Boundary String Field Theory of the $D\bar{D}$ System

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Outline

- Define BSFT for $D\bar{D}$ system.
- Derive various terms in action for tachyon + gauge field.
- Show that lower dimensional D-branes are solitons.
- Derive RR-couplings:

$D\bar{D}$:

$$S_{wz} = \int C \wedge \mathrm{Str}\, e^{2\pi i \mathcal{F}}$$

$$\mathcal{F} = d\mathcal{A} - i\mathcal{A} \wedge \mathcal{A}$$

$$\mathcal{A} = \begin{pmatrix} iA^+ & \bar{T} \\ T & iA^- \end{pmatrix}$$

non-BPS Dp:

$$S_{wz} = \int C \wedge \mathrm{tr}\, e^{2\pi (iF - T^2 + DT)}$$

- Show that solitons have correct RR-charge.

$$Q_{RR} = \mathrm{ind}(Q) = \mathrm{ind}\begin{pmatrix} i\not{\partial} + \not{A}^+ & \bar{T} \\ T & i\not{\partial} + \not{A}^- \end{pmatrix}$$

BSFT approach

(Witten; Shatashvili; Gerasimov, Shatashvili; Kutasov, Marino, Moore)

- Theory on space of all two dimensional field theories conformal in bulk but with arbitrary boundary interactions.

- Classical spacetime action for superstring is disk partition function:

$$S(\lambda_i) = Z(\lambda_i) = \int D\phi \, e^{-S_{bulk} - S_{bndy}}$$

$$S_{bndy} = \sum_i \lambda_i \mathcal{O}^i$$

- Closely related to σ-model approach

(Fradkin, Tseytlin; Callan et. al. Andreev, Tseytlin)

⋮

Organization of computations

1) Integrate over fields in bulk with fixed boundary conditions $\phi_i(\tau)$

- We'll consider only NS-NS and RR vacua.

$$\Psi_{bulk}[\phi_i(\tau)] = \int D\phi_{bulk}\, e^{-S_{bulk}}$$

2) Include boundary interaction:

$$\Psi_{bndy}[\phi_i(\tau)] = e^{-S_{bndy}}$$

3) Integrate over boundary values:

$$Z = \int D\phi\, e^{-S_{bulk} - S_{bndy}}$$

$$= \int D\phi(\tau)\, \Psi_{bulk}\, \Psi_{bndy}$$

4) Renormalize by zeta-function

$$S = Z_{ren}$$

Bulk wavefunctional

- bulk action for NSR string: $(\alpha' = 1)$

$$S_{bulk} = \frac{1}{4\pi}\int d^2z \left(2\partial X^\mu \bar\partial X^\mu + \psi^\mu \bar\partial \psi^\mu + \tilde\psi^\mu \partial \tilde\psi^\mu\right)$$

- boundary conditions (NS sector)

$$X^\mu(\tau) = x_0^\mu + \sqrt{\tfrac{1}{2}}\sum_{n=1}^{\infty}\left(X_n^\mu e^{in\tau} + X_{-n}^\mu e^{-in\tau}\right)$$

$$\psi^\mu(\tau) = \tilde\psi^\mu(\tau) = \sum_{r=\frac{1}{2}}^{\infty}\left(\psi_r^\mu e^{ir\tau} + \psi_{-r}^\mu e^{-ir\tau}\right)$$

- S_{bulk} for solution with these boundary conditions,

$$S_{bulk} = \frac{1}{2}\sum_{n=1}^{\infty} n X_{-n}^\mu X_n^\mu + i\sum_{r=\frac{1}{2}}^{\infty}\psi_{-r}^\mu \psi_r^\mu$$

$$\Psi_{bulk} = e^{-S_{bulk}}$$

Boundary interaction

- Demand spacetime gauge invariance:

$$\delta A^{\pm}_{\mu} = \partial_\mu \alpha^{\pm} + i[\alpha^{\pm}, A^{\pm}_{\mu}]$$

$$\delta T = -iT\alpha^{+} + i\alpha^{-}T$$

and worldsheet supersymmetry

$$X^{\mu} = x^{\mu} + \theta\psi^{\mu}$$

$$D = \partial_\theta + \theta\partial_\tau$$

- Gauge fields and tachyons naturally packaged as

$$M(X) = \begin{pmatrix} iA^{+}_{\mu}DX^{\mu} & \overline{T} \\ T & iA^{-}_{\mu}DX^{\mu} \end{pmatrix}$$

- option 1: path ordering:

$$e^{-S_{bndy}} = \mathrm{tr}\, \hat{P}\, e^{i\int d\tau d\theta\, M(X)}$$

susy path ordering

Path ordering is awkward!

Boundary fermions (Marcus, Sagnotti; Witten; Harvey, Kutasov, Martinec; Kutasov, Marino, Moore)

- Consider $N = 2^{m-1}$ $D9\overline{D9}$ pairs.

$$M(X) = \sum_{k=0}^{2m} \frac{1}{2k!} M^{I_1 \cdots I_k} \gamma^{I_1 \cdots I_k}$$

$\uparrow$ $SO(2m)$ γ-matrices

- Introduce $2m$ boundary fermion superfields:

$$\Gamma^I = \eta^I + \theta F^I, \qquad I = 1 .. 2m$$

$$S(\Gamma^I) = -\int d\tau d\theta \, \tfrac{1}{4} \Gamma^I D \Gamma^I$$

$$\Rightarrow \{ \eta^I, \eta^J \} = 2\delta^{IJ}$$

- So replace $\gamma^I \to \Gamma^I$

$$\boxed{S_{bndy} = -\int d\tau d\theta \left(\tfrac{1}{4} \Gamma^I D \Gamma^I + \sum \tfrac{1}{2k!} M^{I_1 \cdots I_k} \Gamma^{I_1} \cdots \Gamma^{I_k} \right)}$$

- Manifest gauge symmetry is

$$U(1) \times SO(2m) \subset U(2^{m-1}) \times U(2^{m-1})$$

- No path ordering needed.

- Representation of matrices in terms of boundary fermions changes rule for matrix multiplication.

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \sum_k M^{I_1 \cdots I_k} \gamma^{I_1 \cdots I_k}$$

Use: $\gamma^I = \begin{pmatrix} 0 & \tilde{\gamma}^I \\ \tilde{\gamma}^I & 0 \end{pmatrix}$

with, e.g. $A = \begin{cases} \text{bosonic for} & (-)^a = +1 \\ \text{fermionic for} & (-)^a = -1 \end{cases}$

$$M \longrightarrow \sum M^{I_1 \cdots I_k} \Gamma^{I_1} \cdots \Gamma^{I_k}$$

- Multiply M, M' keeping Γ's to the right:

$$MM' = \sum M^{I_1 \cdots I_k} \Gamma^{I_1 \cdots I_k} M'^{J_1 \cdots J_{k'}} \Gamma^{J_1 \cdots J_{k'}}$$

$$= \sum (-)^{km'} M^{I_1 \cdots I_k} M'^{J_1 \cdots J_{k'}} \Gamma^{I_1 \cdots I_k} \Gamma^{J_1 \cdots J_{k'}}$$

where: $M'^{J_1 \cdots J_{k'}} = \begin{cases} \text{bosonic for} & (-)^{m'} = +1 \\ \text{fermionic for} & (-)^{m'} = -1 \end{cases}$

- In terms of matrices:

$$MM' = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} AA' + (-)^{c'} BC' & AB' + (-)^{d'} BD' \\ DC' + (-)^{a'} CA' & DD' + (-)^{b'} CB' \end{pmatrix}$$

- Will be important for getting correct tachyon covariant derivative.

- Now integrate out auxilliary fields F^I $(\Gamma^I = \eta^I + \theta F^I)$

$$S_{bndy} = -\int d\tau d\theta \left(\frac{1}{4} \dot{\eta}^I \eta^I + \sum \frac{1}{2k!} (M_1 - M_0^2)^{I_1 \cdots I_k} \eta^{I_1} \cdots \eta^{I_k} \right)$$

defined by earlier matrix mult. rule

$$M = M_0 + \theta M_1$$

- Explicitly:

$$M_0 = \begin{pmatrix} i A^+_\mu \psi^\mu & \bar{T} \\ T & i A^-_\mu \psi^\mu \end{pmatrix}$$

$$M_1 = \begin{pmatrix} i(A^+_\mu \dot{x}^\mu + \frac{1}{2}(dA^+)_{\mu\nu}\psi^\mu\psi^\nu) & \partial_\mu \bar{T} \psi^\mu \\ \partial_\mu T \psi^\mu & i(A^-_\mu \dot{x}^\mu + \frac{1}{2}(dA^-)_{\mu\nu}\psi^\mu\psi^\nu) \end{pmatrix}$$

- M_0^2 terms give $[A_\mu, A_\nu]$ in non-abelian field strength as well as well as correct non-abelian tachyon covariant derivative

Examples

- Take single $D9\,\overline{D9}$ pair $(m=1)$

$$S_{bndy} = -\int d\tau \Big(-\tfrac{1}{4} T^I \dot{T}^I + \tfrac{1}{4} \eta^I \dot{\eta}^I + \tfrac{1}{2} D_\mu T^I \psi^\mu \eta^I$$
$$+ \tfrac{i}{2}\left(\dot{x}^\mu A_\mu + \tfrac{1}{2} F_{\mu\nu} \psi^\mu \psi^\nu\right)$$
$$+ \tfrac{i}{4}\left(\dot{x}^\mu A_\mu^{IJ} + \tfrac{1}{2} F_{\mu\nu}^{IJ} \psi^\mu \psi^\nu\right) \eta^I \eta^J \Big)$$

$(I, J = 1, 2)$

- T charged with respect to relative gauge field A_μ^{IJ}.

- Interacting theory, so we can't work out partition function in closed form. Consider special cases:

1) $T^I =$ constant, $F_{\mu\nu} =$ constant, $F_{\mu\nu}^{IJ} = 0$

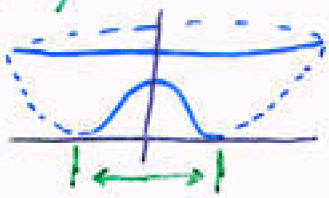
- tachyon and gauge fields decouple.

$$S = Z = 2T_{D9} \int d^{10}x \, e^{-2\pi T \bar{T}} \sqrt{\det(\eta_{\mu\nu} + 2\pi F_{\mu\nu})}$$

(Sen)

$T = \frac{1}{2}(T^1 + iT^2)$

- $V(T, \bar{T}) = 2T_{D9}\, e^{-2\pi T \bar{T}}$



coords. cover this region

3) Linear tachyon profiles, $A_\mu = A_\mu^{IJ} = 0$

- Relevant for describing D-branes as solitons
(Kutasov, Marino, Moore)

- By spacetime and gauge rotations take

$$T^I = u^I X^I , \quad I = 1, 2$$

- Path integral is Gaussian and yields:

$$S = \int \frac{d^{10}x}{(2\pi)^5} e^{-2\pi T\bar{T}} \prod_{I=1}^{2} F(\pi y^I)$$

$y^I = (u^I)^2$

$$F(x) = \sqrt{2\pi} \frac{\prod_{r=1/2}^{\infty} (1 + x/r)}{\prod_{n=1}^{\infty} (n+x)} \underset{\text{zeta}}{=} \frac{4^x \cdot x \cdot \Gamma(x)^2}{2\Gamma(2x)}$$

- Stationary point correspond to $y^I = 0$ or ∞.

a) $y^1 = \infty$, $y^2 = 0$, $\quad T = y^1 x^1$

- describes kink $\Longrightarrow$ non-BPS D8-brane (Sen)

$$S = 2T_{D9} \int d^{10}x \, e^{-\frac{\pi}{2} y^1 (x^1)^2} F(\pi y^1) \Big|_{y^1=\infty}$$

$$= 2\pi\sqrt{2}\, T_{D9} \int d^9x \Longrightarrow T_{\text{non-BPS } D8} = 2\pi\sqrt{2}\, T_{D9} \checkmark$$

b) $y^1 = y^2 = \infty$, $\quad T = y^1 x^1 + i y^2 x^2$

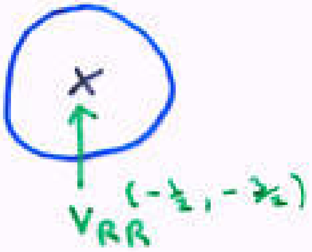
- describes vortex $\Longrightarrow$ BPS D7-brane (Sen)

RR couplings

- Turn on constant C field and find coupling generalizing

$$\int C \wedge e^{2\pi F}$$

- Bulk wavefunctional now obtained from



$$V_{RR}^{(-\frac{1}{2},-\frac{3}{2})} = S^a C_{ab} S^b e^{-\frac{1}{2}\phi(0)} e^{-\frac{3}{2}\tilde{\phi}(0)}$$

- Fermions become integer moded, and $C_{\mu_0 \cdots \mu_p}$ label zero mode wavefunctions:

$$\Psi^{RR}_{bulk} = \exp\left[-\frac{1}{2}\sum_{n=1}^{\infty} n X^\mu_{-n} X^\mu_n - i \sum_{n=1}^{\infty} \psi^\mu_{-n}\psi^\mu_n\right] \cdot \sum_{p\ odd} \frac{(-i)^{(9-p)/2}}{(p+1)!} C_{\mu_0 \cdots \mu_p} \psi_0^{\mu_0} \cdots \psi_0^{\mu_p}$$

- Want to compute

$$Z_{RR} = \int DX\, D\psi\, D\eta\, e^{-S_{bndy}} \Psi^{RR}_{bulk}$$

$$S_{bndy} = -\int d\tau \left(\frac{1}{4}\dot{\eta}^I \eta^I + \sum \frac{1}{2k!}(M_1 - M_0^2)^{I_1 \cdots I_k} \eta^{I_1} \cdots \eta^{I_k}\right)$$

- Focus on particular $C_{01\cdots p}$. Let all fields be independent of $X^0 \cdots X^p$. Partition function factorizes:

$$Z_{RR} = \left[\int \prod_{\mu=0}^{p} DX^\mu D\psi^\mu (-i)^{\frac{9-p}{2}} C_{0\cdots p} \psi_0^0 \cdots \psi_0^p e^{-S^{(1)}_{bulk}} \right]$$

$$\cdot \left[\int \prod_{\mu=p+1}^{9} DX^\mu D\psi^\mu D\eta \, e^{-S^{(2)}_{bulk} - S_{bndy}} \right]$$

- First factor easily computed.
Second is Witten index $\Rightarrow$ nonzero modes cancel by susy.

- Restricting to zero modes: $S_{bulk} = 0$,

$$M_0 = \begin{pmatrix} iA^+ & \bar{T} \\ T & iA^- \end{pmatrix} \equiv i\mathcal{A} = \text{superconnection}$$
(Quillen)

$$M_1 = \begin{pmatrix} idA^+ & d\bar{T} \\ dT & idA^- \end{pmatrix} \equiv i d\mathcal{A}$$

where, e.g. $dT = \partial_\mu T \psi_0^\mu$

- Combination appearing in action is curvature of $\mathcal{A}$

$$M_1 - M_0^2 = i(d\mathcal{A} - i\mathcal{A}\wedge\mathcal{A}) = i\mathcal{F} = \begin{pmatrix} iF^+ - T\bar{T} & D\bar{T} \\ DT & iF^- - \bar{T}T \end{pmatrix}$$

- Integrating over periodic η^I gives

$$Z_{RR} = T_{D9} \int C \wedge \underset{\uparrow}{\mathrm{Str}}\, e^{2\pi i \mathcal{F}}$$
(conjectured by Kennedy, Wilkins)

$\mathrm{Str}\, M = \mathrm{tr} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} M$

- Can now compute RR charge of solitons.

ABS: $\gamma^i = SO(2m)$ γ-matrices

$$\gamma^{i=1\ldots 2m-1} = \begin{pmatrix} 0 & \tilde{\gamma}^i \\ \tilde{\gamma}^i & 0 \end{pmatrix}, \quad \gamma^{2m} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Tachyon profile: $\begin{pmatrix} 0 & \bar{T} \\ T & 0 \end{pmatrix} = u\gamma^i x^i$

- Nonvanishing RR coupling is:

$$S = T_{D9} \int C \wedge \frac{1}{(2m)!} e^{-2\pi u^2 \vec{x}^2} \, \mathrm{Str}\,(2\pi u \gamma^i dx^i)^{2m}$$

$$= T_{D(9-2m)} \int C_{10-2m}$$

RR coupling for non-BPS D-brane

$(-)^{F_L}$ orbifold: $T = \bar{T}$, $A^+ = A^-$, omit η^{2m}
(Sen)

$$S = \frac{T_{D9}}{\sqrt{2}} \int C \wedge \mathrm{Str}\, e^{2\pi i \mathcal{F}}$$

$$\mathcal{F} = \begin{pmatrix} iF - T^2 & DT \\ DT & iF - T^2 \end{pmatrix}, \quad \mathrm{Str}\, M = \mathrm{tr}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M$$

or $$S = \sqrt{2}\, T_{D9} \int C \wedge \mathrm{tr}\, e^{2\pi(iF - T^2 + DT)}$$

- dT term computed by (Billo, Craps, Roose; Kutasov, Marino, Moore)

RR charge as index

- S_{WZ} came from path integral in susy QM with periodic boundary conditions.

$$Z = \int_{\text{per. b.c.}} D\phi\, e^{-S} = \mathrm{Tr}(-)^F e^{-\beta H} = \mathrm{ind}(Q)$$

- Find index theorem by writing operator form of Q (Witten; Alvarez-Gaumé; Friedan, Windey)

- Use action:

$$S = \frac{1}{4}\int d\tau\, d\theta\, DX^\mu D^2 X^\mu$$

$$-\int d\tau \left(\frac{1}{4}\dot{\eta}^I \eta^I + \sum \frac{1}{2k!}(M_1 - M_0^2)^{I_1 \cdots I_k} \eta^{I_1} \cdots \eta^{I_k} \right)$$

- Susy transformations:

$$\delta X^\mu = \epsilon \psi^\mu$$

$$\delta \psi^\mu = \epsilon \dot{X}^\mu$$

$$\delta \eta^{I_1} = F^{I_1} = \sum_{k=1}^{2m} \frac{(-)^k}{(k-1)!} M_0^{I_1 \cdots I_k} \eta^{I_2} \cdots \eta^{I_k}$$

auxilliary field

- Canonical quantization

$$[x^\mu, p_\nu] = i\delta^\mu{}_\nu$$

$$\{\psi^\mu, \psi^\nu\} = -2\delta^{\mu\nu}$$

$$\{\eta^I, \eta^J\} = 2\delta^{IJ}$$

- Supercharge works out to be

$$Q = i\psi^\mu p_\mu - \sum \frac{1}{2k!} M_0^{I_1 \cdots I_k} \eta^{I_1} \cdots \eta^{I_k}$$

$$= i\psi^\mu p_\mu - i\mathcal{A}$$

- Represent commutation relations by

$$p_\mu \to -i\partial_\mu, \quad \psi^\mu \to i\gamma^\mu, \quad \eta^I \to \gamma^I$$

$$\Longrightarrow \quad Q = \begin{pmatrix} i\not{\partial} + \not{A}^+ & \bar{T} \\ T & i\not{\partial} + \not{A}^- \end{pmatrix}$$

- Keeping track of zero mode norm., index theorem is:

$$\mathrm{ind}\begin{pmatrix} i\not{\partial} + \not{A}^+ & \bar{T} \\ T & i\not{\partial} + \not{A}^- \end{pmatrix} = \left(\frac{-i}{4\pi^2}\right)^n \int \mathrm{Str}\, e^{2\pi i \mathcal{F}}$$

2n dim. manifold

- Index counts zero eigenvalues weighted by

$$(-)^F = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$$

- RR charge of solitons given by index.

Conclusion

- Three approaches to constructing D-branes as solitons

1) Level truncation
    - violates gauge invariance
    - involves infinite number of fields

2) NC geometry
    - Solitons have gauge field which sets covariant derivatives to zero (finite B).

3) BSFT
    - Solitons have vanishing gauge field

(1) vs. (2), (3): Need transformation involving full string field.

(2) vs. (3): Generalization of Seiberg-Witten map which can take solutions with $A_\mu = 0$ to solutions with $A_\mu \neq 0$.