SUPERGRAVITY DUAL OF A CASCADING CONFINING GAUGE THEORY

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Talk at Strings 2001

Mumbai, Jan. 9

OUTLINE

1. D3-branes on the conifold.
$AdS_5 \times T^{11}$ and its $SU(N) \times SU(N)$ dual.

2. Adding wrapped D5-branes to get
$SU(N+M) \times SU(N)$
Warped conifold and RG cascade.

3. Infrared dynamics $\rightarrow$ warped deformed conifold!

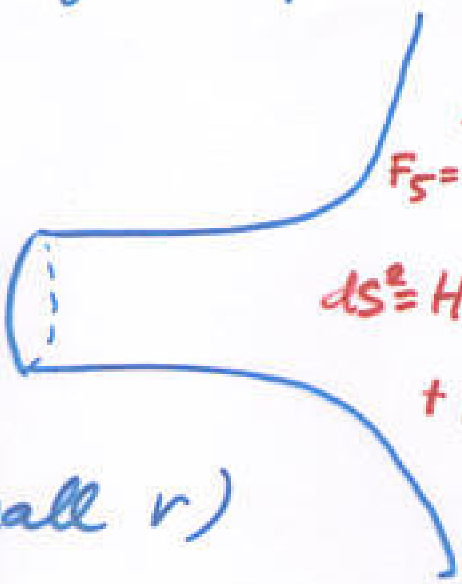
4. Chiral symmetry breaking, confinement, baryons, glueballs, domain walls, etc.

5. 2-point function: dimensional transmutation

6. Gravity duals of fractional Dp-branes with $p \neq 3$.

Brief review of the AdS/CFT duality.

Compare a stack of many coincident D3-branes with its supergravity background (Gubser, IK, Tseytlin, ...)



$\Phi = \text{const}$;

$F_5 = N \, \mathrm{vol}(S^5) + *$;

$ds^2 = H^{-\frac{1}{2}}(r)\, dx_\mu^2 + H^{\frac{1}{2}}(r)(dr^2 + r^2 d\Omega_5^2)$,

$H(r) = 1 + \frac{L^4}{r^4}$.

N D3-branes

In the low energy (small r) limit we expect duality between SU(N) SYM with $\mathcal{N}=4$ SUSY and the $AdS_5 \times S^5$ throat of the 3-brane (Maldacena).

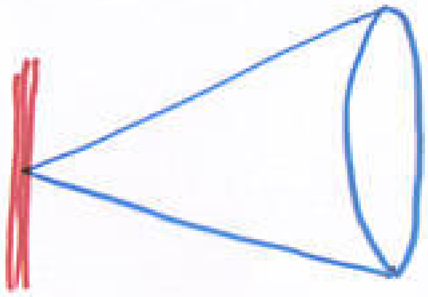
Comparison of tensions in gravity and D-brane theory gives (Gubser, IK, Peet) $L^4 = \frac{\kappa N}{2\pi^{5/2}} = (2 g_{YM}^2 N)\alpha'^2$.

Gravity gives exact predictions in the limit $L \gg l_s$; i.e. $g_{YM}^2 N \gg 1$ (large 't Hooft coupling).

String σ-model corrections proceed in powers of $\frac{\alpha'}{L^2} \sim (g_{YM}^2 N)^{-\frac{1}{2}}$; string loops in powers of $\frac{\kappa^2}{L^8} \sim \frac{1}{N^2}$.

It is possible to break some of the SUSY without destroying conformal invariance.

Place D3-branes at the apex of a Ricci-flat 6-d cone Y_6, whose base is X_5 (Kachru, Silverstein; ...)



X_5 is an Einstein manifold:

$$R_{ij} = 4 g_{ij} ;$$

The metric of Y_6 is

$$ds_6^2 = dr^2 + r^2 g_{ij} d\theta^i d\theta^j .$$

The metric produced by N D3-branes is

$$ds^2 = H(r)^{-\frac{1}{2}} dx_{\parallel}^2 + H(r)^{\frac{1}{2}} \left(dr^2 + r^2 g_{ij} d\theta^i d\theta^j \right) ,$$

$$H(r) = 1 + \frac{L^4}{r^4} ; \qquad L^4 \sim g_s N \alpha'^2 ;$$

In the throat ($r \Rightarrow 0$) limit we find

$$ds^2 \rightarrow \frac{r^2}{L^2} dx_{\parallel}^2 + \frac{L^2}{r^2} dr^2 + L^2 g_{ij} d\theta^i d\theta^j ,$$

which describes the space $AdS_5 \times X_5$.

Type IIB theory on this background is expected to be dual to the IR limit of the field theory of N D3-branes at the conical singularity.

A rather simple example of Y_6 is the non-compact Calabi-Yau manifold known as the conifold.

It is defined in $\mathbb{C}^4$ by

$$\sum_{i=1}^{4} z_i^2 = 0 .$$

Before inserting D3-branes we have $N=2$ SUSY in $D=4$; with the D3-branes we find $N=1$.

The base of the conifold is Einstein space

$T^{11} = (SU(2) \times SU(2))/U(1)$;

$U(1)$ is generated by $\sigma^3_L + \sigma^3_R$.

T^{11} is a $U(1)$ bundle over $S^2 \times S^2$, and its metric is

$$g_{ij}\, d\theta^i d\theta^j = \frac{1}{9}\left(d\psi + \cos\theta_1\, d\varphi_1 + \cos\theta_2\, d\varphi_2\right)^2 + \frac{1}{6}\sum_{a=1}^{2}\left(d\theta_a^2 + \sin^2\theta_a\, d\varphi_a^2\right),$$

$\theta_a \in [0, \pi]$; $\varphi_a \in [0, 2\pi)$; $\psi \in [0, 4\pi)$.

Type IIB on $AdS_5 \times T^{11}$ should be dual to the IR limit of the $N=1$ field theory on N D3-branes placed at $z_i = 0$ (IK, Witten).

$$ds^2_{\text{throat}} = \frac{r^2}{L^2}\left(-dt^2 + dx^2\right) + L^2\left(\frac{dr^2}{r^2} + g_{ij}\, d\theta^i d\theta^j\right)$$

Construction of the dual field theory is motivated by describing the conifold as

$$z_1 z_2 - z_3 z_4 = 0$$ (after a linear change of variables). This equation is "solved" by

$z_1 = A_1 B_1$; $z_2 = A_2 B_2$; $z_3 = A_1 B_2$; $z_4 = A_2 B_1$;

For N D3-branes we find a $U(N) \times U(N)$ gauge theory with fields   I.K., Witten; Morrison, Plesser

A_1, A_2 in $(N, \bar{N})$ and

B_1, B_2 in $(\bar{N}, N)$,

and an <u>exactly</u> marginal superpotential

$$W = \frac{\lambda}{2} \epsilon^{ij} \epsilon^{kl} \operatorname{Tr}\left(A_i B_k A_j B_l\right) = \lambda \operatorname{Tr}\left[A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1\right]$$

W has $SU(2) \times SU(2)$ symmetry, with $SU(2)_L$ acting on A_i and $SU(2)_R$ acting on B_k.

In the infrared the $U(1)$ factors decouple, and the interacting gauge theory is $SU(N) \times SU(N)$.

The A's and the B's have R-charge $\frac{1}{2}$ which insures that the R-symmetry is anomaly free.

W has $R = 2$ which is the marginal value.

Topologically, $T^{11} \sim S^2 \times S^3$.

If we add M D5-branes wrapped over the S^2 to the N D3-branes at the singularity, then the gauge group changes to $SU(N+M) \times SU(N)$. (Gubser, IK).

To implement the SUGRA dual of this non-conformal $\mathcal{N}=1$ gauge theory, we add M units of the 3-form R-R flux

$$\int_{S^3} H^{RR} = M$$ (produced by the D5's)

to the N units of 5-form flux produced by the regular D3-branes

$$\int_{T^{11}} F_5 = N.$$

H^{RR} induces the radial dependence of

$\int_{S^2} B^{NS-NS}$, which translates into flow of $\frac{1}{g_1^2} - \frac{1}{g_2^2}$

(Nekrasov, IK)

It is further possible to find exact SUGRA solution including the back-reaction of the 3-form field strengths (IK, Tseytlin)

$$B_{NS-NS} = 3 g_s M \ln(r/r_0)\, \omega_2$$

$$H_{RR} = M e^4 \wedge \omega_2$$

$$ds_{10}^2 = h^{-\frac{1}{2}}(r)(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + h^{\frac{1}{2}}(r)(dr^2 + r^2 ds_{T^{11}}^2)$$

where $h(r) = b_0 + \dfrac{4\pi g_s N + \text{const}\,[(g_s M)^2 \ln(r/r_0) + (g_s M)^2/4]}{r^4}$

The "improved" 5-form field strength

$$F_5 = dC_4 + B_{NS-NS} \wedge H_{RR} = \mathcal{F}_5 + *\mathcal{F}_5$$

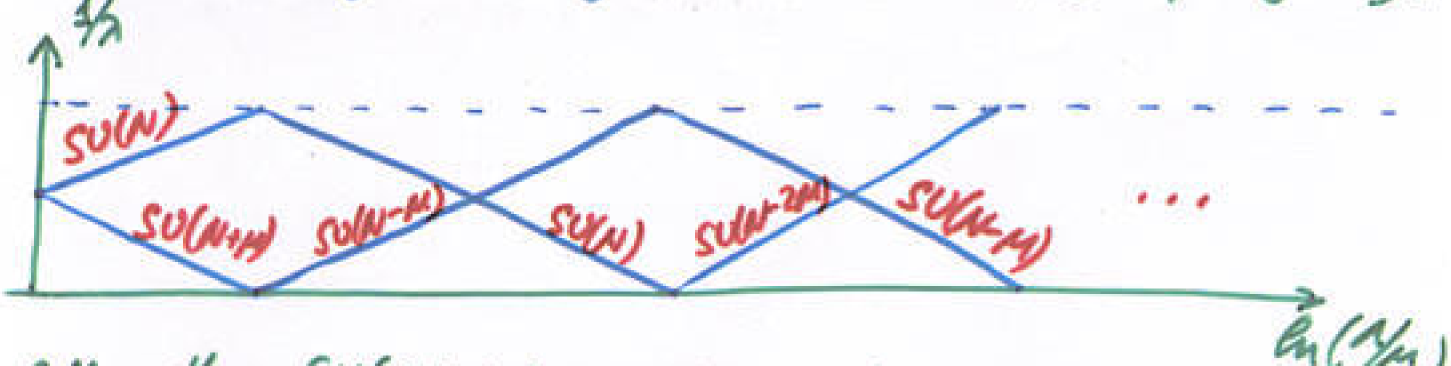
$$\mathcal{F}_5 = \left[N + \frac{\text{const}}{4\pi} g_s M^2 \ln(r/r_0)\right] \text{vol}(T^{11}).$$

This is the background of N D3's and M D5's wrapped over the S^2 at the conifold singularity. For $M=0$ it reduces to the standard 3-brane solution.

The effective 3-brane charge $\int_{T^{11}} F_5$ has become r-dependent!

As $\int_{S^2} B_{NS-NS}$ goes through a period, $N \to N - M$

On the FT side the jumps are due to the $\mathcal{N}=1$ Seiberg duality (IK, Strassler; in progress).



After the $SU(N+M)$ coupling diverges, application of duality gives $SU(N_f - N_c) = SU(N-M)$ group.

Suppose that $N = 10^6 M + p$; $p \sim \mathcal{O}(1)$.

After 10^6 jumps we reach $SU(p) \times SU(M+p)$, and further jumps are impossible.

If $g_s M \gg 1$ then the SUGRA background

$$h(r) \sim \frac{(g_s M)^2 \ln(r/r_*)}{r^4}$$

appears to have small curvatures.

Note also the appearance of the $SU(M)$ 't Hooft coupling $g_s M$.

However, there is a naked singularity at $r = r_*$.

Consider the case $p=1$: only 1 regular brane is left at the bottom of the cascade.

Find $SU(M+1)$ gauge theory coupled to C_1, C_2 in $M+1$; D_1, D_2 in $\overline{M+1}$

Define gauge invariants $N_{ij} = C_i D_j$.

$$W = \lambda \underbrace{N_{ij} N_{ke} \epsilon^{ik} \epsilon^{je}}_{2 \det N_{ij} = \mathcal{D}} + (M-1) \left[\frac{2\Lambda^{3M+1}}{N_{ij} N_{ke} \epsilon^{ik} \epsilon^{je}} \right]^{1/M-1}$$

ADS term.

Varying W, find $\mathcal{D}^M = \dfrac{2\Lambda^{3M+1}}{\lambda^{M-1}}$

The M different solutions are related by the non-anomalous $\mathbb{Z}_{2M}$ R-symmetry which acts as $\mathcal{D} \longrightarrow \mathcal{D} e^{2\pi i/M}$: $\mathbb{Z}_{2M}$ is broken to $\mathbb{Z}_2$.

The deformed equation for SUSY vacua is

$\det N_{ij} = \text{const}$

This is the deformed conifold!

For large M, $\mathbb{Z}_{2M}$ is essentially $U(1)$.

Define $z_{ij} = \frac{1}{\sqrt{2}} \begin{pmatrix} z_3 + i z_4 & z_1 + i z_2 \\ z_1 - i z_2 & -z_3 + i z_4 \end{pmatrix}$

det $z_{ij} = -\frac{\epsilon^2}{2} \Longrightarrow \sum_{a=1}^{4} z_a^2 = \epsilon^2$;

The deformation breaks the $U(1)_R$ symmetry

$z_a \to e^{i\alpha} z_a$ down to Z_2 (χSB)

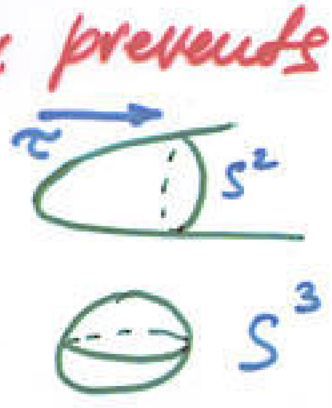
but preserves the global $SU(2) \times SU(2) \sim SO(4)$

Replacing conifold → deformed conifold cures the naked singularity problem!

I.K., M. Strassler

Geometrical reason: at the apex S^2 shrinks but S^3 remains finite.

Conserved RR F_3 flux prevents the S^3 from collapsing.



The 10-d metric is a

WARPED DEFORMED CONIFOLD

$$ds_{10}^2 = h^{-\frac{1}{2}}(\tau)\, dx_{11}^2 + h^{\frac{1}{2}}(\tau)\, ds_6^2$$

ds_6^2 is the CY metric on $\sum_{a=1}^{4} z_a^2 = \epsilon^2$ and

τ is its radial variable.

The complex 3-form $G_3 = F_3 + \frac{i}{g_s} H_3$ is

a (2,1) form.

This insures $\mathcal{N}=1$ SUSY of the background

Polchinski, Graña
Gubser.

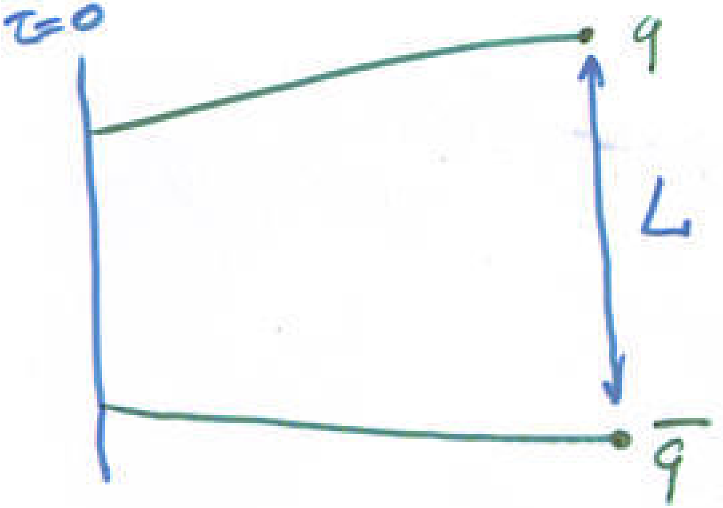
G_3 is known explicitly;

For large τ, $h(\tau) \to \frac{\ln(r/r_*)}{r^4}(g_s M)^2$

small τ, $h(\tau) \to$ const of order $(g_s M)^2$.

$\tau = 0$ is neither a naked singularity nor a horizon!

This implies AREA LAW for Wilson loops.



The $q\bar{q}$ potential is linear since most of the string is near $\tau=0$ where the longitudinal metric is $\sim \frac{1}{g_s M} dx_{\parallel}^2$.

The spectrum of glueballs (normalizable wave functions) on the entire smooth background is DISCRETE (Caceres, Hernandéz; Krasnitz)

The presence of a finite S^3 far in the IR (at $\tau=0$) accounts for much of the "phenomenology" of a confining $\mathcal{N}=1$ theory.

1. A D3-brane wrapped over the S^3 is a BARYON VERTEX connecting M strings (external quarks).

2. A D5-brane wrapped over the S^3 is a DOMAIN WALL in $R^{3,1}$ separating two adjacent vacua.

For large $g_s M$ curvatures are small everywhere $\Rightarrow$ we find a reliable SUGRA dual of a cascading, confining theory.

Is the cascading theory a local QFT?

Study the 2-point function of the operator which corresponds to a massless minimally coupled scalar ϕ (M. Krasnitz).

For large τ the equation is determined by the asymptotic metric:

$$\left[r^{-5}\partial_r r^5 \partial_r - A^2 k^2 \frac{r_*^4}{r^4} \ln \frac{r}{r_*}\right] \phi_k(r) = 0;$$

$A \sim g_s M$; k_μ is the 4-momentum.

Introducing $y = \frac{A k r_*^2}{r}$, we have

$$\left[y^3 \partial_y y^{-3} \partial_y - \ln \frac{Y}{y}\right] \phi(y) = 0;$$

$Y = A k r_*$;

The boundary is at $y = 0$.

As for $m=0$ field in AdS_5 we impose the boundary condition $\phi(0) = 1$.

For $y \ll \frac{1}{\sqrt{\ln Y}}$, the solution is

$$\phi_{UV} = 1 - \frac{1}{4} y^2 \ln \frac{Y}{y} + y^4 \left[\frac{1}{48} \ln^3 \frac{Y}{y} + \frac{1}{64} \ln^2 \frac{Y}{y} + \frac{1}{128} \ln \frac{Y}{y} + c\right]$$

C is determined by matching to the form of the solution for $y \gg \frac{1}{y}$;

$$C = -\frac{1}{16}\ln^3(Akr_*)\,; \quad \text{for } Akr_* \gg 1\,;$$

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim \phi r^5 \partial_r \phi\big|_{r=\infty} \sim g_s^2 M^4 k^4 \ln^3(Akr_*)$$

In position space, for $|x| \ll Ar_*$;

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \sim g_s^2 M^4 \frac{\ln^2\left(\frac{|x|}{Ar_*}\right)}{|x|^8} \sim N_{eff}^2(x)\,|x|^{-8};$$

where $N_{eff}(x) \sim g_s M^2 \ln\frac{Ar_*}{|x|}$;

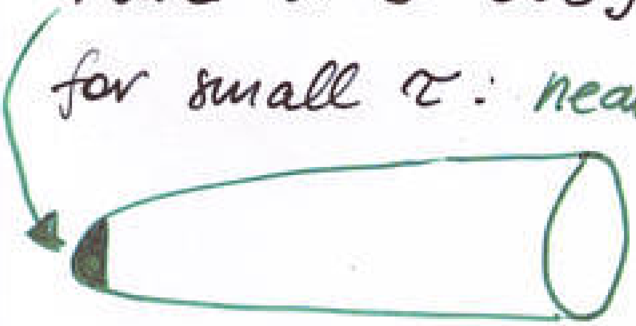
This suggests that the cascading theory <u>is</u> a local gauge theory with a scale dependent number of colors.

The glueball masses are determined by normalizable solutions to $\Box\phi = 0$ in the full metric!

$m_n = \frac{c_n}{Ar_*}$; c_n is a <u>discrete</u> spectrum.

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \sim e^{-m_0|x|} \quad \text{for } |x| \gg Ar_*;$$

Pure $\mathcal{N}=1$ SUSY $SU(M)$ theory is found for small τ: near the apex.



Cascade jumps occur every $\Delta\tau \sim \frac{1}{g_s M}$

To zoom in on the pure glue region we need to take the limit $g_s M \to 0$.

This necessitates studying full STRING σ-MODEL on the WARPED DEFORMED CONIFOLD WITH R-R (and NS-NS) 3-form field strengths.

SUGRA gives the strong coupling limit of our exotic cascading $\mathcal{N}=1$ theory which flows to pure $SU(M)$ in the IR.

Gravity Dual of N Dp-branes + M wrapped D(p+2)-branes (C. Herzog, I.K.)

Related solutions were found by Cvetic, Lü, Pope.

$$ds_{10}^2 = h(r)^{-\frac{1}{2}} dx_{11}^2 + h(r)^{\frac{1}{2}} (dr^2 + r^2 ds_{8-p}^2) \; ;$$

$$e^{4\Phi} = h(r)^{3-p} \; ; \qquad F_{0\ldots pr} = \partial_r h^{-1} .$$

ds_{8-p}^2 is the Einstein metric on the base of the 9-p dimensional Ricci flat cone, X_{8-p}.

$$B_{NS-NS} = Q \frac{r^{p-3}}{p-3} \omega_2 \; ; \qquad F_{6-p} \sim Q *_{8-p} \omega_2$$

ω_2 is the harmonic 2-form dual to the wrapped cycle.

$$h(r) = \frac{P}{r^{7-p}} - \frac{Q^2}{(3-p)(10-2p) r^{10-2p}} \; ; \quad p<5.$$

$P \sim g_s N \; ; \quad Q \sim g_s M .$

For $p<4$ there is a naked singularity in the IR, where h vanishes.

An Explicit Example of Duality

$$p = 1 \; ; \quad X_7 = Q^{111} = \frac{SU(2) \times SU(2) \times SU(2)}{U(1) \times U(1)}$$

Q^{111} has $SU(2)^3 \times U(1)$ symmetry.

The dual $\mathcal{N}=2$ 1+1 dimensional theory has gauge group $U(N+M) \times U(N) \times U(N)$ coupled to

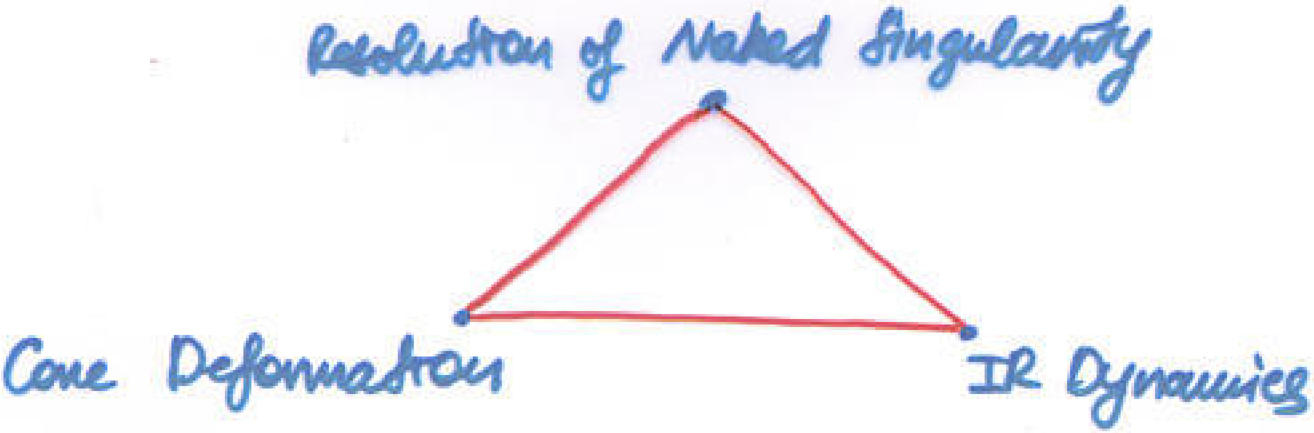
$$A_1, A_2 \quad \text{in} \quad (N+M, \bar{N}, 1)$$
$$B_1, B_2 \quad \text{in} \quad (1, N, \bar{N})$$
$$C_1, C_2 \quad \text{in} \quad (\overline{N+M}, 1, N).$$

These chiral superfields realize the three $SU(2)$'s, respectively.

Varying RR flux $\Rightarrow$ reduction of N in the IR.

This may be due to Higgsing (see Aharony, hep-th/0101013).

We expect



To Be Continued...