de Sitter Entropy & String Theory

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work in progress, with V. Balasubramanian & D. Minic

Recent astrophysical/cosmological observations suggest that we may live in a universe well approximated by spacetime with small, but non-zero, positive cosmological constant – de Sitter space.

String theory is supposed to be a complete quantum theory of gravity; it is expected to revolutionize cosmology, just like it has other areas of physics:

- quantum field theory (Seiberg-Witten solutions, D-brane technology, new concepts at strong coupling, ...)
- high-energy phenomenology (large extra dimensions, brane-worlds, ...)

and
- mathematics (quantum geometry, mirror symmetry, non-commutative geometry, K-theory, ...)

But: As a part of this process of uncovering new concepts inspired by string theory, we should at least understand the old ones:

CENTRAL QUESTION:

is it possible to describe the quantum mechanics of de Sitter space (or, generally, spaces with $\Lambda > 0$) in string theory?

Maldacena, Strominger, Hawking, Banks, Bousso, ...

... if so, lessons about:
- entropy in cosmology
- holography "
- susy breaking
- and, perhaps, real world

de Sitter spacetime

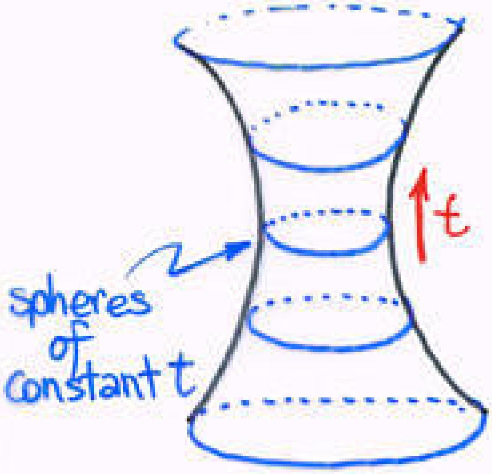
Maximally symmetric solution of Einstein's equations with positive Λ, in D spacetime dimensions.

dS_4:

$$\boxed{-v^2+w^2+x^2+y^2+z^2=\ell^2} \qquad \ell^2=\frac{3}{\Lambda}$$

$\Downarrow$

isometry group $SO(4,1)$,
generally $SO(D,1)$.



$$ds^2=-dt^2+\ell^2\cosh^2\left(\frac{t}{\ell}\right)d\Omega_3^2$$

cosmological t-dependent factor

space is round S^3

There are other useful coordinate systems on de Sitter:

$$ds^2=-d\hat{t}^2+e^{2\hat{t}/\ell}\left(d\tilde{x}^2+d\tilde{y}^2+d\tilde{z}^2\right)$$

covers half the hyperboloid relevant for inflation;

cosmological expansion

foliation by flat $\mathbb{R}^3$

Another coordinate system:

$$ds^2=-d\tilde{t}^2+\sinh^2\left(\frac{\tilde{t}}{\ell}\right)d\tilde{\Omega}_3^2$$

slicing by the hyperbolic space H^3

Quantum mechanics requires observers.

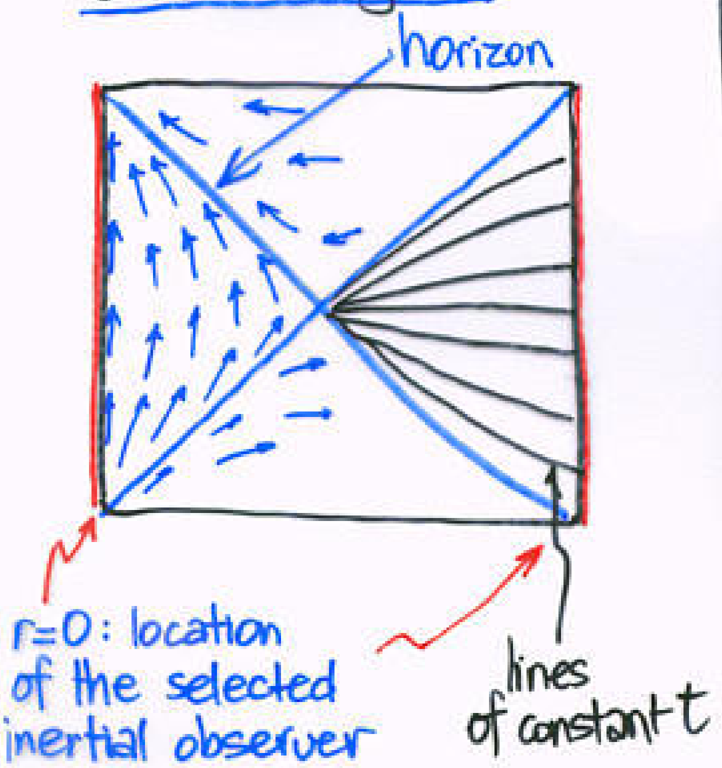
Consider the $\boxed{\text{inertial observer}}$, say, at the pole of the sphere.

There is a natural coordinate system, associated with this observer, in which the metric appears <u>static</u>:

$$ds^2 = -\left(1-r^2/\ell^2\right)dt^2 + \frac{dr^2}{\left(1-r^2/\ell^2\right)} + r^2 d\Omega_2^2 .$$

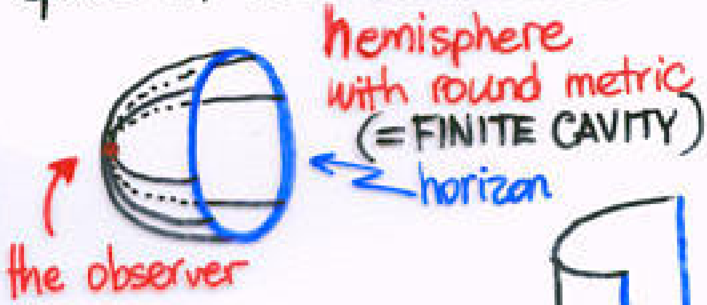
This static patch coordinate system has an obvious Killing vector $\partial/\partial t$, timelike at $r<\ell$ but null at $r=\ell$: horizon!

<u>Penrose diagram</u>:

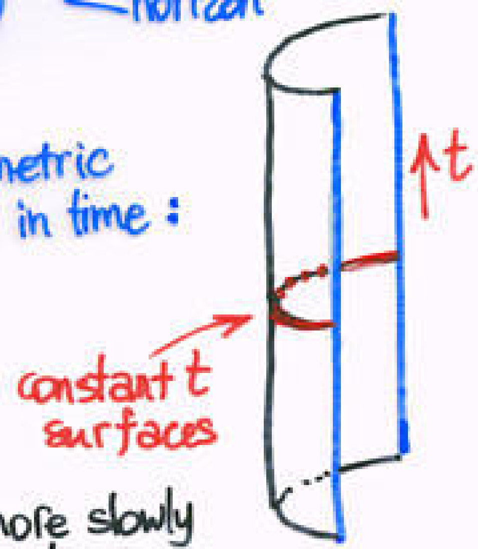


<u>A useful picture to keep in mind</u>:

space in these coordinates:



This space metric is constant in time:



time flows more slowly for larger r

According to the Bekenstein-Hawking formula,

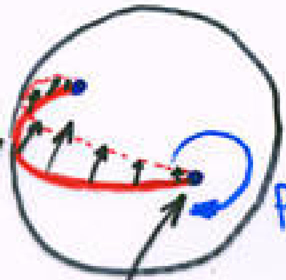
horizon implies entropy: Gibbons & Hawking

$$\boxed{\begin{aligned} &S = \frac{\mathcal{A}}{4G_N} \sim \Lambda^{-\frac{(D-2)}{2}} \\ \text{at temperature}\quad &T \sim \sqrt{\Lambda} \sim \frac{1}{\ell}. \end{aligned}}$$

As first shown by Gibbons & Hawking, the cosmological horizon implies same thermod. effects as black hole horizons: (particle creation in thermal bath at T; laws of thermod. hold.)

One approach to seeing this: Euclideanize.

Euclidean de Sitter (EdS): $t \to i\tau$



$\frac{\partial}{\partial \tau}$: angular rotations

periodicity determines $\frac{1}{T}$.

horizon: sphere of codimension two: $S^{D-2} \subset S^D$

Thermodynamics of horizons immediately brings issues:

- stat-mech interpretation?
- holography?
- analogs of Bekenstein bound?

String theory has been instrumental for (some) black holes.

Quantum mechanics of de Sitter space?

Successes of string theory in black hole physics: more often than not tied with supersymmetry.

de Sitter spacetime is believed to be incompatible with supersymmetry, in the following sense:

- de Sitter superalgebra does exist;
- it does not have highest-weight unitary rep's;
- it can formally be represented by fields described by a supergravity Lagrangian.

Pilch & Sohnius & van Nieuwenhuizen [PSvN]

Comments: ☹

- instead of $\sum_i \{Q_i, \bar{Q}_i\} = H$ as in AdS case (or flat space), satisfies $\sum_i \{Q_i, \bar{Q}_i\} = 0$

  $\Rightarrow$ cannot prove a positive energy theorem using standard susy arguments

- multi-particle Fock space clearly cannot exist
- the Lagrangian has ghosts: $\int (R + \frac{1}{4} F^2 + \ldots)$

wrong relative sign

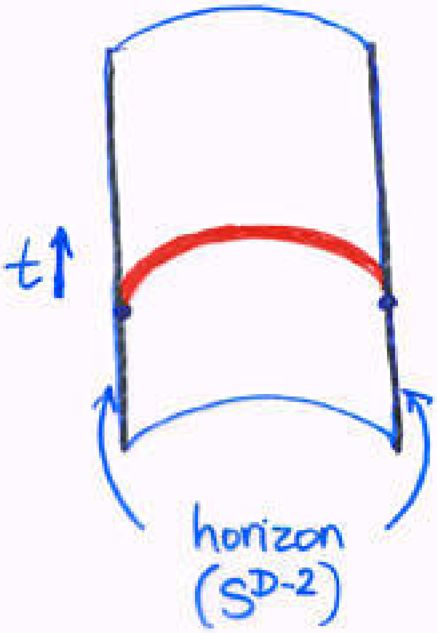
Comments to comments: ☺?

- (for example, for dS_4) the R-symmetry [PSvN] in the maximally supersymmetric case is $SO(6,2)$!
- holography & related ideas imply that only finite N of d.o.f. should describe everything $\Rightarrow$ no multi-particle states

Banks

- the ghosts in Lagrangian, as well as $\sum \{Q, \bar{Q}\} = 0$, reminiscent of topological theory ... reduction of d.o.f.

Various facts suggest that we should perhaps take the observer and the associated horizon seriously, and try to make sense of the quantum mechanics of the static patch – finite cavity.



Some supporting arguments:

- finite cavity surrounded by finite-size horizon; natural extensions of Bekenstein bound suggest finite # of d.o.f., in accord with our previous comments Banks, ...
- the Killing vector $\partial/\partial t$ allows one to define Killing energy; somewhat surprisingly, one can prove a (perturb.) positive-energy theorem for it, Abbott & Deser assuming one stays inside the horizon – all violations come from modes that naively live behind the horizon (holography, complementarity)
- the Killing energy is conserved, and could in principle be used to define a Q.M. hamiltonian – after all, we do believe in quantum mechanics!

How to accomplish this?

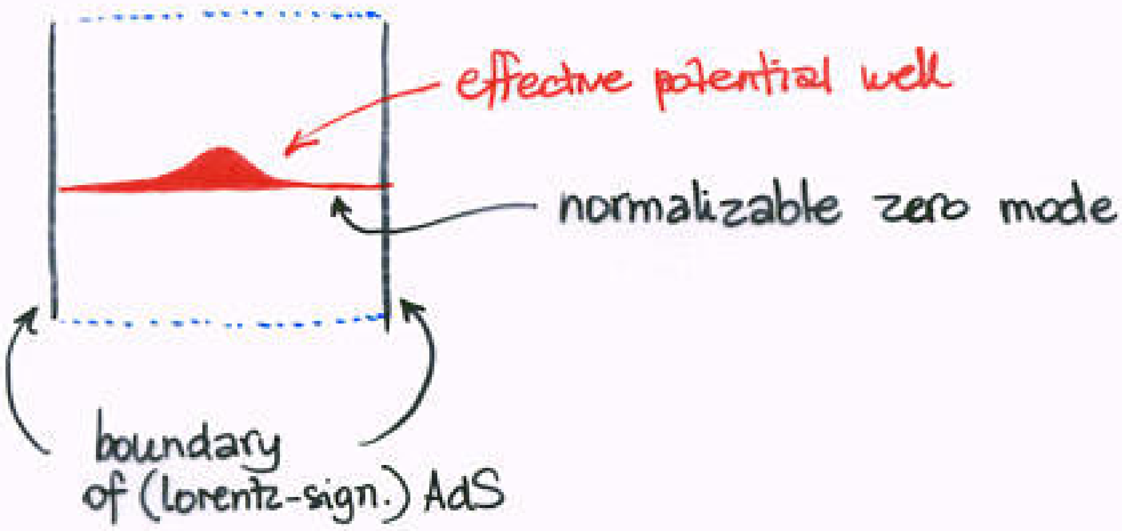
... presently unknown.

Hints:
- repulsive conditions at the horizon?
- breaking $SO(D,1)$ to a smaller subgroup (symm's of observer)?
- consider only modes with same p^μ as observer?

"Compactifying" on AdS down to Euclidean de Sitter

We would like to isolate the degrees of freedom in AdS/CFT that only excite the sphere, but not AdS.

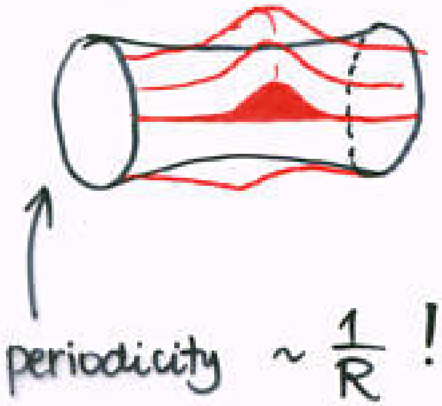
First of all, these modes should be in their ground state on the (space-like slice of) AdS:



Now, we still have the Lorentz-signature time direction in AdS...

↓

The supergravity modes project down to the AdS with closed time-like loops: (frequencies are correspondingly quantized)



we will only consider one selected "inertial observer" in AdS; alternatively, cut off AdS_p at smallest reasonable volume $\sim R^{p-1}$...

Impose symmetry-breaking periodicity conditions.

MASS GAP $\sim \frac{1}{R}$. Witten

According to the proposal, the number of such d.o.f. should count $\boxed{\text{de Sitter entropy}}$.

Consider $AdS_5 \times EdS_5$.

EdS_5 is S^5, the horizon is a large S^3;

Euclidean time translations = rotations of sphere.

(cf. giant gravitons!)

The area of the horizon, in units measured by the effective five-dimensional Newton constant G_{N5}, should ~~be given by~~ give the entropy $S = \dfrac{A}{4G_{N5}}$.

Now, $A \sim R^3$, but what is G_{N5}?

Despite the fact that we have "compactified" on a non-compact, Lorentz-signature space, G_{N5} will be finite:

$$G_{N5} \sim \frac{G_N}{R^4 \cdot R} \sim \frac{G_N}{R^5}.$$

# of d.o.f. zero modes CFT

(g_s canceled out)

Thus, for the entropy we obtain $S \sim \dfrac{R^3}{G_{N5}} \sim \dfrac{R^8}{G_N}$.

In terms of the parameters of AdS/CFT,

$R \sim (\alpha'^2 g_s N)^{1/4}$, $G_N \sim g_s^2 \alpha'^4$:

$$\boxed{S \sim \frac{\alpha'^4 g_s^2 N^2}{g_s^2 \alpha'^4} \sim N^2}!$$

Consider other canonical examples:

- $EdS_4 \times AdS_7 \longleftrightarrow$ (2,0)-theory of N M5-branes

Entropy of EdS_4:

$$S = \frac{R^2}{4G_{N4}}, \quad G_{N4} \sim \frac{G_N}{R^7},$$

$$S \sim \frac{R^9}{G_N}.$$

AdS/CFT duality implies $R \sim \ell_P N^{1/3}$;
in those variables, we get

$$\boxed{S \sim N^3}$$

- $EdS_7 \times AdS_4 \longleftrightarrow$ N M2-branes CFT

$$\boxed{S \sim N^{3/2}}$$

- $EdS_3 \times AdS_3 \times \mathcal{M}$

$$S = \frac{R}{4G_{N3}}, \quad G_{N3} \sim \frac{G_{N6}}{R^3}$$

$$\boxed{S \sim \frac{R^4}{g_s^2 \alpha'^2} \sim N}$$

(using duality $R^2 \sim g_s \alpha' \sqrt{N}$)

In all the canonical cases, the Gibbons-Hawking entropy of the EdS factor scales exactly like the number of degrees of freedom of the corresponding CFT on the boundary of the associated AdS space, truncated to the zero modes — a matrix model picture for EdS?

$\boxed{\text{The matrix model: dual CFT, at finite } T \sim \frac{1}{R}, \text{ truncated to a point — } 0{+}1 \text{ dim. Q.M. on } S^1}$

suggestive of a topological theory on boundary of AdS...

this will need further testing...

On the AdS side, we reduce to zero modes.
What is the picture on the EdS side?

$AdS_5 \times S_5$ group theory:

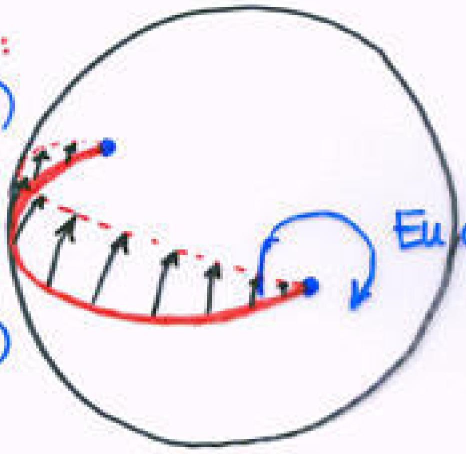
$SO(6) \times SO(4,2)$

$\downarrow$

$SO(4) \times SO(2)$

$\times \cancel{SO(4)} \times SO(2)$

singlets



Euclidean time; $\frac{\partial}{\partial \tau} \equiv \frac{\partial}{\partial \phi}$

fixed ang. momentum — microcanonical ensemble?

angular coordinate on sphere...

Myers; McGreevy, Susskind, Toumbas

Giant gravitons on the sphere ...
expand into a codimension-two sphere, and at the max angular momentum given by the exclusion principle, they fill out the Euclidean de Sitter horizon.

Lorentz-signature de Sitter?

We already have a Lorentzian-signature time in the theory, on the AdS factor.

Options: (a) keep it; select a new time coordinate, such that the Killing vector is a sum of $\partial/\partial t_{AdS}$ and $\partial/\partial\phi$, the EdS time.

This is interesting for several reasons:

- group theory

$$SO(6)\times SO(4,2)\to \underbrace{SO(4)\times SO(2)}_{EdS}\times\overbrace{\cancel{SO(4)}\times SO(2)}^{AdS}$$

truncate to singlets

$$SO(2)_{new}=\mathrm{diag}\left(SO(2)_{EdS}\times SO(2)_{AdS}\right)$$

← TOPOLOG. TWISTING!?

- inspired by giant gravitons ("null" in AdS × S)
- seems to provide "repulsive boundary conditions" at the edge of the finite cavity...

(b) Wick-rotate it; this "double-Wick-rotation" of AdS × S has been tried before but never worked...

(c) keep the Lorentzian-signature AdS time, Wick-rotate the EdS time, and interpret the result as a compactification of M*-theory with two times (or IIB*, ...) Hull

(d) do nothing, interpret calculations in Euclidean de Sitter, and rotate to Lorentz signature afterwards

⋮