On the thermodynamics of Little String Theory and 2d String Theory

* D. Sahakyan, D.K. ; hep-th/0012258
* V. Kazakov, I. Kostov, D.K. ; hep-th/0101011

The problem: dynamics at singularities in string theory

$R^{d-1,1} \times$ 

(equivalently, NS5-branes wrapped around surfaces)

- At small g_s get a non-trivial non-local theory (LST) decoupled from bulk physics.

- Degrees of freedom at singularity?
  (branes wrapped around vanishing cycles)

- Main tool for studying LST:
  "near-horizon geometry" of singularity.

ABKS

Many applications

Berkooz
Rozali
Seiberg

* Matrix theory.

* QCD strings (NS5-branes wrapped around Riemann surfaces)

Antoniadis
Dimopoulos
Giveon

* Brane worlds.

A closely related model

Two dimensional string theory:
$R^{1,1}$ with linear dilaton along spatial direction
($d=1$ version of general story)

Here, there is an independent formulation of the dynamics:
Matrix Quantum Mechanics in a double scaling limit

A very interesting toy model of LST.

In this lecture I will describe the high energy thermodynamics of these systems.

Main results:

High energy thermodynamics

One expects thermodynamics to be dominated by black holes (or black branes):

<u>Example 1:</u> N flat NS5-branes

Near-extremal, near-horizon geometry:

Callan-Harvey-Strominger
Horowitz-Strominger
Maldacena-Strominger

$$\mathcal{M} = \frac{SL(2)}{U(1)} \times S^3 \times R^5$$

dilaton: $e^{2\Phi_0} = \frac{N}{\mu}$ $\quad \xrightarrow{\quad} \quad e^{\Phi} \to 0$

$\mu \simeq$ energy density on fivebranes.

Example 2: 2d string theory

Euclidean black hole:

Witten $\qquad \mathcal{M} = SL(2)/U(1)$

So, as usual in BH thermodynamics, string loop expansion is a $\frac{1}{E}$ expansion.

* High energy density of states is:

$$\rho(E) \sim E^{\alpha} e^{\beta_H E}$$

β_H, α will be computed; α depends in an interesting way on the parameters defining the model.

- - - - - - - - - - - - -

* Thermodynamics is unstable; there is a finite temperature phase transition at $T \approx \frac{1}{\beta_H}$.

- - - - - - - - - - - - - - - - -

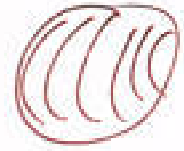
* There is an unstable mode (tachyon) in near-horizon geometry associated with above instability.

The canonical free energy

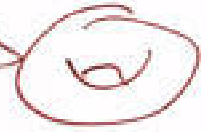
$$-\beta F \equiv \log Z$$

is related to the single string partition sum on $\mathcal{M}$ as follows:

$$-\beta F = Z_{\text{string}} = e^{-2\Phi_0} Z_0 +$$



$$+ Z_1 +$$



Z_h = genus h string partition sum

$$+ e^{2\Phi_0} Z_2 +$$

$e^{2\Phi_0} \sim \frac{1}{E}$

$$+ \dots$$

Genus zero free energy, Z_0

$$Z_0(\quad \times S^3 \times R^5) = Z_0(\quad) \times \underbrace{Z_0(S^3)}_{(\sqrt{N\alpha'})^3} \times \underbrace{Z_0(R^5)}_{V_5}$$

$$Z_0\left(\frac{SL(2)}{U(1)}\right) = ?$$

Bosonic zero modes:

- Divergence due to volume of $SL(2)$
- Have to divide by $\mathrm{vol}(SL(2))$ due to conformal Killing group of sphere.

finite result

Fermionic zero modes make $Z_0 = 0$ due to "accidental" $N=2$ superconform. symmetry of the cigar $N=1$ SCFT.

So, classically we find $F=0$.

This is consistent with thermodynamics:

$e^{2\Phi_0} = \frac{N}{\mu}$

$\beta_H = 2\pi\sqrt{N\alpha'}$

Temperature, $T_H = \frac{1}{\beta_H}$, independent of energy density on fivebranes:

$$\beta = \frac{\partial S}{\partial E} = \beta_H \quad \Rightarrow \quad S = \beta_H E$$

and:

$$-\beta F = S - \beta_H E = 0$$

Since to leading order in $\frac{1}{E}$ the thermodynamics is degenerate, it is of interest to compute $\frac{1}{E}$ corrections.

One expects: $S(E) = \beta_H E + \alpha \log E + \ldots$

$\left(\rho(E) \cong E^{\alpha} e^{\beta_H E} \right)$

which implies:

$$Z(\beta) = e^{-\beta F} \approx \int_0^\infty dE \, \rho(E) \, e^{\beta_H E} \approx (\beta - \beta_H)^{-\alpha - 1}$$

$$\Rightarrow \quad \beta F \sim (\alpha+1) \log(\beta - \beta_H)$$

$$E \approx \frac{\alpha+1}{\beta - \beta_H}$$

or: $$\boxed{\beta F \approx -(\alpha+1)\log E + O\left(\frac{1}{E}\right)}$$

Computing α is of interest for thermodynamics:

* sign of α determines thermodynamic (in)stability.

* Dependence of α on parameters (e.g. N, V_5) gives hint regarding microscopic d.o.f.

Calculation of α

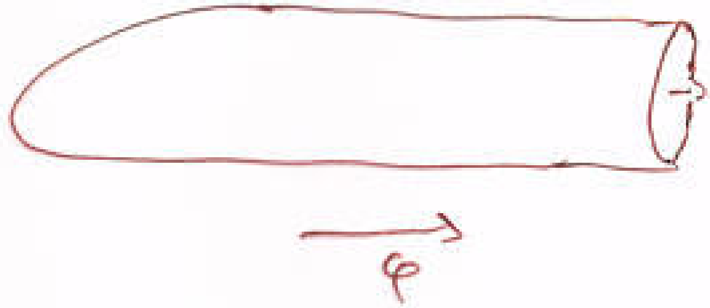
From the point of view of perturbative ($\frac{1}{E}$) expansion, $F \sim \log E$ corresponds to the torus (genus 1) contribution.

In particular, we expect Z_1 to go like $\log E$.

We next explain the origin of this "logarithmic scaling violation".

Consider the torus partition sum on the cigar:

$Z_{cigar}(\tau,\bar{\tau}) = ?$

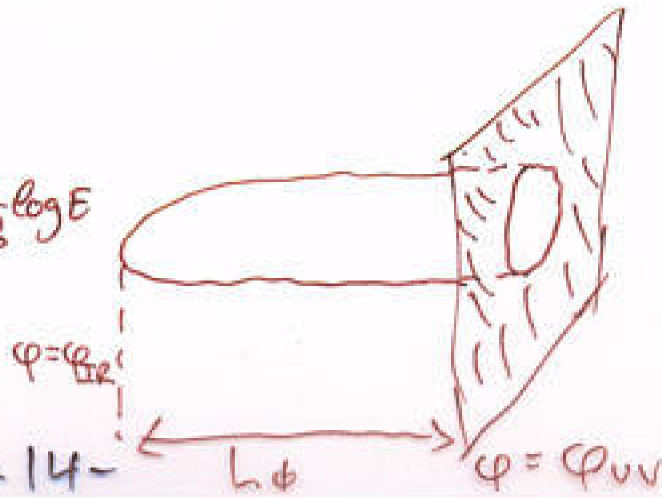


It is divergent due to zero mode integral over φ (from region far from tip).

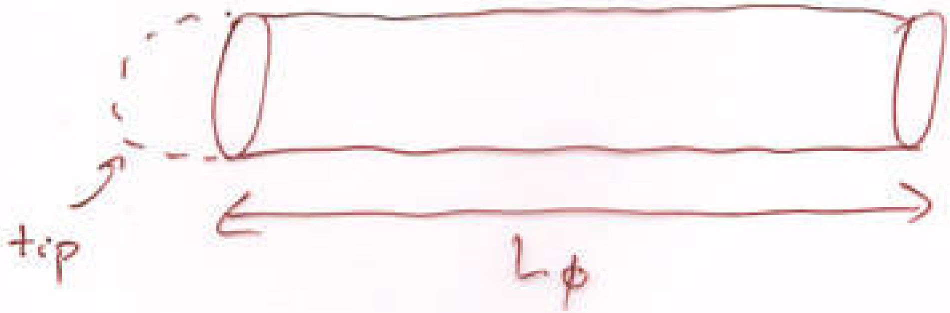
To regularize, cut-off the cigar at some $\varphi = \varphi_{UV}$:

$L_\phi = \varphi_{UV} - \varphi_{IR} \sim const - \frac{1}{Q}\log E$

$\left(Q = \frac{2}{\sqrt{N\alpha'}} \right)$



$Z_{cigar}(\tau,\bar{\tau})$ has a "bulk" contribution proportional to L_ϕ, coming from long cylinder:



and a "boundary" contribution from tip.

We are interested in former, which goes like $(-)\log \epsilon$.

The bulk contribution is given by:

$$Z_1 = \beta V_5 \frac{L_\phi}{4} \int_{\substack{\text{strip} \\ |\tau_1| \le \frac{1}{2} \\ 0 < \tau_2 < \infty}} \frac{d^2\tau}{\tau_2} \left(\frac{1}{4\pi^2 \alpha' \tau_2}\right)^{\frac{7}{2}} \frac{1}{|\eta(\tau)|^{10}}$$

$$\times Z_{N-2}(\tau,\bar{\tau}) \sum_{m \in \mathbb{Z}} \left| \sum_{\mu=2}^{4} U_\mu(m)\, \delta_\mu \left(\frac{\theta_\mu}{\eta}\right)^4 (\tau) \right|^2 \times$$

$$\times e^{-\frac{\beta_H^2 m^2}{4\pi \alpha' \tau_2}}$$

* δ_μ, $U_\mu(n,m)$ are phases (± 1) associated with GSO projection. ($\mu = 2,3,4$ labels spin structures)
* $Z_{N-2}(\tau,\bar{\tau})$ is the appropriate $SU(2)$ WZW partition sum.

By focusing on the coefficient of $\log E$ in F, we can determine α:

$$\alpha + 1 = -\frac{\beta_H V_5}{4 Q} \int_{\text{strip}} \frac{d^2\tau}{\tau_2} \frac{1}{(4\pi^2 \alpha' \tau_2)^{\frac{7}{2}}} \frac{Z_{1-2}(\tau,\bar\tau)}{|\eta(\tau)|^{18}} \times$$

$$\times \sum_{m \in \mathbb{Z}} \left| \sum_{\mu=2}^{4} U_\mu(m)\, \delta_\mu\, \theta_\mu^4(\tau) \right|^2 e^{-\frac{\beta_H^2 m^2}{4\pi \alpha' \tau_2}}$$

* recall:

$$\beta_H = 2\pi \sqrt{N\alpha'}$$

$$Q = \frac{2}{\sqrt{N\alpha'}}$$

* Note: integral is only a functn of N.

Comments

1) α is a large negative number (proportional to V_5). Thus, thermodynamics is unstable:

$$E = \frac{\alpha+1}{\beta-\beta_H}\,;\quad \alpha+1<0 \Rightarrow \beta<\beta_H$$

specific heat is negative

2) <u>What happens below Hagedorn temperature?</u>

As we heat up the system, T_H is reached at a finite energy density. $\Rightarrow$ phase transition at T_H.

3) What happens above Hagedorn temperature?

Negative specific heat suggests the existence of a negative mode in Euclidean cigar Background.

Since thermodynamics is marginal classically and becomes unstable at one loop, expect a classical <u>mass less</u> mode which becomes <u>tachyonic</u> quantum mechanically

- can show that in sugra on cigar there is no suitable candidate

- However, in string theory on the cigar, there is a mode which lives near the tip and is exactly massless. It corresponds to a mode of fermionic string tachyon with winding number one (around the cigar).

We believe that at one loop the mass of this mode receives a tachyonic correction:

$$M^2 \approx C(\beta - \beta_H) \qquad (C > 0)$$

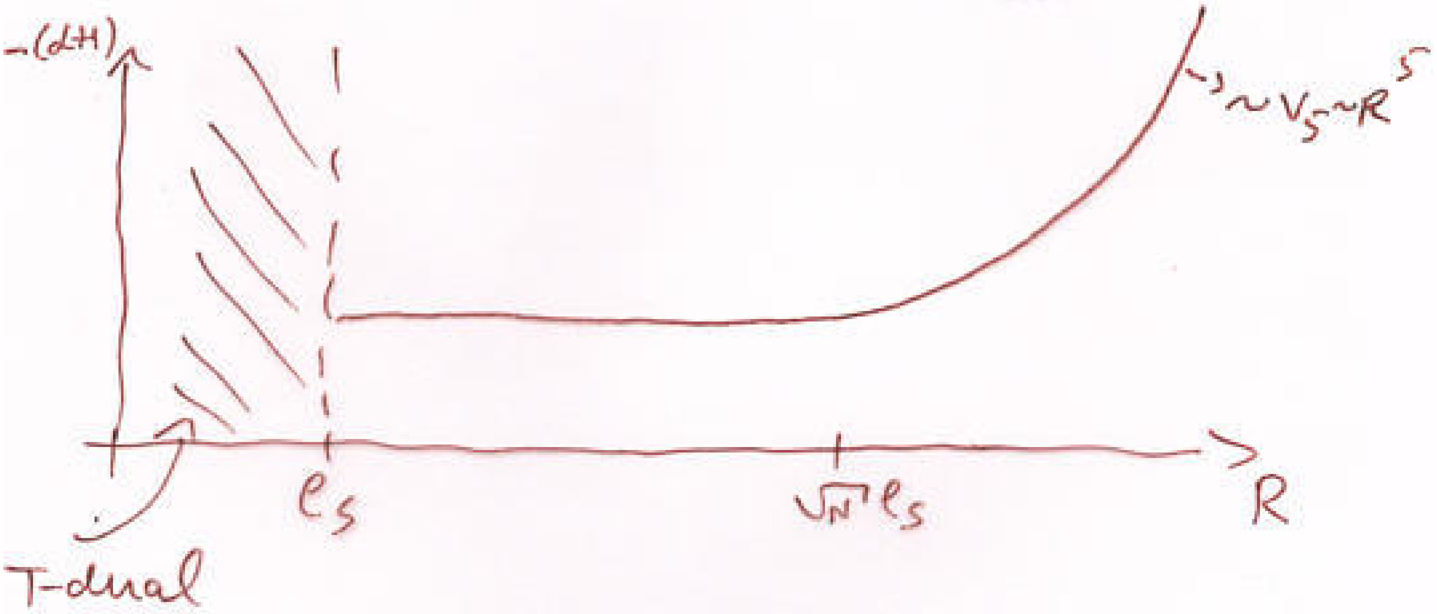
- This needs to be checked explicitly.

- The high temperature phase of LST involves condensation of this mode.

4) What happens for compact NS5-branes?

E.g., replace $R^5 \longrightarrow (S^1)^5$

S^1: circle of radius R.



For large N:

* $R \gg \sqrt{N}\,\ell_s$: $\quad \alpha + 1 = -\frac{24}{\pi^{12}} (\alpha' N)^{-\frac{5}{2}} V_5 \left(1 - \frac{1}{2^{10}}\right) \zeta(10)$

* $R \ll \sqrt{N}\,\ell_s$: $\quad \alpha + 1 = -\frac{24}{\pi^4} \frac{31}{32} \zeta(5)$

As R decreases, thermodynamic fluctuations near Hagedorn temperature increase;
in the microcanonical ensemble the thermodynamics is always unstable.

5) 2d string theory

Essentially the same discussion can be repeated for 2d strings:

* At low T (or E), thermodynamics corresponds to perturbative string modes (= massless "tachyon"), or eigenvalues of matrix in MQM:

$$S \sim \sqrt{\frac{2\pi E \cdot V_L}{6}} \quad ; V_L = \text{volume of 1d space}$$

* At high energies, find:

$$S \sim \beta_H E + s_1 \log E + \ldots$$

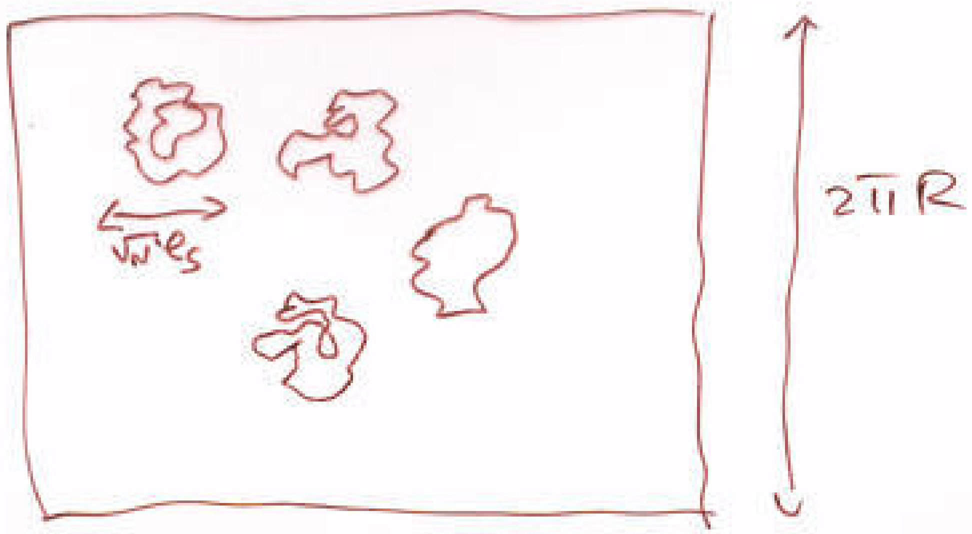
$$\beta_H = 3\pi\sqrt{\alpha'}$$

$$1 + s_1 = -\frac{13}{288}$$

An obvious next step:

How can one understand the Hagedorn densities of states found above, from a microscopic analysis?

* In LST this seems to have to do with strings living in fivebranes. These strings seem to have property that unlike conventional strings, they never become arbitrarily long: high energy entropy appears to be dominated by configurations like:



* In 2d string theory, Hagedorn density of states should come from non-singlet states of MQM, which are non-perturbative states in 2d string description.

This is a well defined problem that remains to be analyzed.