

Tunneling
Mediated

~~SUSY~~

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(a)

Introduction / Motivation

For many years, a common expectation in the string theory community was:

Nature = $E_8 \times E_8$ Het. String
on CY 3-fold K

One can then use the "hidden" E_8 for e.g. ~~SUSY~~, which can be communicated to us by 4d gravity.

In light of Horava-Witten, can modify this to:



1b

The oldest scenarios had a metric of form:

$$dS_{10}^2 = dS_{M^4}^2 + dS_K^2$$

to a very good approximation, while in the Horava-Witten case \exists mild warping as a function of X_{11} . More recently, it has been realized that rich new physical ideas emerge if one studies strings in more general 'warped' backgrounds:

$$dS_{10}^2 = f(X_{int})^2 dS_{M^4}^2 + dS_K^2 \leftarrow \text{only } X_{int} \text{ dependent}$$

Of course the most prominent "new" examples come from AdS_5 / CFT_4 duality & the closely related "Randall-Sundrum" scenarios.

②

There, to start with, one should take

$$dS_5^2 = e^{-2kX_5} \eta_{\mu\nu} dx^\mu dx^\nu + dX_5^2$$

For $X_5 \in (-\infty, \infty)$, this \rightarrow AdS₅ (Poincaré coords)

Via AdS/CFT

Field theory

IR physics

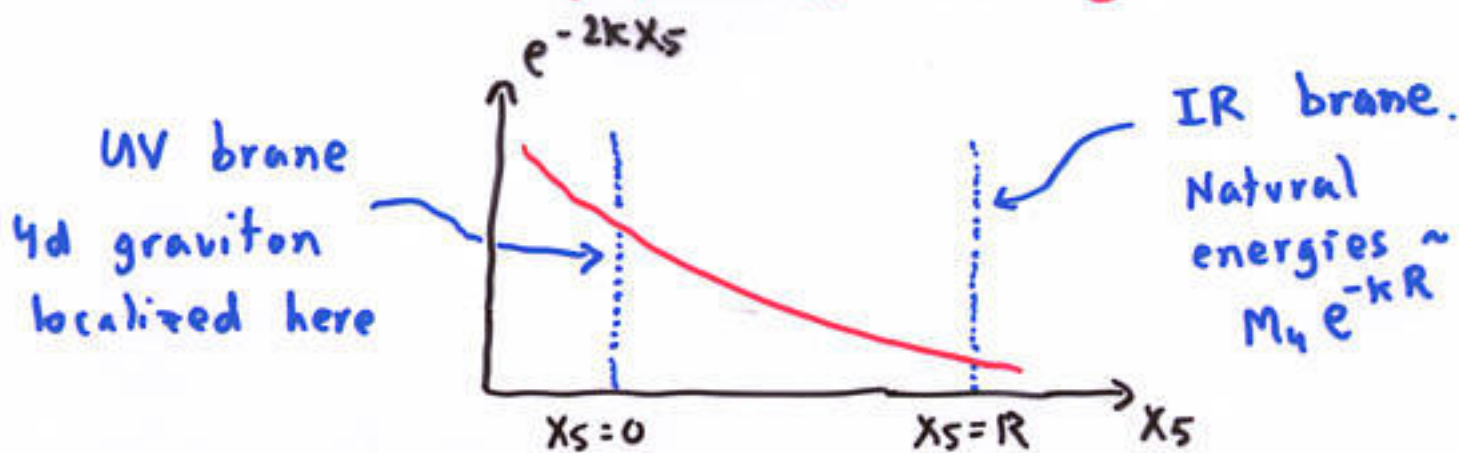
UV physics

Gravity

$X_5 \rightarrow +\infty$ (warp factor $\rightarrow 0$)

$X_5 \rightarrow -\infty$ (warp factor large)

In RS scenarios, one chops the X_5 interval on the left (and, possibly, the right):



③

So for "natural" $k (\equiv \frac{1}{L_{\text{AdS}}})$ and R , one can generate hierarchies $\Lambda_{\text{IR}} \ll M_4$.

This is the AdS dual manifestation of RG running in the strongly coupled QFT.

H. Verlinde

How does this fit into string theory?

Consider M-theory on a Calabi-Yau fourfold X_4 . Tadpole cancellation for the C-field \rightarrow

$$\frac{\chi(X_4)}{24} = N_{M2} + \frac{1}{8\pi^2} \int_{X_4} G \wedge G \quad \text{SVW}$$

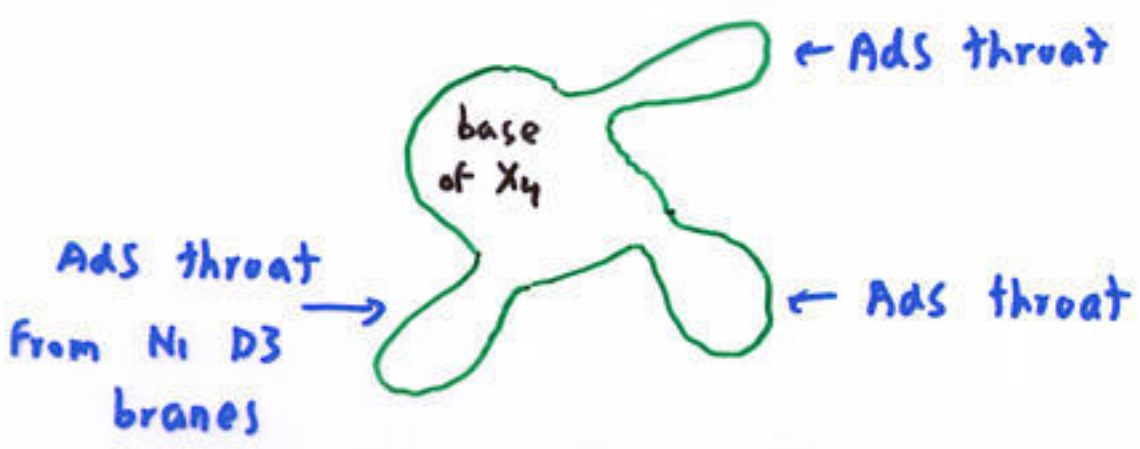
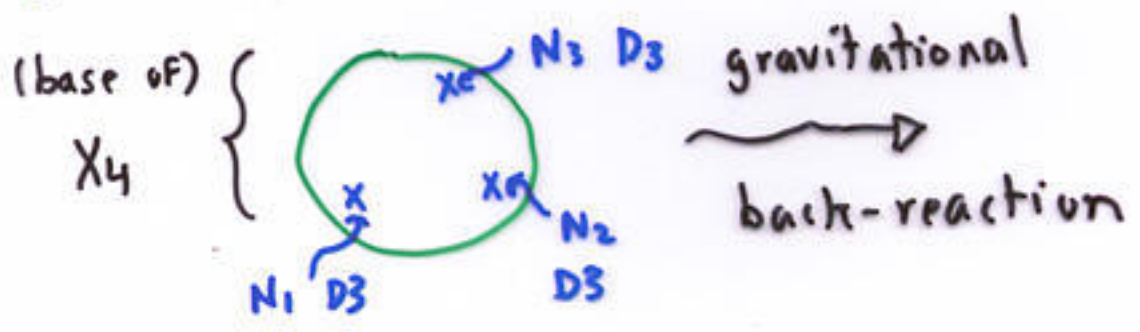
Very simple constructions of fourfolds yield $\chi > 10^5 \rightarrow$ can imagine a "large N" collection of branes transverse to X_4 .

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IF X_4 is elliptic w/a section, then one can take the "F-theory limit", yielding (for suitable choices of G) a theory with 4d Poincaré inv. and N D3 branes. DRS; GYW

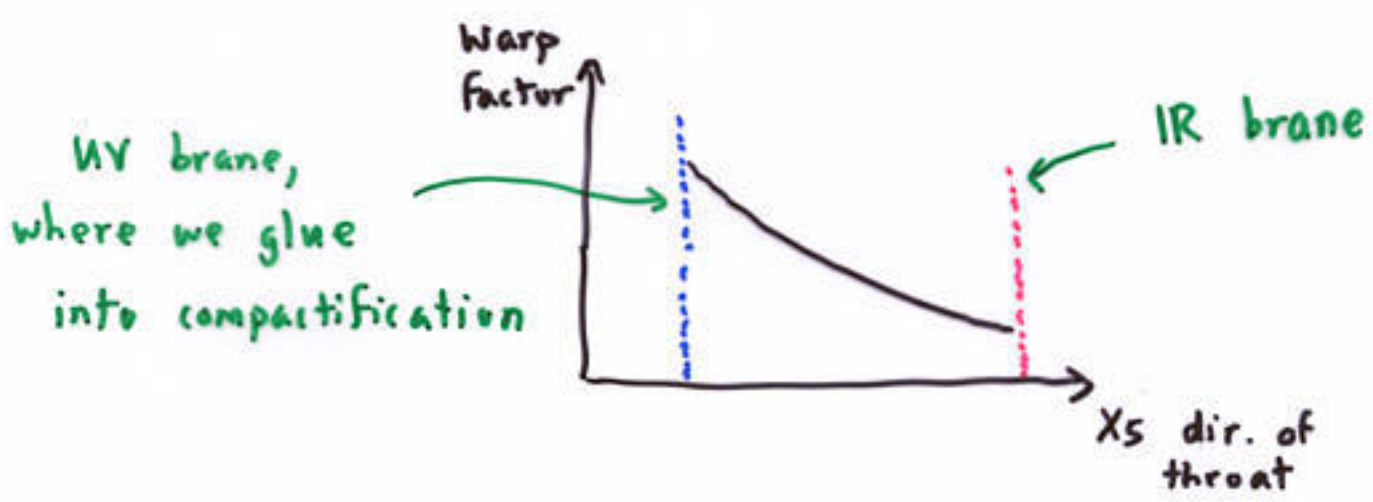
IF one now uses AdS/CFT ideas (since N can be $\gg 1$, this might be justified)

then :

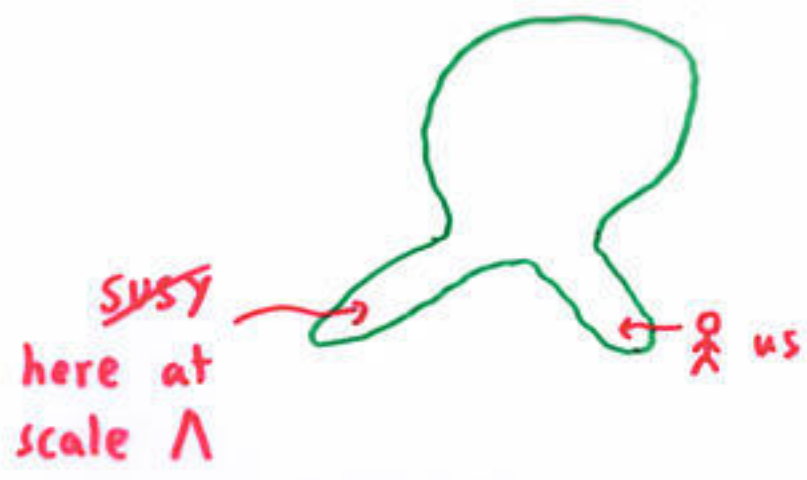


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A generic such throat could well be cut-off in the IR by a mass gap or IR free QFT (\rightarrow highly curved gravity sol'n) :

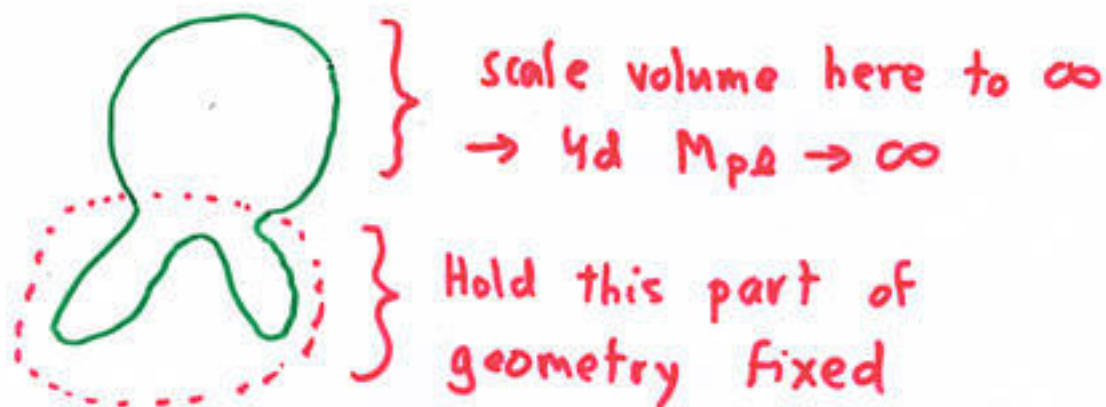


Natural Question: Is there any novel physics in the communication between the modes in such throats? E.g. imagine :



What do we see? Obviously NOT 4d gravity mediation, since:

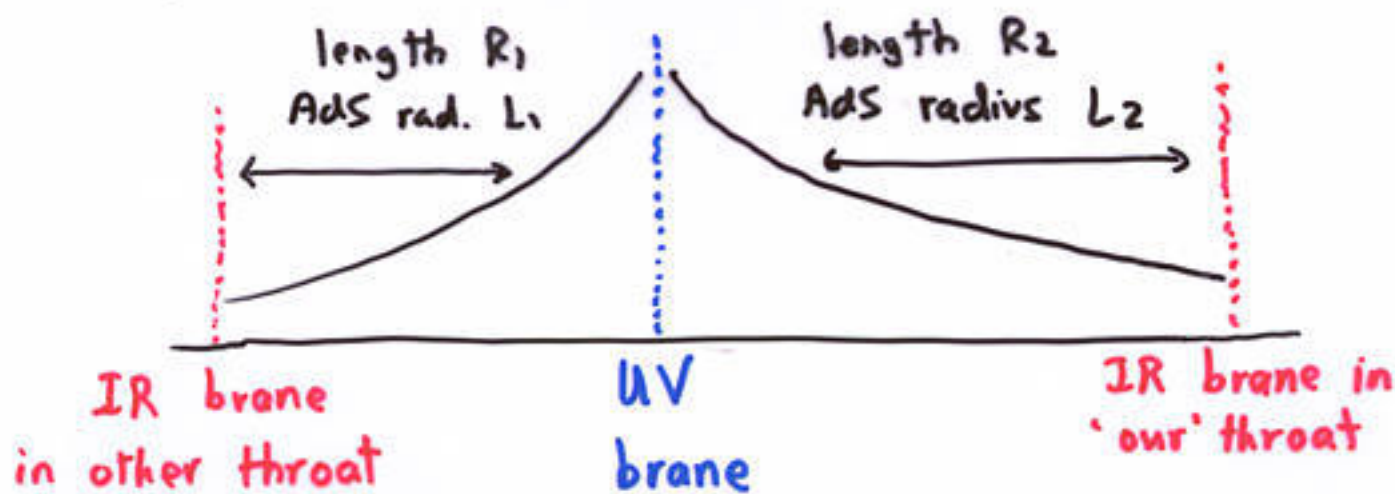
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We find that the communication that persists in such a limit can be a $>$ effect than 4d gravity mediation even when M_4 is finite.

This communication is fruitfully thought of as a "tunneling" between the throats ...

To facilitate calculations, consider a toy-model of the above:

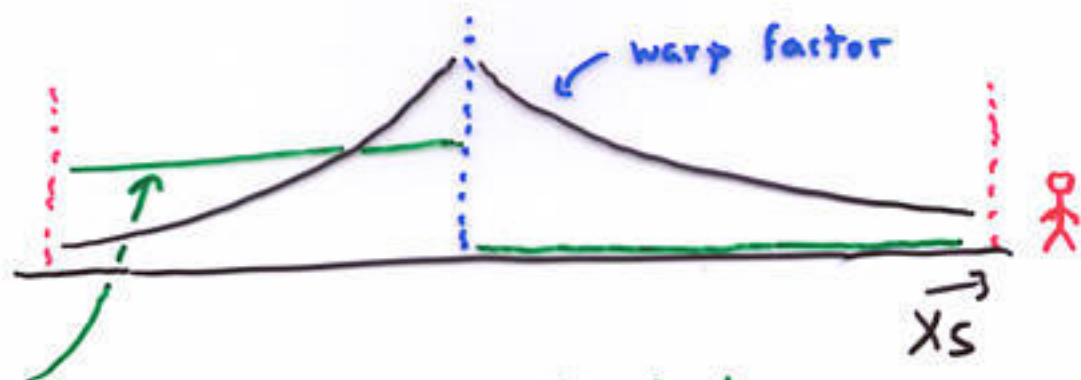


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To see what changes if ~~SUSY~~ in a different throat, we should compare two calculations:

I: Start with a SUSY single throat theory containing bulk scalar ϕ (+ superpartners). Add ~~SUSY~~ 5d mass to ϕ & compute the splittings in 4d after KK reduction.

II: Same as above, except:



step function mass for ϕ which
→ ~~SUSY~~ in left throat

How do splittings on right IR wall compare to case I?

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Calculation I:

Scalar field in background metric:

$$ds^2 = e^{-2kX_5} \eta_{\mu\nu} dX^\mu dX^\nu + dX_5^2$$

$0 \leq X_5 \leq R$ (+ b.c. we'll get to)

Scalar wave eqn is just

$$\frac{1}{\sqrt{-G}} \partial_\mu (\sqrt{-G} G^{\mu\nu} \partial_\nu) \phi = m_0^2 \phi$$

If we look for modes w/ 4d mass μ :

$$\left(\sum_{i=1}^3 \frac{\partial^2}{\partial X_i^2} - \frac{\partial^2}{\partial t^2} \right) \phi = \mu^2 \phi$$

and define:

$$\hat{\phi} = e^{-\frac{3}{2}kX_5} \phi, \quad z = \frac{e^{kX_5} - 1}{k}$$

then scalar EOM \rightarrow

$$\frac{\partial^2}{\partial z^2} \hat{\phi} - \frac{\left(\frac{15}{4} k^2 + m_0^2 \right)}{(kz+1)^2} \hat{\phi} = -\mu^2 \hat{\phi}$$

④

And after some more massaging

$$X \equiv \mu (z + 1/k)$$

$$\hat{\phi} \equiv (\kappa z + 1)^{1/2} \psi$$

one finds Bessel's equation

$$\left\{ \left[\frac{1}{x} \frac{d}{dx} \right]^2 + 1 - \frac{(m_0^2 + 4k^2)}{\kappa^2 x^2} \right\} \psi = 0$$

$$\rightarrow \psi = a J_\nu(x) + b Y_\nu(x)$$

\swarrow Bessel fcn \swarrow Neumann fcn

$$\nu \equiv \sqrt{4 + \left(\frac{m_0}{\kappa}\right)^2}$$

$$Y_\nu \equiv \frac{\cos(\nu\pi) J_\nu - J_{-\nu}}{\sin(\nu\pi)}$$

Impose standard b.c. at $x = \frac{\mu}{\kappa}$ (UV brane)

and $x = \frac{\mu}{\kappa} e^{kR}$ (IR brane), solve for allowed

4d KK masses $\mu \Rightarrow$ you find

$$\mu \sim k e^{-kR} \times (1 + \alpha \nu + \beta n)$$

no dep. here

$n = 0, 1, 2, \dots$

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It follows that (assuming $M_0 \rightarrow 0$ is the SUSY case for our bulk multiplet) the 4d mass splitting for bulk KK modes is

$$\Delta M \sim \left(\sqrt{4 + \left(\frac{M_0}{k} \right)^2} - 2 \right) k e^{-kR}$$

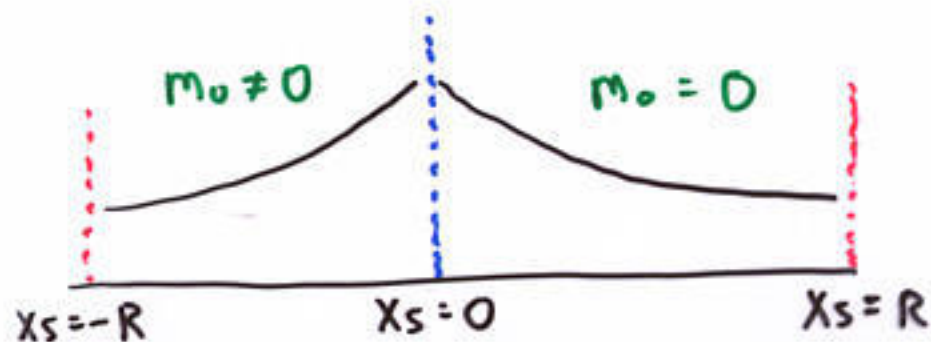
We can then do simple computations to find the induced splitting of IR wall localized fields, due to the bulk SUSY. This step is the same in both cases, so proceed now to:

Calculation II: Tunneling mediated SUSY

Now imagine:

$$M_0(X_5) =$$

$$m_0 \theta(-X_5)$$



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• Bulk calculations same as previous case.

However, index of Bessel/Neumann functions differs on left & right:

$$\Psi = \begin{cases} \Psi_R(x) = a_R J_\nu + b_R Y_\nu & x > \frac{\mu}{k} \\ \Psi_L(x) = a_L J_\nu + b_L Y_\nu & x < \frac{\mu}{k} \end{cases}$$

$$\nu \equiv \sqrt{4 + \left(\frac{m_0}{k}\right)^2}$$

Imposing b.c. @ the left/right IR branes

$(x = \frac{\mu}{k} e^{\pm kR})$ & the UV brane $(x = \frac{\mu}{k})$

we find again a discrete spectrum. Compared to the previous splittings $\Delta\mu$ in Calc. I, we find:

$$(\Delta\mu)_{II} = (\Delta\mu)_{I} \times e^{-4kR}$$

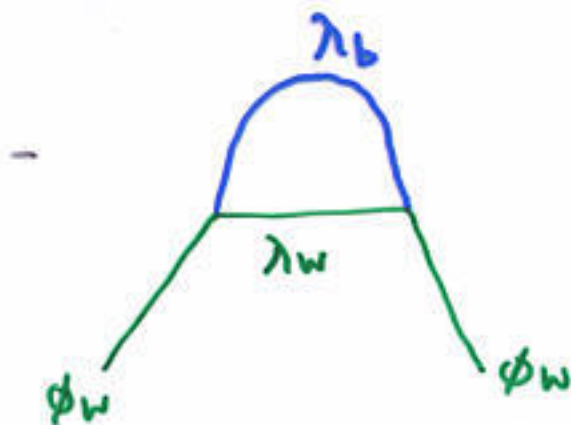
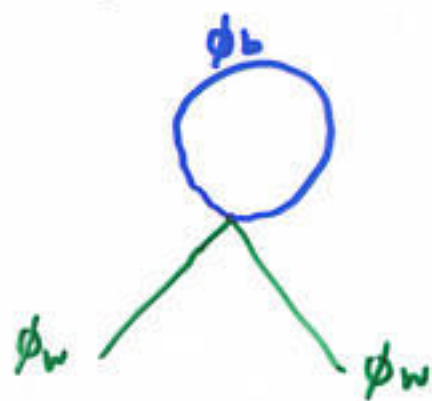
\uparrow scale of syby in left throat \uparrow tunneling suppression

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$(\Delta\mu)_{II}$ is splitting of KK modes of bulk fields. But we still have to deal with:

Transmission to wall fields

Suppose \exists bosons/fermions ϕ_w, λ_w localized on the right IR brane, in addition to the bulk fields ϕ_b, λ_b . At what level do the bulk splittings \rightarrow splittings for brane localized matter? Generic quartic & Yukawa couplings \rightarrow mass shift for ϕ_w :



⑬

A very naive estimate of the Σ over k/k modes in the loop yields:

$$(\Delta m^2)_{\text{wall}} \sim \frac{1}{M_{\text{pl}} L^3} e^{-5R/L}$$

] a wide range of parameters for which this effect is much larger than 4d gravity mediation.

Several important questions:

1. Can one actually make very concrete models of e.g. CY 4-fold compactifications, where:

UV brane = bulk of CY

"IR branes" = dually represented by

G_2 -flux through cycles in the CY,

ala Klebanov-Strassler

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Using known results, e.g. superpotential for complex structure moduli in presence of Flux:

$$W = \int_{CY} \Omega_{(4,0)} \wedge G \quad GVW$$

should be possible to explicitly construct such models with (almost all) moduli stabilized, and with hierarchies (due to discrete fine tuning of fluxes).

Work in prog.
w/ Giddings and Polchinski

2. Can one use features of these warped models (like the SUSY mechanism discussed here) in real model building? Advantages / disadvantages

relative to $\left\{ \begin{array}{l} \text{gauge} \\ \text{4d gravity} \\ \text{anomaly} \end{array} \right.$ mediation?