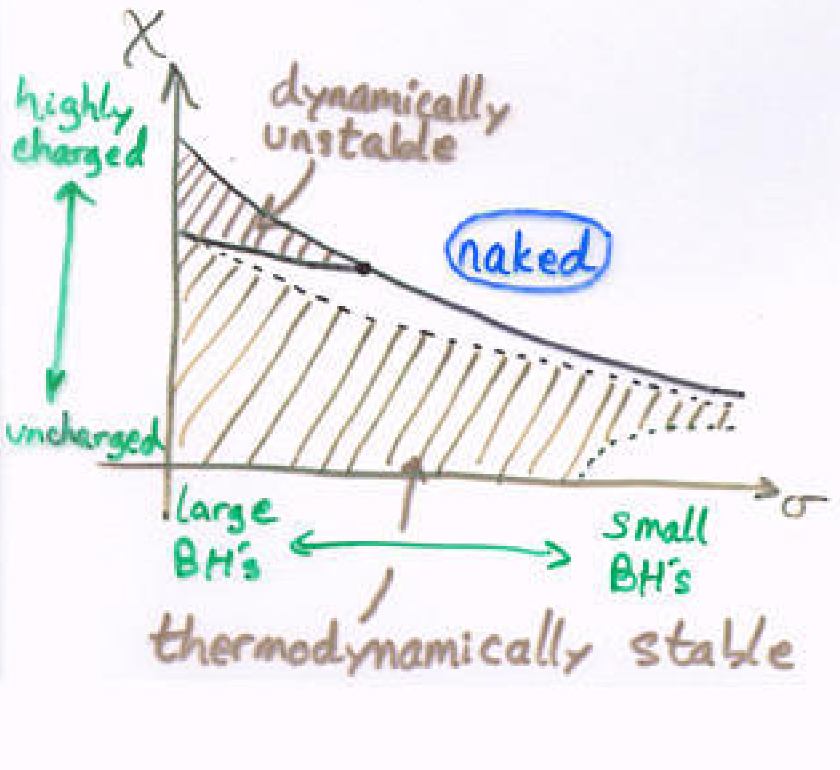
(1) dynamical (Lorentzian time)

(2) static (thermodynamics, Euclidean time)

→ For branes of infinite extent, I claim

(1) $\Leftrightarrow$ (2)

→ For finite size BH's, finite volume effects can get in the way; and for BH's in flat space, (1) and (2) are uncorrelated.



$$\chi = \frac{Q}{M^{2/3} L^{4/3}}$$

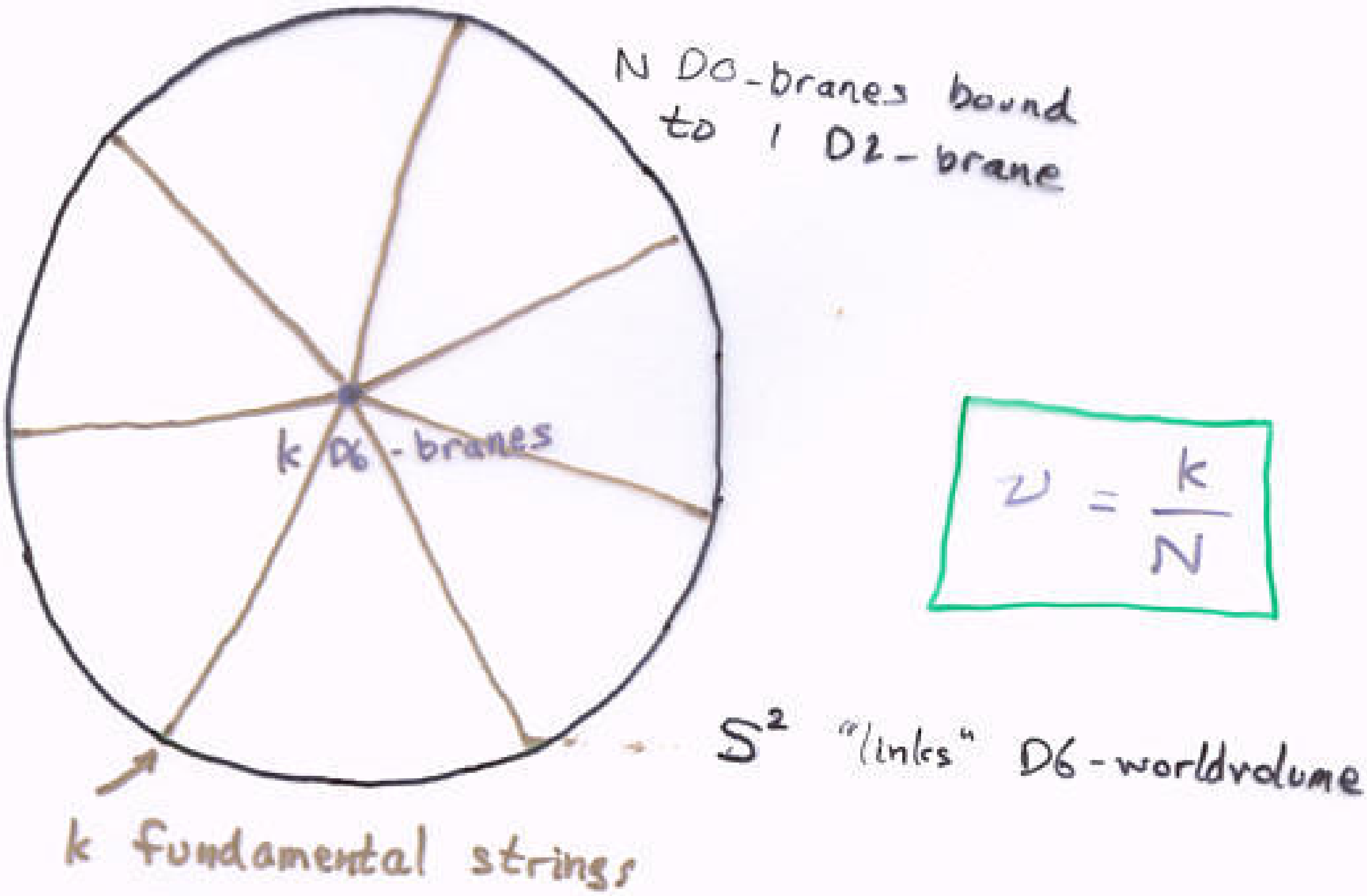
$$\sigma = \frac{L^{4/3}}{M^{2/3}}$$

$$G_4 = 1$$

Part II: work with M. Rangamani

Bernevig, Brodie, Susskind, & Toumbas introduced a D-brane configuration which is claimed to exhibit fractional quantum Hall behavior.

A sketch of the setup:



$$\boxed{\nu = \frac{k}{N}}$$

Points of interest:

- D2-D0 system has a locally stable equilibrium radius.
- D0-brane <u>density</u> on D2 stays ~fixed as N or k is varied.
- SUSY is broken on D2-worldvolume on a scale comparable to circumference of S^2.
- Inclusion of strings is necessitated by Hanany-Witten effect.
- These strings are fermions in their ground state, but since it seems natural to tie them up with $\epsilon_{i_1 \cdots i_k}$ on D6, their ends on D2-brane are probably bosons.
- String ends $\longleftrightarrow$ "electrons"

  D0-brane density $\longleftrightarrow$ magnetic field

Questions:

- Does this system have a ground state well-approximated by a Laughlin wave-function?

$$\Psi_{1/m} = \prod_{i<j} (z_i - z_j)^m \, e^{-\sum_{k=1}^{N} |z_k|^2/4\ell^2}$$

$\nu = \frac{1}{m}$

m even for bosons...

- Do we expect quantum Hall phenomenology, eg quantized transverse conductance?

- Do we expect inhomogeneous phases, eg Wigner crystal or stripe phases?

We have to understand inter-electron forces to address these questions.

Effective action on D2-brane:

$$S_{eff} = \int d^3\xi \sqrt{g} \left[J^0 A_0 - \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} m_\gamma \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + \frac{1}{2}(\partial\phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 + \text{fermions} + \text{interactions} \right]$$

$$+ \sum_{i=1}^{K} q \int_{\gamma_i} (A + \phi \, ds)$$

$$m_\gamma R = \frac{3}{4\sqrt{2}} \qquad m_\phi R = 4 \qquad R = \frac{3}{\sqrt{2}} (\pi N)^{1/3} \ell_s$$

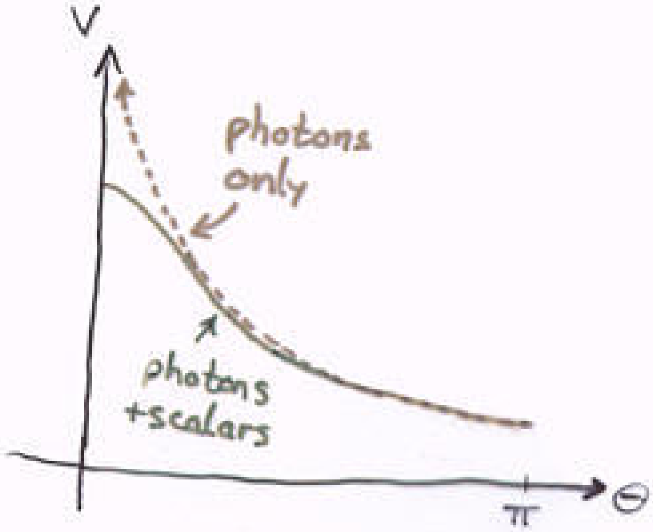
$$ds^2_{open} = \frac{(\pi N)^{1/3}}{\sqrt{2}} \left(d\tau^2 - R^2 d\Omega_2^2 \right) = \frac{(\pi N)^{1/3}}{\sqrt{2}} g_{\mu\nu} d\xi^\mu d\xi^\nu$$

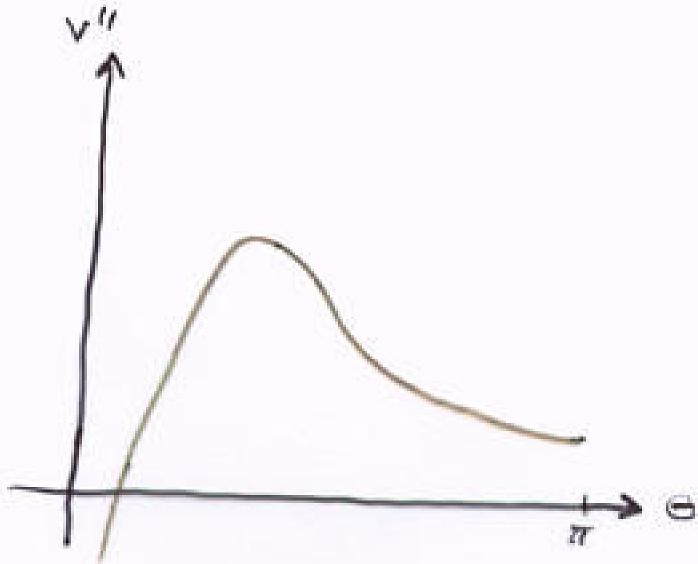
$$g_{YM}^2 \ell_s = 2^{1/4} \sqrt{\pi} / \nu \sqrt{N} \qquad q^2 \ell_s = \frac{1}{2\nu} \left(\frac{\pi}{N} \right)^{1/3}$$

weak coupling as $N \to \infty$

To obtain inter-electron force, integrate out A and ϕ. Closed string exchange is negligible by comparison — this is "enough" of a decoupling limit to guarantee that.

$m_\gamma < m_\phi$, So there is a net <u>repulsion</u>:





For $\theta \ll 1$, $V(\theta) \approx V_0 + q^2 \theta^2 \log \theta$

Recall $q^2 \sim N^{-1/3}/\nu$. Also $\omega_{\text{cyclotron}} \sim N^{-1/3}$.

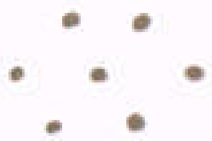
"Figure of merit" $\eta = \dfrac{V_{\text{typical}}}{\omega_{\text{cyc}}} \sim \dfrac{q^2}{\omega_{\text{cyc}}} \sim \dfrac{1}{\nu}$

should be small for projection onto LLL to be valid.

→ $\eta \sim \frac{1}{\nu} \sim O(N^0)$ at least, so Laughlin wave-function has a reasonable chance for $\nu \lesssim 1$ but not too small.

Elaboration:

If repulsions are <u>strong</u>, many LL's are involved, and likeliest ground state is Wigner crystal:



For small ν this surely wins out over FQHE.

$\lambda \sim \frac{1}{\nu} \gtrsim 1$ seems bad, but the <u>short-range</u> repulsions are weaker: FQHE is more favored than naive analysis suggests.

More systematic methods exist to distinguish between W-C and FQHE: a direction we have begun to explore.

Potential energy for a continuous charge distribution, $\rho = \sum_{\ell m} \rho_{\ell m} Y_{\ell m}(\theta, \phi)$:

cf Bena-Nudelman

$$V = \frac{1}{2} \sum_{\ell, m} \left(\frac{1}{\ell(\ell+1) + m_\gamma R^2} - \frac{1}{\ell(\ell+1) + m_\phi^2 R^2} \right) |\rho_{\ell m}|^2$$

$$\sim \sum_{\ell, m} \frac{1}{\ell^3} |\rho_{\ell m}|^2$$

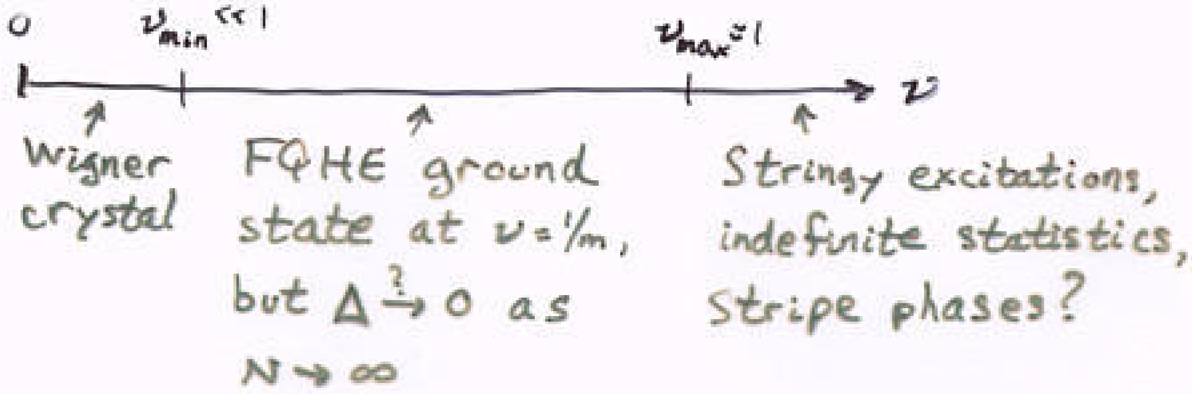
positive, so <u>stable</u> against small S^2-fluctuations

The $1/\ell^3$ fall-off indicates that uniform state is only just stable against high-ℓ fluctuations; moreover, this stability requires distant-neighbor repulsion.

→ Thus, even assuming a nearly-Laughlin ground state, it seems doubtful that the "quantum Hall fluid" will be incompressable.

→ Besides, no dirt means no localization of quasi-particles : ρ_{xy} won't be quantized.

A reasonable guess for the dynamics:



At any ν, the near-flatness of V around spherically uniform state suggests that electrons/strings may want to clump locally (ie into BIons).

→ Need some grasp of non-~~linear~~ gaussian dynamics to be sure.

The situation would be better, vis a vis quantum Hall, if the scalar were very heavy or absent. cf recent preprint by Susskind...

Summary

- Proposed thermodynamic criterion for existence of Gregory-Laflamme instability. Checked it for near-extremal spinning M2-brane. Got unstable black holes in AdS as a bonus.

- Investigated inter-electron forces for "quantum Hall soliton." May well have Laughlin ground state for $\nu = 1/m$, m not too large. Mass gap likely to be small.