ADS, CFT and Cosmology

Matter Effective Action

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{g}\left(R + W[\mathbf{g}]\right)$$

$$W[\mathbf{g}] = \int d[\phi]\exp\left(-S_m[\phi,\mathbf{g}]\right)$$

Einstein Equations

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi G \left\langle T_{ij} \right\rangle$$

Trace Anomaly

$$\langle T \rangle = aF - cG + d\nabla^2 R$$

$F = C_{ijkl}C^{ijkl} =$ Weyl squared

$G = R_{ijkl}R^{ijkl} - 4R_{ij}R^{ij} + R^2 \propto$ Euler density

$$a = \frac{1}{120\,(4\pi)^2}\left(N_S + 6N_F + 12N_V\right)$$

$$c = \frac{1}{360\,(4\pi)^2}\left(N_S + 11N_F + 62N_V\right)$$

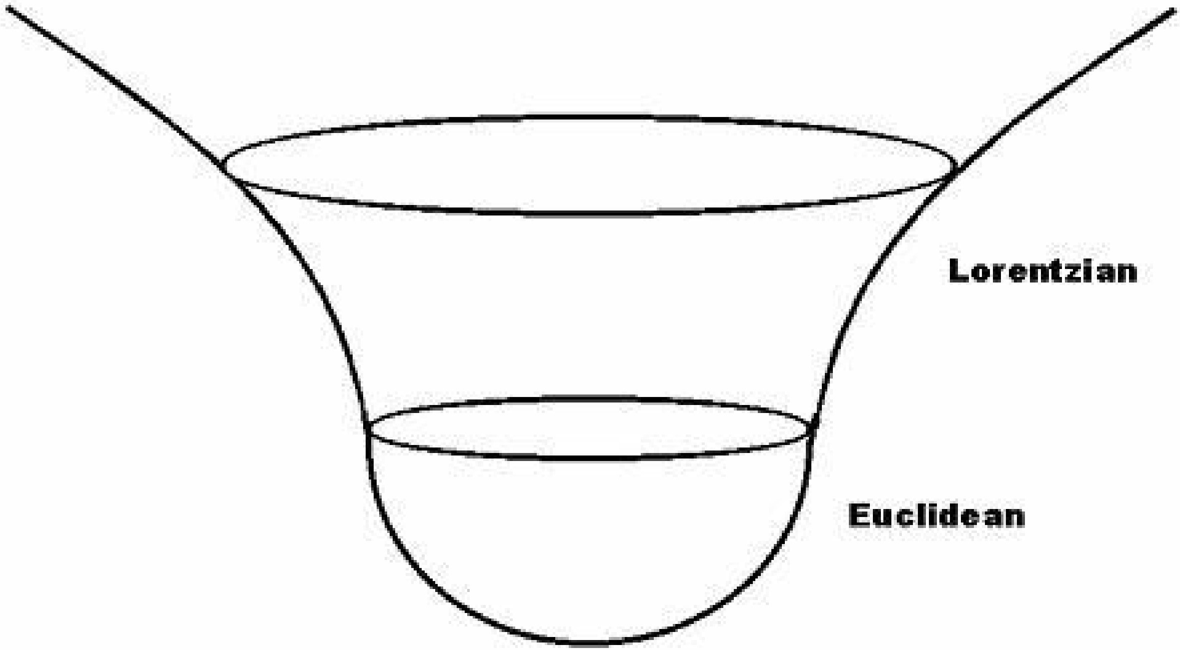
$$d = \frac{1}{180\,(4\pi)^2}\left(N_S + 6N_F - 18N_V\right)$$

N_S=number of real scalars

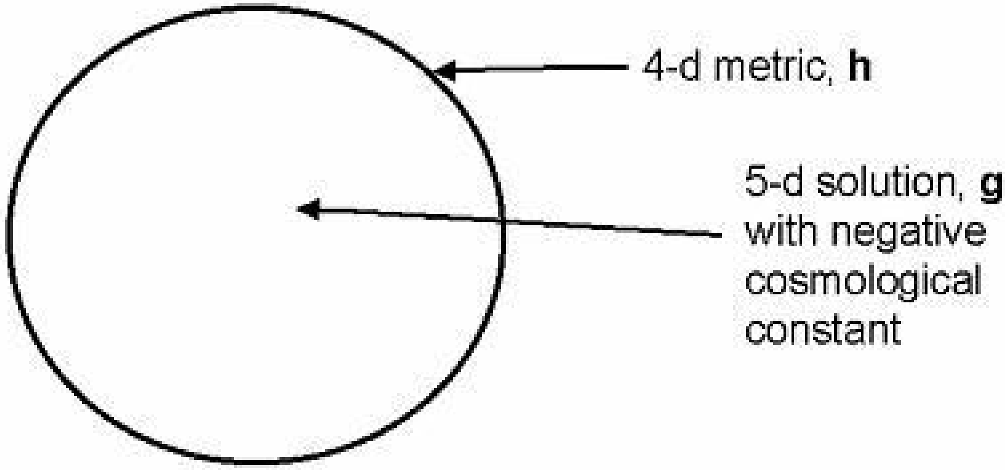
N_F=number of Dirac fermions

N_V=number of vectors

Quantum Creation of the Universe



ADS-CFT



$$W[\mathbf{h}] = -\frac{1}{16\pi G_5}\int d^5x\sqrt{g}\left(R+\frac{12}{l^2}\right) + \text{surface counter terms}$$

O4 Metrics

$$ds^2 = d\sigma^2 + b(\sigma)^2 d\Omega^2$$

The only stationary point metrics of the combined action are flat space and the round four sphere, Euclidean de Sitter space

Combined Action

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{h}R + W[\mathbf{h}]$$

Where

$$W[\mathbf{h}] = -\frac{1}{16\pi G_5}\int d^5x\sqrt{g}\left(R_5 + \frac{12}{l^2}\right) - \frac{1}{8\pi G_5}\int d^4x\sqrt{h}K$$
$$+\frac{3}{8\pi G_5 l}\int d^4x\sqrt{h} + \frac{l}{32\pi G_5}\int d^4x\sqrt{h}R$$
$$-\frac{N}{32\pi^2}\left(\log\frac{l}{r}+\beta\right)\int d^4x\sqrt{h}\left(R_{ij}R^{ij} - \frac{1}{3}R^2\right) + \frac{\alpha N}{192\pi^2}\int d^4x\sqrt{h}R^2$$

and $\left(\frac{l^3}{G_5} = \frac{2N}{\pi}\right)$

Stationary Point Metric

$$ds^2 = d\sigma^2 + r^2 \sin^2 \frac{\sigma}{r} d\Omega^2$$

where $$r = \sqrt{\frac{NG}{4\pi}}$$

Two point function <hh>

1. Pick a perturbation h_{ij} of four sphere metric,
2. Solve five dimensional Einstein equations with four sphere as boundary,
3. Calculate second variation of action in terms of h_{ij} and invert to get two point function.

Transverse traceless modes

$$ds^2 = l^2\left(dy^2 + \sinh^2 y\left(d\sigma^2 + \sin^2 \sigma d\Omega^2\right)\right)$$

Expand δg_{ij} in tensor spherical harmonics

$$\delta g_{ij}(y,x) = \sum_{p=2}^{\infty} f_p(y) H_{ij}^{(p)}(x)$$

Solve for $f_p(y)$ to get

$$f_p(y) = \frac{\sinh^{p+2} y}{\cosh^p y} F\left(\frac{p}{2}, \frac{(p+1)}{2}, p+\frac{5}{2}, \tanh^2 y\right)$$

Scalar part of combined action

Scalar perturbation $\delta h_{ij} = \varphi h_{ij}$ has action

$$S = \frac{3}{8NG^2} \int d^4x \sqrt{\gamma}\, \varphi \left(2\alpha \nabla^2 - 1\right)\left(\nabla^2 + 4\right)\varphi$$

where γ_{ij} and ∇ are the metric and connection on a unit four sphere.

Instability of de Sitter

$$\varphi \sim \exp\left(\frac{t}{12|\alpha|r}\right)$$

Transverse Traceless Action

$$S = \frac{1}{NG^2} \sum_p \left(\int d^4 x \sqrt{\gamma}\, \delta h^{ij}(x) H_{ij}^{(p)}(x) \right)^2 M(p, \alpha, \beta)$$

Where

$$M(p,\alpha,\beta) = \Psi(p) + p^2 + 3p + 6 + 2\beta p(p+1)(p+2)(p+3) - 4\alpha p(p+3)$$

and

$$\Psi(p) = p(p+1)(p+2)(p+3)\left[\psi\left(\frac{p+5}{2}\right) + \psi\left(\frac{p+4}{2}\right) - \psi(2) - \psi(1)\right]$$
$$+ p^4 + 2p^3 - 5p^2 - 10p - 6$$

where $\psi(z) \equiv \frac{d}{dz} \log \Gamma(z)$

Integral representation of two point function

$$\left\langle \delta h_{ij}(x)\delta h_{kl}(y)\right\rangle \sim \int dq\left(\text{continuum of modes}\right)+\text{discrete poles}$$

Tensor Fluctuations in Microwave

Observational limit satisfied if $N=10^{10}$

(not taking matter effective

action in to account)

or

$N=10^{4}$ and $\beta=10^{3}$

(with matter effective action)

Features that should be present in ultimate model

1. Large N approximation valid
2. Eternally inflating de Sitter state
3. Ghosts

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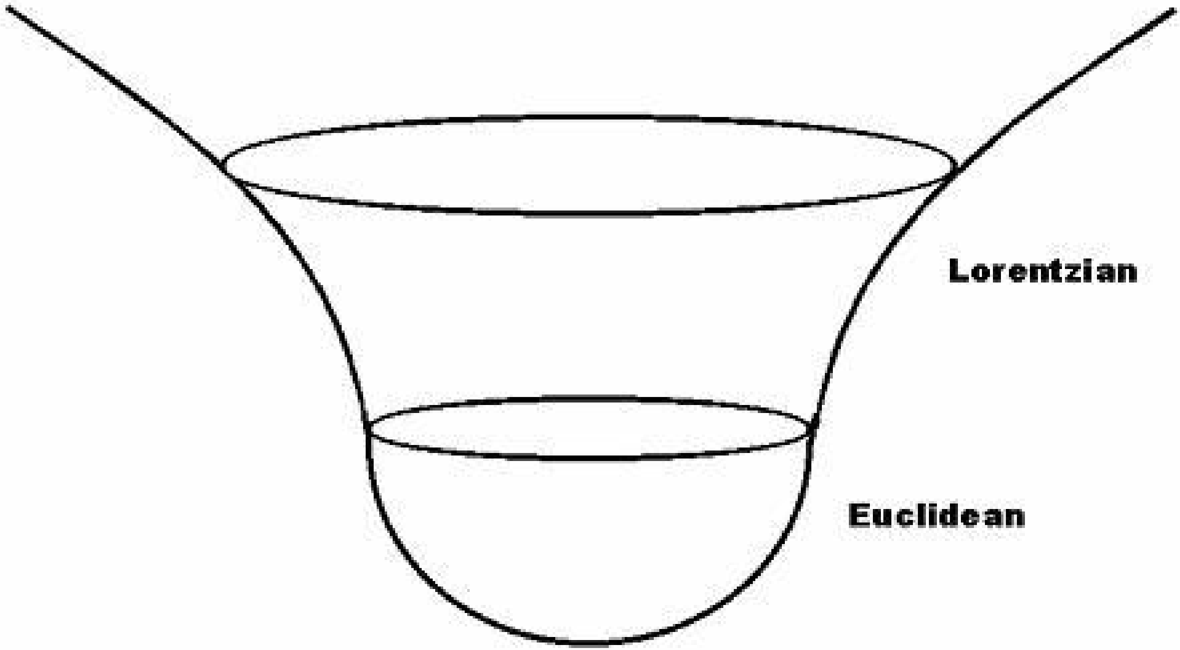

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