Non-BPS D-branes on a Calabi-Yau Orbifold

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Based on :

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Results

- Start with IIA on a CY 3-fold orbifold with $(h^{1,1}, h^{1,2}) = (11, 11)$

- Take a system containing a pair of B$\not{P}$S D-strings on this background.
  ↳ (A single D2-brane wrapped on a non-susy 2-cycle of the CY_3)

  * We determine the region of stability of the moduli space for this system.

  * We show that —

    i) beyond this region of stability, the system can decay into a pair of B$\not{P}$S D3-branes.

    ii) At one point on the bdry. of this region of stability, $\exists$ an exactly marginal bdry. deformation, which connects the BCFT of the B$\not{P}$S D-strings to the BCFT of the B$\not{P}$S D3-branes.

  * Discuss the phase diagram in the moduli space.

* Start with its 4-fold cover T^6 spanned by $X^4, \ldots, X^9$.

* Then mod it out by two $\mathbb{Z}_2$ transformations g_4 and g_4' as described below:

	X^4	X^5	X^6	X^7	X^8	X^9
g_4	$+$	$+$	$-$	$-$	$-$	$-$
g_4'	$-$	$-$	$-, \frac{1}{2}$	$-$	$+, \frac{1}{2}$	$+$
$g_4' g_4$	$-$	$-$	$+, \frac{1}{2}$	$+$	$-, \frac{1}{2}$	$-$

$$\psi^{6,7,8,9} \xrightarrow{g_4} -\psi^{6,7,8,9} \;; \quad \psi^{4,5} \xrightarrow{g_4} \psi^{4,5}$$

$$\psi^{4,5,6,7} \xrightarrow{g_4'} -\psi^{4,5,6,7} \;; \quad \psi^{8,9} \xrightarrow{g_4'} \psi^{8,9}$$

$$X^{0,\ldots,3} \xrightarrow{g_4,\, g_4'} X^{0,\ldots,3}$$

$$\psi^{0,\ldots,3} \xrightarrow{g_4,\, g_4'} \psi^{0,\ldots,3}$$

$\longrightarrow$ Calabi-Yau 3-fold Orbifold $T^6/g_4 \otimes g_4'$ with $(h^{(1,1)}, h^{(1,2)}) = (11, 11)$

$\rightarrow$ Preserves only $1/4$ th of the spacetime susy.

The Configuration :

* Consider IIA theory on $T^6/\mathcal{I}_4 \otimes \mathcal{I}_4'$

* Take a ~~BPS~~ D-string of this theory and wrap it along X^9 s.t. it stretches over a fundamental interval $[0, 2\pi R_9]$. Other 8 spatial coordinates of this D-string are as given below :

$$(X^1, X^2, X^3, X^4, X^5, X^6, X^7, X^8) = (\underbrace{b_1, b_2, b_3, a_4, a_5}_{\text{Arbitrary no.s.}}, 0, 0, 0)$$

$\rightarrow$ $\mathcal{I}_4$- invariant config$^{\underline{n}}$ but not $\mathcal{I}_4'$- invariant.

* $\mathcal{I}_4'$- image of the above D-string is located at :

$$(X^1, X^2, X^3, X^4, X^5, X^6, X^7, X^8)$$

$$= (b_1, b_2, b_3, -a_4, -a_5, \pi R_6, 0, \pi R_8)$$

$\Rightarrow$ (D-string + $\mathcal{I}_4'$-image)

$\rightarrow$ An $\mathcal{I}_4 \otimes \mathcal{I}_4'$-invariant configⁿ

$\rightarrow$ Non-compact coordinates of the D-string and its $\mathcal{I}_4'$-image are identical.

$\hookrightarrow$ A single object in the orbifold theory.

Physical Interpretation :

Examine the charge quantum no.s. of the object.

→ BPS (stable) D-string ⇒ does not carry any bulk RR-charge.

→ Passes through the orbifold fixed pts. and hence carries twisted RR charge associated with those fixed pts.

Twisted RR charge → charge corresponding to the vector field obtained by reducing RR 3-form on the collapsed 2-cycles.

Blown-up limit : The object is a BPS 2-brane wrapped on a non-susy 2-cycles (of minimal area) which collapses to a line in the orbifold limit.

Region of Instability in the Moduli space

(Restricting to tree level of open string theory)

Typical instability is due to the tachyon present in the spectra of open strings stretched between the D-string and its various images; e.g.

i) Images under $\mathcal{I}_4'$,

ii) Images under translation by $2\pi R_i$ along the compact directions, X^i; $4 \le i \le 8$.

Tachyon from here gives rise to BPS decay channels ($\Rightarrow$ End products are BPS D-branes)

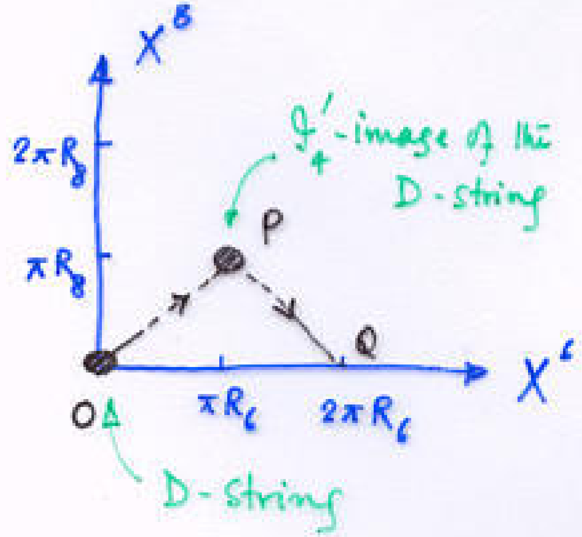
[Sen; Bergman + Gaberdiel]

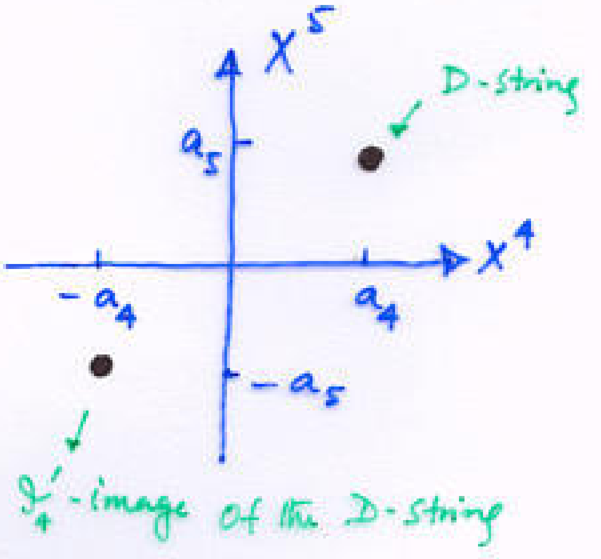
* We shall restrict ourselves to that region of the moduli space, s.t. $\nexists$ any tachyon from this sector.

- $R_i \geq \frac{1}{\sqrt{2}}$, $6 \le i \le 8$
- $R_9 \le \sqrt{2}$

* Concentrate on the instability arising from the tachyon in (2).

* In this case, the effective moduli space is <u>2-dimensional</u>, spanned by R_6, R_8, controlling the distance between the D-string and its $\mathcal{I}'_4$-image.





* Mass formula for the winding mode tachyon coming from the open string $\overrightarrow{OP}$ & $\overrightarrow{PQ}$
($\alpha' = 1$) (We shall work with the 4-fold cover of the orbifold)

$$M^2_{T(1)} = \left(\frac{a_4}{\pi}\right)^2 + \left(\frac{a_5}{\pi}\right)^2 + \left(\frac{1}{2} R_6\right)^2 + \left(\frac{1}{2} R_8\right)^2 - \frac{1}{2}$$

$\because$ $a_4, a_5 \to$ d.o.f. of the D-string,

Require : $\nexists$ tachyon for any values of a_4 and a_5.

Most stringent condn. comes from :

$$a_4 = a_5 = 0$$

$$\Rightarrow M^2_{T(1)} \geq 0 \Rightarrow \boxed{R_6^2 + R_8^2 \geq 2}$$

On the critical curve : $R_6^2 + R_8^2 = 2$, the tachyonic mode is massless.

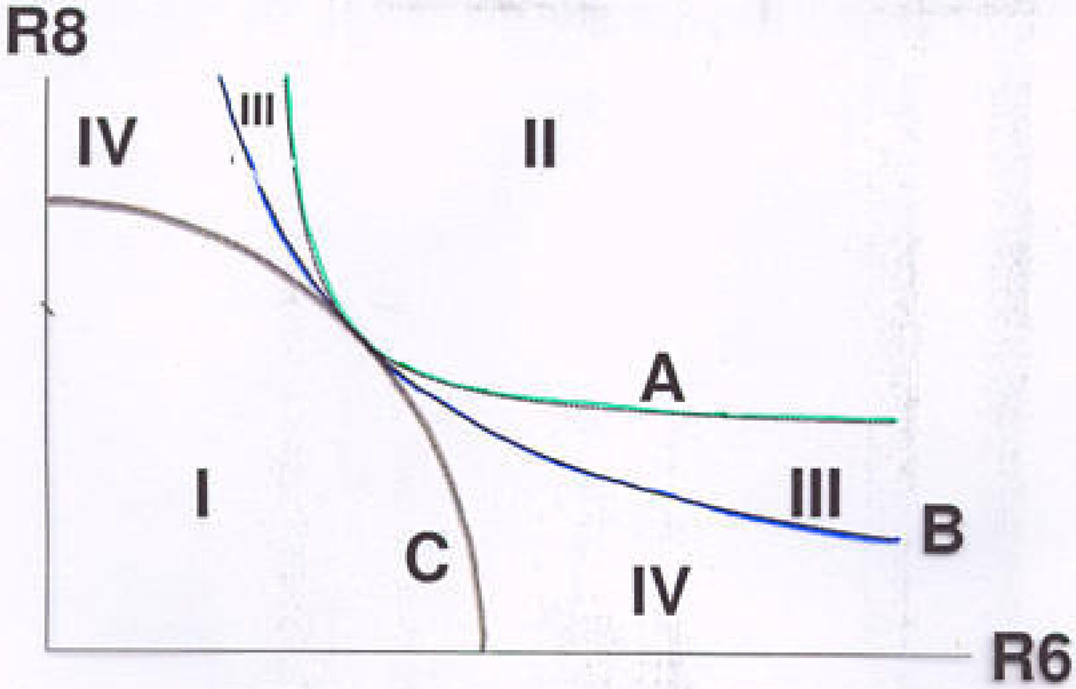
* Similar condn. for the absence of the tachyon in the spectra of open strings joining the pair of ~~BPS~~ D3-brane is :

$$\frac{1}{R_6^2} + \frac{1}{R_8^2} \geq 2$$

* Compare the mass of the D1-brane and D3-brane system :

$$M_{D1} \leq M_{D3} \quad \text{iff} \quad \boxed{R_6 R_8 \geq 1}$$

L9.



where

- Curve A $\leftrightarrow R_6^{-2} + R_8^{-2} = 2$
- Curve B $\leftrightarrow R_6 R_8 = 1$
- Curve C $\leftrightarrow R_6^2 + R_8^2 = 2$

All 3 curves A, B and C intersect at $(R_6, R_8) = (1, 1)$.

* Region I : $R_6^2 + R_8^2 < 2$

Also $R_6^{-2} + R_8^{-2} > 2$

D3-brane pair has no tachyon.

And $R_6 R_8 < 1$

$\Rightarrow$ D3-brane pair is lighter than the D1-brane pair.

$\Downarrow$

D3-brane pair is stable.

D1-brane pair has tachyon in this region and hence unstable.

* Region II : $R_6^{-2} + R_8^{-2} < 2$

Also $R_6^2 + R_8^2 > 2$

D1-brane pair has no tachyon $\rightarrow$ Stable.

And $R_6 R_8 > 1$

$\Rightarrow$ D1-brane pair $\rightarrow$ lighter.

D3-brane pair $\rightarrow$ unstable.

* Region III : $R_6 R_8 > 1$ and

$$R_6^{-2} + R_8^{-2} > 2$$

Also $R_6^2 + R_8^2 > 2$

$\Rightarrow$ Both D1 and D3-brane pairs

$\rightarrow$ free from tachyonic mode

But $M_{D3} > M_{D1}$

$\Rightarrow$ D-string pair $\rightarrow$ Stable

D3-brane pair $\rightarrow$ Metastable

* Region IV : $R_6 R_8 < 1$ and $R_6^2 + R_8^2 > 2$

Also $R_6^{-2} + R_8^{-2} > 2$

$\Rightarrow$ Both D-string and D3-brane pairs

$\rightarrow$ free from tachyonic mode

But $M_{D1} > M_{D3}$

$\Rightarrow$ D3-brane pair $\rightarrow$ stable

D-string pair $\rightarrow$ Metastable

* All 3 curves A, B and C meet at one pt. viz. at the critical radii

$$R_6 = R_8 = 1$$

* At the critical radii,

$\exists$ an exactly marginal bdry. deformation through which the D-string BCFT can be deformed smoothly to the BCFT of the D3-brane pair.

* Determination of Decay Products :

Basic Principle :

Compare the quantum no.s of the initial system and the final decay products.

The quantum no.s. are :

i) Mass

ii) Bulk RR charge → Zero for the initial system and hence for the final system

iii) Twisted RR charge

- X^9 → Common directions for both the initial and final system
- g_4' and $g_4' g_4$ do not produce any new fixed pts. (and hence no new massless state and twisted sector RR charge)

To make calculation simpler, we shall

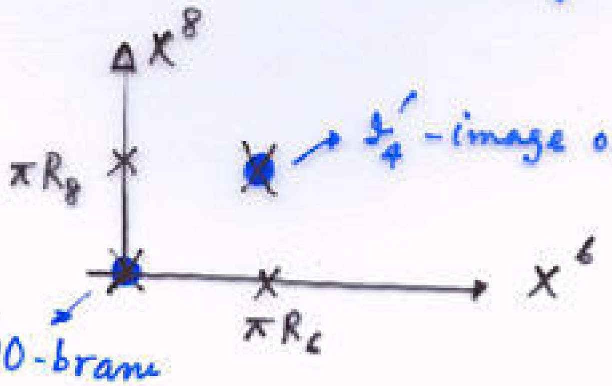
i) Work with 2-fold cover of the CY-orbifold viz. we shall mod out T^6 by $\mathcal{I}_4$ but not by $\mathcal{I}_4'$.

ii) Perform a T-duality along x^9.

$\Rightarrow$ Background: IIB on $T^6/\tilde{\mathcal{I}}_4\cdot(-1)^{F_L}\otimes\mathcal{I}_4'$

(Work with its 2-fold cover $T^6/\tilde{\mathcal{I}}_4\cdot(-1)^{F_L}$)

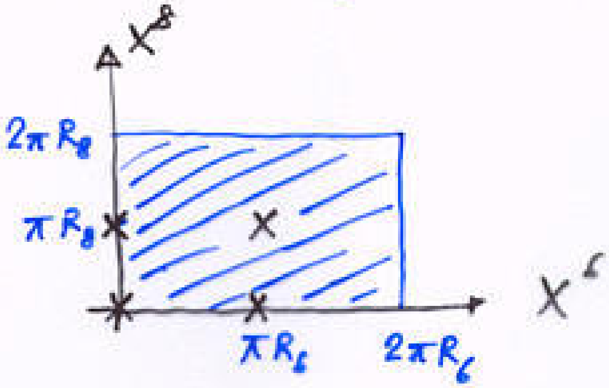
<u>Initial System</u>: D0-brane and its $\mathcal{I}_4'$-image located as shown in fig. below:



<u>Twisted RR charge</u>:

- $q^{(in)}_{(0,0)} = q^{(in)}_{(\pi R_6,\pi R_8)} = 1$
- $q^{(in)}_{(0,\pi R_8)} = q^{(in)}_{(\pi R_6,0)} = 0$

Final System : A pair of D2-brane stretched along X^6-X^8, as shown in fig. below :



$\theta_6, \theta_8 \rightarrow$ Wilson lines along X^6 and X^8, respectively

Twisted RR charge associated with various fixed pts. ($\times$) :

$$q_{(0,0)} = \frac{1}{2}\epsilon \quad ; \quad q_{(0,\pi R_8)} = \frac{1}{2}\epsilon e^{i\theta_8}$$

$$q_{(\pi R_6,0)} = \frac{1}{2}\epsilon e^{i\theta_6} \quad ; \quad q_{(\pi R_6,\pi R_8)} = \frac{1}{2}\epsilon e^{i(\theta_6+\theta_8)}$$

where :

- $\epsilon = \pm 1$
- $\mathcal{I}_4$ - invariance $\Rightarrow \theta_6, \theta_8 = 0, \pi$
- $\mathcal{I}_4'$ - invariance $\Rightarrow \begin{cases} \theta_6 = \theta_8 = 0 \\ \text{or} \\ \theta_6 = \theta_8 = \pi \end{cases}$

For N such D2-branes,

Conservation of twisted RR charge

$$\Rightarrow \quad q^{(fin)}_{(0,0)} = \frac{1}{2}\sum_{k=1}^{N} \epsilon^{(k)} = 1 \quad \Rightarrow N \to \text{Even integer}$$

$$q^{(fin)}_{(0,\pi R_8)} = \frac{1}{2}\sum_{k=1}^{N} \epsilon^{(k)} e^{i\theta_8^{(k)}} = 0$$

$$q^{(fin)}_{(\pi R_6,0)} = \frac{1}{2}\sum_{k=1}^{N} \epsilon^{(k)} e^{i\theta_6^{(k)}} = 0$$

$$q^{(fin)}_{(\pi R_6,\pi R_8)} = \frac{1}{2}\sum_{k=1}^{N} \epsilon^{(k)} e^{i(\theta_6^{(k)} + \theta_8^{(k)})} = 1$$

Demand :

Mass of the initial System > Mass of the final decay products

$$\Rightarrow \quad \boxed{R_6 R_8 \leq \frac{2}{N}}$$

↳ Less stringent for $N = 2$

For $N = 2$, it is automatically satisfied

whenever : $\boxed{R_6^2 + R_8^2 \leq 2}$ → Region of instability of the initial system

For $N \geq 4$ in the eqn.

$$R_6 R_8 \leq \frac{2}{N} \,,$$

$\rightarrow$ Addl. decay channels open up

$\rightarrow$ Final decay products contain 4 or more ~~BPS~~ D3-branes.

Note : For $R_6 R_8 \leq \frac{2}{N}$

and $N > 2$,

there is a chance that

$$R_i \ngtr \frac{1}{\sqrt{2}} \quad \forall\, i = 6,8$$

$\Rightarrow$ Addl. BPS decay channels open up.

$\boxed{\text{For } N = 2}$: $\epsilon^{(1)} = \epsilon^{(2)} = +1$

And

$$\left\{ \begin{array}{ll} \text{Either} & \theta_6^{(1)} = \theta_8^{(1)} = 0 \,;\; \theta_6^{(2)} = \theta_8^{(2)} = \pi \\ \text{Or} & \theta_6^{(1)} = \theta_8^{(1)} = \pi \,;\; \theta_6^{(2)} = \theta_8^{(2)} = 0 \end{array} \right.$$

$\Rightarrow$ One of the D2-branes carries $\mathbb{Z}_2$-Wilson lines along X^6 and X^8

Interpretation of the final state : (IIA description)

Look at the RR charges carried by the final decay products.

→ Carries no bulk RR charge

→ Carries twisted RR charge associated with each fixed pt. it passes.

Twisted RR charge ↔ BPS D2-branes wrapped on homologically nontrivial 2-cycles

Then

BPS D3-brane $\xrightarrow[\text{limit}]{\text{Blown up}}$ BPS D2-brane wrapped on some complicated non-susy 2 cycle (?)

Wrong

Why ?

Take $R_4, \ldots, R_9$ → Large but finite

⇒ ~~BPS~~ D3-brane unstable,

but continues to exist as a classical solu$^{\underline{n}}$ of the E.O.M.

→ In this region, it can not be interpreted as a BPS D2-brane wrapped on some non-susy 2-cycle, ∵ in this region it occupies a 3-dim. subspace.

Then ?

In the blown-up limit, this ~~BPS~~ D3-brane represents BPS D4-brane wrapped on a homologically trivial 4-cycle, but carries non-trivial magnetic flux through various homology 2-cycles.