MATRIX MODEL, NONCOMMUTATIVE GAUGE THEORY AND BRANE-ANTIBRANE SYSTEMS

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Based on:

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MOTIVATION

- TACHYON CONDENSATION IS STUDIED USING
  - OPEN SFT
  - BSFT
  - DUAL GAUGE THEORY
  - CFT MARGINAL DEFORM$^{\underline{N}}$S
  - NC GAUGE THEORY

- MATRIX MODEL NATURALLY CONNECTED TO NC GAUGE THEORY

- MATRIX MODEL HAS A SIMPLE HAMILTONIAN AND SUPERSYMMETRIC GROUND STATE

- BRANE-ANTIBRANE SYSTEM IS SIMPLY DESCRIBED IN MATRIX THEORY

⟼ WHAT IS THE DESCRIPTION OF TACHYON CONDENSATION IN MATRIX THEORY ?

- MATRIX MODEL CAN DESCRIBE BRANES, FUNDAMENTAL STRINGS (MATRIX STRINGS) ⋯
  SO CAN OSFT.

⟼ HOW DO THE CONSTRUCTIONS COMPARE ? ($B \neq 0$ VS $B = 0$)

⟼ HOW DO THE VACUA, HAMILTONIANS COMPARE ?

- MATRIX MODEL ⟷ NCGT FOR BPS BRANES IS NATURAL, BUT NCGT FOR NON-BPS BRANES (AND $D\bar{D}$) USE REAL (AND COMPLEX) MATRIX T

⟼ HOW DO THESE APPEAR FROM (SAY) BFSS MATRIX VARIABLES X^M ?

(1.2)

PLAN

2• CONSTRUCTION OF NCGT FOR $D\bar{D}$ SYSTEMS USING MATRIX MODEL

3• GAUGE FIXING : FIND PHYSICAL VARIABLES (COMPLEX T, U(1) × U(1))

4• SOLVE G.F. CONDITION EXACTLY, WRITE EXACT TACHYON POTENTIAL

5• FIND LOCAL MINIMA CORRES-PONDING TO VORTEX SOLUTIONS

6• FIND EXACT GROUND STATE AND PATH CONNECTING TO IT

7• FIND CLOSED STRING EXCITATIONS IN THE TACHYONIC VACUUM
- MATRIX STRING (DVV)
- FLUX TUBE

Related work :

H. Awata, S. Hirano, Y. Hyakutake
9902158

P. Kraus, A. Rajaraman, S. Shenker
0010016

M. Li 0010058

Also

O. Aharony, M. Berkooz
9611215

G. Lifschytz, S. Mathur
9612087

2. WE CONSIDER D2-$\overline{\text{D2}}$ SYSTEM

- MATRIX MODEL DESCRIPTION

RECALL BFSS MATRIX MODEL ($A_0 = 0$ GAUGE)

$$H = \frac{\sqrt{2\pi}}{g_s} \mathrm{Tr}\left(\frac{(\dot{X}^M)^2}{2} - \frac{[X^M, X^N]^2}{4} \right)$$

GAUSS LAW

$$[X^M, \dot{X}^M] = 0 \qquad M = 1, 2, \cdots 9$$

SINGLE D2 BRANE

$$X^1 = x_1 \qquad X^2 = x_2$$

$$[x_1, x_2] = i\theta$$

$$X^i = 0 \qquad i = 3, 4, \cdots 9$$

2 COINCIDENT D2-BRANES

$$X^1 = \begin{pmatrix} x_1 & 0 \\ 0 & x_1 \end{pmatrix} \quad X^2 = \begin{pmatrix} x_2 & 0 \\ 0 & x_2 \end{pmatrix}$$

$$X^i = 0$$

COINCIDENT D2-$\overline{D2}$

$$X^1 = \begin{pmatrix} x_1 & 0 \\ 0 & x_1 \end{pmatrix} \quad X^2 = \begin{pmatrix} x_2 & 0 \\ 0 & -x_2 \end{pmatrix}$$

$$X^i = 0$$

SATISFIES E.O.M., GAUSS LAW

$$0 = [X^M, [X^N, X^M]]$$

• NON-COMMUTATIVE GAUGE THEORY DESCRIPTION

KNOWN FOR 2 COINCIDENT D2

$$X^a = \begin{pmatrix} x^a & 0 \\ 0 & x^a \end{pmatrix} + \theta^{ab} \mathcal{A}_b$$

$a = 1, 2$

$$X^j = \phi^j$$

$j = 3, 4, \cdots 9$

$$H = H_A + H_\phi$$

$$H_A = \frac{\sqrt{2\pi}}{g_s} \mathrm{Tr}\Big[\frac{1}{2} G^{ab} \dot{\mathcal{A}}_a \dot{\mathcal{A}}_b$$

$$+ \frac{1}{4} G^{ac} G^{bd} \left(\mathcal{F}_{ab} - \theta^{-1}_{ab}\right)\left(\mathcal{F}_{cd} - \theta^{-1}_{cd}\right)\Big]$$

$$G^{ab} = \theta^2 \delta^{ab} \qquad \theta^{ab} = \theta \epsilon^{ab}$$

GAUSS: $G^{ab} \mathcal{D}_0 \dot{\mathcal{A}}_b - i[\phi^i, \dot{\phi}^i] = 0$

(2.3)

- $D2\text{-}\overline{D2}$ AS FLUX

$$X^2 = \begin{pmatrix} x^2 & 0 \\ 0 & -x^2 \end{pmatrix}$$

$$= \begin{pmatrix} x^2 & 0 \\ 0 & x^2 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & -2x^2 \end{pmatrix}}_{-\theta \mathcal{A}_1}$$

$$\therefore \mathcal{A}_1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{2x^2}{\theta} \end{pmatrix}$$

$$\mathcal{F}_{12} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{\theta} \end{pmatrix}$$

CLASSICAL SOLUTION OF THE $U(2)$ NC GAUGE THEORY

- WORK IN THE "HIGGS BRANCH" FOR NOW:

$$\phi^i = 0 \qquad i = 3, 4, \cdots 9$$

- PARAMETERISE

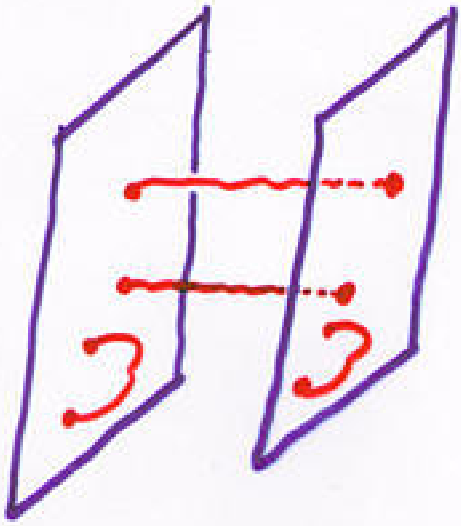
$$\mathcal{A}_a = \begin{pmatrix} A_a & T_a \\ T_a^\dagger & \tilde{A}_a \end{pmatrix} \qquad a = 1, 2$$

$$\mathcal{F}_{12} = \begin{pmatrix} F_{12} - i(T_1 T_2^\dagger - T_2 T_1^\dagger) & D_1 T_2 - D_2 T_1 \\ (D_1 T_2)^\dagger - (D_2 T_1)^\dagger & \tilde{F}_{12} - i(T_1^\dagger T_2 - T_2^\dagger T_1) \end{pmatrix}$$

D2-D2 BGD. $\quad A_a = \tilde{A}_a = 0 = T_a$

D2-$\overline{\text{D2}}$ BGD. $\quad \tilde{A}_1 = \dfrac{2x^2}{\theta}$, REST $= 0$

3. DEGREES OF FREEDOM



TWO U(1) GAUGE BOSONS = 2

ONE COMPLEX TACHYON = 2

$$\overline{4}$$

RIGHT NOW WE HAVE

$$A_a = 2$$

$$\tilde{A}_a = 2$$

$$T_a, T_a^+ = 4$$

$$\overline{8}$$

NEED TO FIX GAUGE

(3.1)

- ISOLATION OF REDUNDANT DEGREES OF FREEDOM IS NATURAL IN THE NC GAUGE THEORY FORMULATION AS COMPARED TO THE ORIGINAL MATRIX MODEL FORMULATION.

  IN THE MATRIX MODEL STUDIES, SUCH A SEPARATION IS POSSIBLE ONLY IN THE "ZERO MODE" OF THE FIELDS (ONLY IN THE ZERO MODE SECTOR THE TACHYON CAN BE ISOLATED)

- GAUGE FIXING ISSUES IMPORTANT EVEN IN OSFT

• "UNITARY" GAUGE

$$\mathcal{F}_{12} \longrightarrow \mathcal{U}^\dagger \mathcal{F}_{12} \mathcal{U}$$

$\mathcal{U} \in U(2) \times U(\infty) \equiv U_\infty(2)$

$\mathcal{F}_{12}$ = DIAGONAL

$$D_1 T_2 = D_2 T_1$$

(FORMALLY $T_2 = \frac{1}{D_1} D_2 T_1$)

RESIDUAL GAUGE INVARIANCE

$$U_\infty(1) \times U_\infty(1)$$

$$A_a \longrightarrow U^\dagger A_a U + i U^\dagger \partial_a U$$

$$\tilde{A}_a \longrightarrow \tilde{U}^\dagger A_a \tilde{U} + i \tilde{U}^\dagger \partial_a \tilde{U}$$

$$T_a \longrightarrow U^\dagger T_a \tilde{U}$$

FURTHER GAUGE FIXING

FIX "AXIAL GAUGE"

$$A_1 = 0 \qquad \tilde{A}_1 = \frac{2x^2}{\theta}$$

D2-D2 BGD.

$$\left.\begin{array}{l} A_a = \tilde{A}_a = 0 \\ T_a = 0 \end{array}\right\} \xrightarrow[\tilde{U} = e^{\frac{i\pi}{2\theta}\left((x^1)^2+(x^2)^2\right)}]{U=1} \begin{array}{l} \tilde{A}_1' = \frac{2x^2}{\theta} \\ \tilde{A}_2' = -\frac{2x^1}{\theta} \\ \text{REST}=0 \end{array}$$

D2-$\overline{\text{D2}}$ BGD.

$$\left.\begin{array}{l} \tilde{A}_1 = \frac{2x^2}{\theta} \\ \text{REST}=0 \end{array}\right\} \text{ IS ALREADY IN THIS GAUGE}$$

INDEPENDENT VARIABLES

$$\underbrace{A_2}_{1},\ \underbrace{\tilde{A}_2}_{1},\ \underbrace{T_1}_{2} \quad = 4$$

4. EXACT TACHYON POTENTIAL

SOLVE GAUGE FIXING CONDITIONS EXACTLY :

$$D_1 T_2 = D_2 T_1$$

WITH $\quad A_1 = 0 \quad \tilde{A}_1 = \frac{2x^2}{\theta}$

IN THE MOYAL-PRODUCT NOTATION :

$$D_1 T_2 = \partial_1 T_2 - i A_1 * T_2 + i T_2 * \tilde{A}_1$$

$$= \partial_1 T_2 + i T_2 * \frac{2x^2}{\theta}$$

$$f * g(x^1, x^2) = e^{\frac{i\theta}{2}\epsilon^{ab}\frac{\partial}{\partial x^a}\frac{\partial}{\partial y^b}} f(x) g(y) \Big|_{y=x}$$

$$T_2 * x^2 = x^2 T_2 + \frac{i\theta}{2}\partial_1 T_2$$

$$\therefore \quad D_1 T_2 = \frac{2 i x^2 T_2}{\theta}$$

$$T_2 = \frac{\theta}{2ix^2} D_2 T_1$$

$$= \frac{\theta}{2ix^2}\left(\partial_2 T_1 - iA_2 * T_1 + i T_1 * \tilde{A}_2\right)$$

- WITH THIS WE CAN EXPLICITLY WRITE DOWN THE STATIC HAMILTONIAN IN TERMS OF INDEPENDENT VARIABLES

$T \equiv T_1, \ A_2, \ \tilde{A}_2$

- COMPUTE $\mathcal{F}_{12}$

$$\mathcal{F}_{12} = \begin{pmatrix} \partial_1 A_2 - i\left(T_1 T_2^{+} - T_2 T_1^{+}\right) & 0 \\ 0 & -\partial_1 \tilde{A}_2 - i\left(T_1^{+} T_2 - T_2^{+} T_1\right) \end{pmatrix}$$

• GET

$$\boxed{H_{st} = \frac{\sqrt{2\pi}}{g_s}\theta^4 \int \frac{dx^1 dx^2}{2\pi\theta}\left[h_1 * h_1 + h_2 * h_2 \right]}$$

$$h_1 = \partial_1 A_2 + \frac{\theta}{2}\left(T * \left(\frac{1}{x^2} D_2 T\right)^* + \left(\frac{1}{x^2} D_2 T\right) * T^* \right) - \frac{1}{\theta}$$

$$h_2 = \partial_1 \tilde{A}_2 - \frac{\theta}{2}\left(T^* * \left(\frac{1}{x^2} D_2 T\right) + \left(\frac{1}{x^2} D_2 T\right)^* * T \right) + \frac{1}{\theta}$$

$$\boxed{D_2 T \equiv \partial_1 T - i A_2 * T + i\, T * \tilde{A}_2}$$

• EXPLICIT TACHYON POTENTIAL

• CAN EVALUATE IT FOR ANY CONFIGURATION OF $T, A_2, \tilde{A}_2$

$\longmapsto$ TACHYON CONDENSATION

$\Rightarrow$ "VACUUM", VORTEX

• $A_2, \tilde{A}_2$ APPEAR QUADRATICALLY

(4.3)

- "TOP" AND "BOTTOM" OF THE POTENTIAL

- TOP

$$A_2 = \tilde{A}_2 = 0 = T$$

$$h_1 = -\frac{1}{\theta} \quad , \quad h_2 = \frac{1}{\theta}$$

$$\boxed{\begin{aligned} H_{st} &= \frac{\sqrt{2\pi}}{g_s}\theta^4 \int \frac{d^2x}{2\pi\theta}\left(\frac{1}{\theta^2}+\frac{1}{\theta^2}\right) \\ &= 2\,M_{D2} \end{aligned}}$$

- BOTTOM

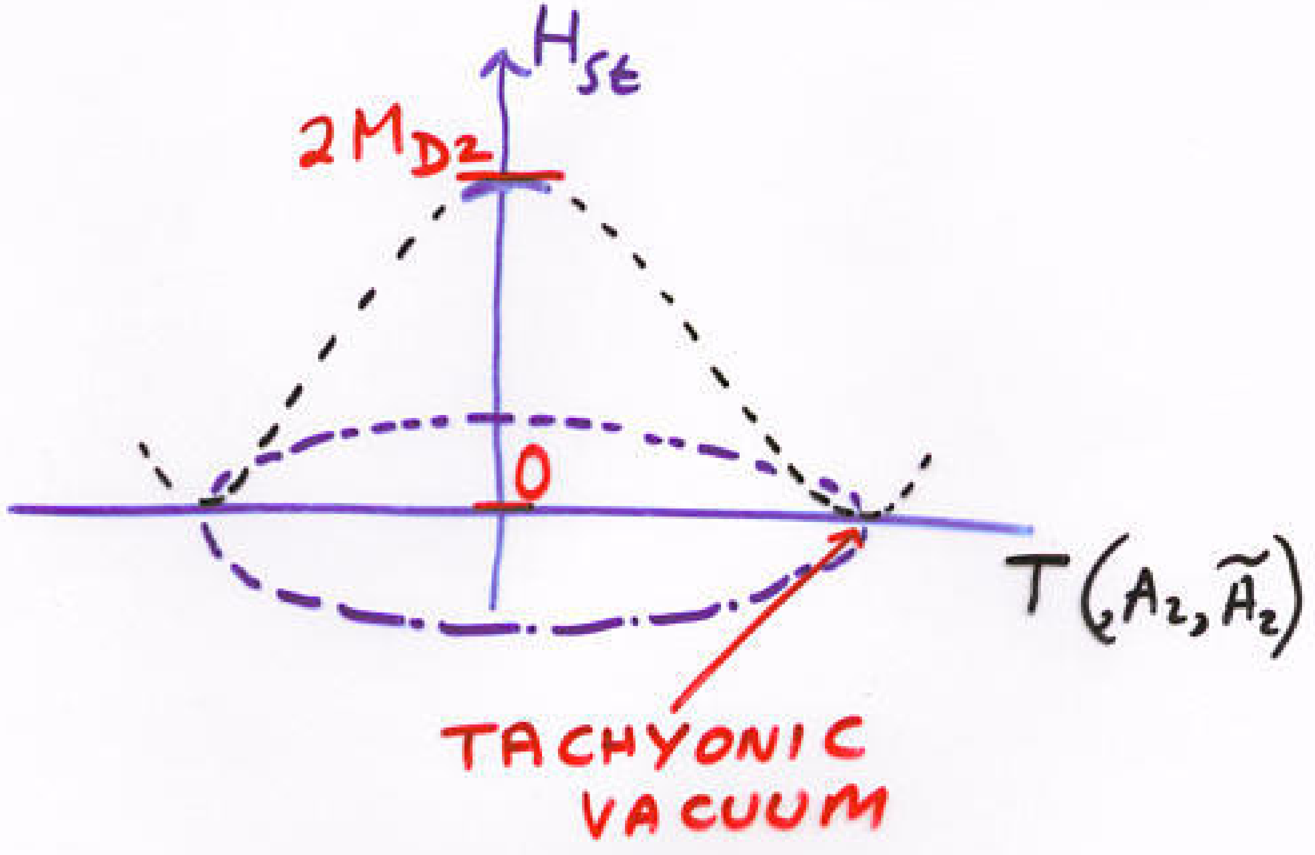
$$A_2 = ? \quad \tilde{A}_2 = ? \quad T = ?$$

$$h_1 = 0 \quad h_2 = 0$$

$$\boxed{H_{st} = 0}$$

• GLOBAL U(1) SYMMETRY:

$$T \to e^{i\theta} T \qquad H_{st} \to H_{st}$$



• PROPERTIES OF "VACUUM"

→ $[X^1, X^2] = 0$

→ SUPERSYMMETRIC GROUND STATE (GIVEN D0-CHARGE SECTOR)

→ $X^a = \begin{pmatrix} \lambda_1^a & & \\ & \ddots & \\ & & \lambda_N^a \end{pmatrix}$ N STATIC D0-BRANES

5. VORTEX SOLUTION

- LET US EVALUATE THE EXACT HAMILTONIAN FOR SOME SIMPLE CONFIGURATIONS

$$T = t\, P_0$$

$$A_2 = \tilde{A}_2 = a\, P_0$$

$$P_0 = 2e^{-\frac{r^2}{\theta}} \quad (= |0\rangle\langle 0|)$$

- GET $\quad T_2 = it\, P_0$

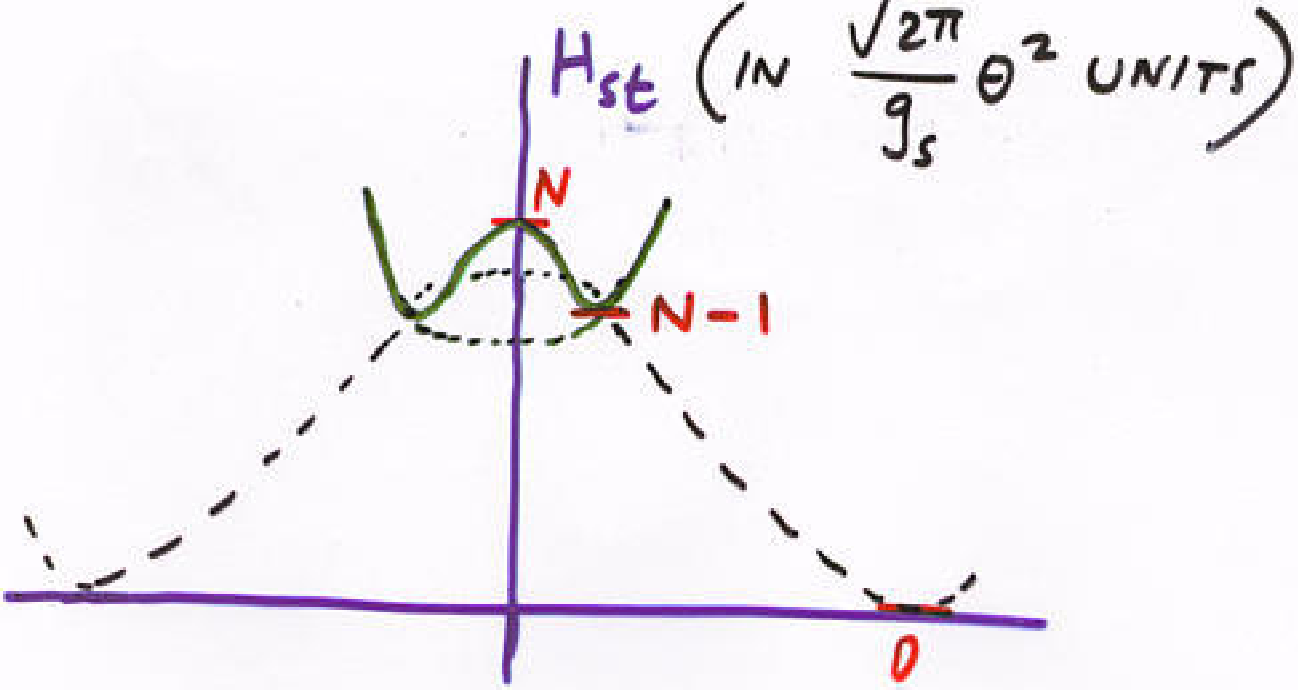
CORRECT $\alpha' m_T^2 = -\frac{1}{\pi 8}$

- $$H_{st} = \frac{\sqrt{2\pi}}{g_s}\theta^2 \left[N - 4\theta |t|^2 + 4\theta^2 |t|^4 + \theta a^2 \right]$$

$$\mathrm{tr}\, \mathbb{1} = \int \frac{dx^1 dx^2}{2\pi\theta}\, \mathbb{1}$$

- MINIMUM

$$|t| = \frac{1}{\sqrt{2\theta}}, \quad a = 0 \qquad (5.1)$$



- IT IS EASY TO CHECK THAT FOR THIS SOLUTION

$$[X^1, X^2] = \left(\begin{array}{cc|cc} 0 & & & \\ & 0 & & \\ \hline & & i\theta \mathbb{1}_{N-1} & \\ & & & -i\theta \mathbb{1}_{N-1} \end{array}\right)$$

RECALL, AT THE <u>TOP</u>

$$[X^1, X^2] = \begin{pmatrix} i\theta \mathbb{1}_N & \\ & -i\theta \mathbb{1}_N \end{pmatrix}$$

AND AT THE <u>BOTTOM</u>

$$[X^1, X^2] = 0_{2N} \qquad (5.2)$$

• WHY IS THIS A VORTEX SOLUTION?

$\longrightarrow$ LOCALIZED CODIMENSION TWO

$\longrightarrow$ IS IDENTIFIABLE AS D0 BRANE (PAIR) SITTING AT THE ORIGIN $\underline{r=0}$ OF THE COMMON WORLD VOLUME OF $D2\overline{D2}$

$\Longrightarrow$ RR CHARGE AND TENSION AS EXPECTED FROM TACHYONIC SOLITONS

$$D2-\overline{D2} \xrightarrow{\text{VORTEX}} D0$$

• SOLUTION COULD HAVE BEEN OBTAINED BY MINIMIZING H_{st} IN THE SPACE

$$\text{RANK}\,[X^1, X^2] = 2N-2 \qquad (5.3)$$

6. D2 - $\overline{\text{D2}}$ ANNIHILATION THROUGH CASCADE OF VORTEX FORMATION :

$$T = \sum_{j=0}^{n-1} t_j P_j \;, \quad A = \tilde{A} = \sum a_j P_j$$

$$P_j = |j\rangle\langle j| = 2(-1)^j L_j(2r^2)\, e^{-\frac{r^2}{\theta}}$$

$$H_{st}(t_j, a_j)$$

min:

$$\boxed{\begin{aligned} |t_{n-1}| &= \frac{1}{\sqrt{2\theta}} \\ |t_{n-2}| &= \frac{1}{\sqrt{\theta}}\left(\frac{1}{\sqrt{2}}+1\right) \\ &\vdots \\ a_j &= 0 \end{aligned}}$$

$$H_{st} = \frac{\sqrt{2\pi}}{g_s}\theta^2 (N-n) \qquad (6.1)$$

• FOR $n = N$ WE REACH THE VACUUM

$$H_{CE} = 0$$

(6.2)

7. CLOSED STRING EXCITATIONS

RESULT:

IF WE COMPACTIFY THE MATRIX MODEL (OR THE U(2) NC GAUGE THEORY) ON A CIRCLE ALONG X^9 (RADIUS R_9) WE FIND TWO TYPES OF CLOSED STRING EXCITATIONS IN THE TACHYONIC VACUUM:

(a) CLOSED STRINGS OF

$$\text{MASS} \sim \frac{1}{g_s^{1/2}} \frac{R_9^{1/2}}{\ell_s^{3/2}}$$

A LA DVV

(b) FLUX STRINGS ALONG X^9 OF TENSION $\sim \dfrac{1}{\alpha'}$

SOME DETAILS

- TACHYONIC VACUUM

$$\underbrace{[X^1, X^2] = 0}_{X^1 = \Lambda^1,\ X^2 = \Lambda^2} \qquad X^i = 0$$

DEGENERATE (LOCATION OF D0'S)

- COMPACTIFY BFSS ALONG X^9

$$X^9 \longrightarrow R_9 (i\partial_\sigma + A(\sigma))$$

$$X^i \longrightarrow Y^i(\sigma) \qquad i = 1, 2, \cdots 9$$

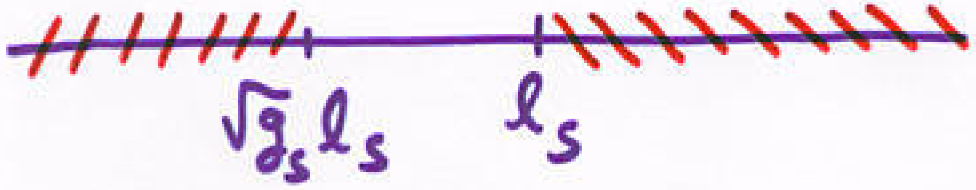
$$H = \frac{1}{l_s} \int_0^{2\pi} d\sigma \, \mathrm{Tr}\Big[\frac{R_9}{l_s} \sum_i \left(\pi_i^2 + (DY^i)^2\right) + \frac{g_s l_s^2}{R_9^2} \left(E - \frac{[Y_i, Y_j]^2}{2}\right) \Big]$$

TACHYONIC VACUUM

$$A(\sigma) = 0, \quad Y^1(\sigma) = \Lambda^1, \quad Y^2(\sigma) = \Lambda^2$$

$$Y^i(\sigma) = 0 \quad i = 3, 4, \cdots 8$$

- CONSIDER TWO REGIONS



(a) $R_q \ll \sqrt{g_s}\, l_s \longrightarrow$ DVV

(b) $R_q \gg l_s \longrightarrow$ FLUX TUBE

- (a) THIS IS THE REGION DESCRIBED BY THE CFT ON $S_N \mathbb{R}^8$

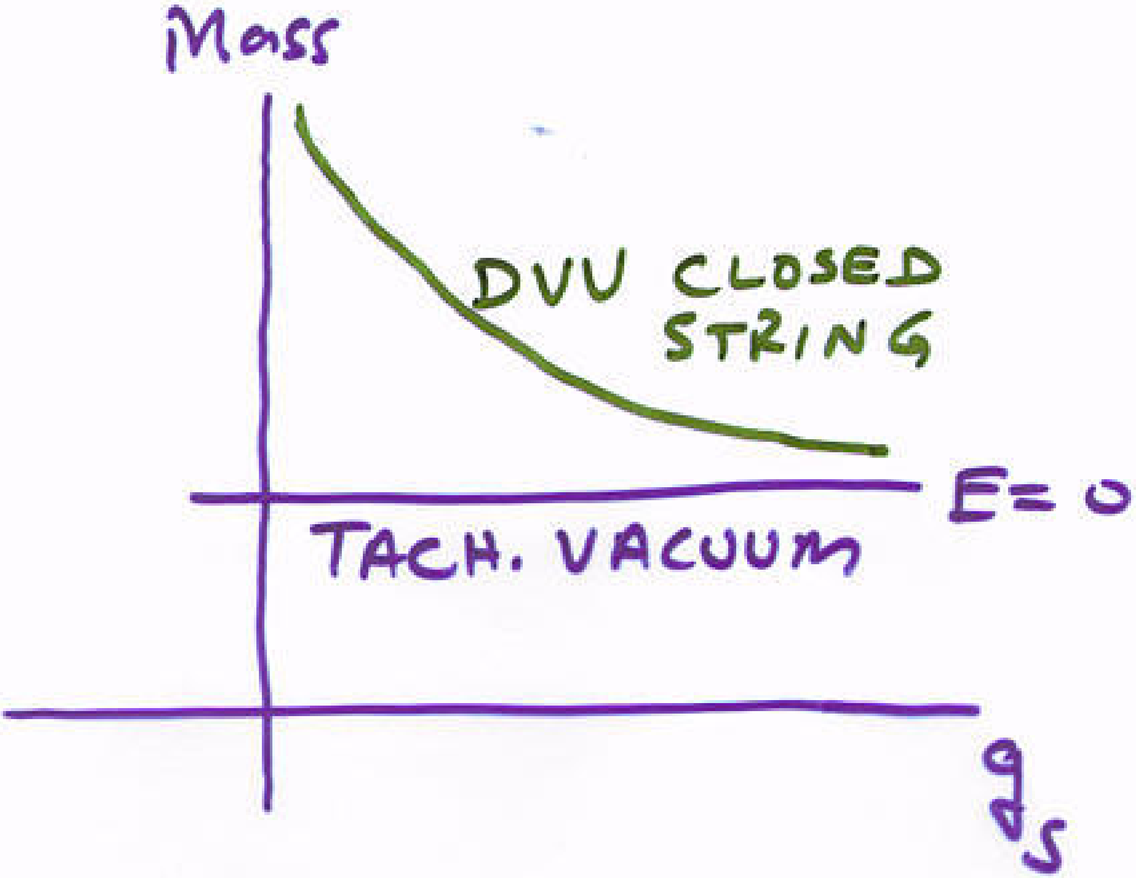
$\Longrightarrow$ MATRIX STRINGS

- THESE STRINGS ARE LIGHT IN THE DUAL THEORY (TST OR 9-10 FLIP)

- SINCE THESE ARE BPS, THEIR MASS FORMULA CAN BE CONTINUED

$$\boxed{\text{MASS} \sim \frac{1}{\sqrt{g_s}}}$$

(7.3)



(7.4)

(b) • IN THIS REGION

AT GENERIC λ^1, λ^2 $(U(1)^N)$

$A = \Lambda$ (DIAGONAL)

GAUSS LAW $\partial_\sigma \dot{\lambda} = 0$

• HAMILTONIAN BECOMES

$$H = \frac{R_9^2}{2 g_s l_s} \sum_{i=1}^{N} \dot{\lambda}_i^2$$

$$= \frac{g_s l_s}{2 R_9^2} \sum_{i=1}^{N} e_i^2$$

e_i = ELECTRIC FIELD EIGENVALUE

• IN IIB DESCRIPTION,

D0's $\longrightarrow$ D1's

• FLUX CARRIED BY D-STRINGS ALONG CIRCLE $\longrightarrow$ FUNDAMEN-TAL STRING

(7.5)

- n F-STRINGS BOUND TO 1 D-STRING

$$E = \begin{pmatrix} n & & & \\ & 0 & & \\ & & 0 & \\ & & & \ddots \end{pmatrix}$$

$|0\rangle\langle 0|$ LOCALIZED

- TENSION $= \dfrac{1}{\alpha'}$

- (n, k) SUBTLE !

8. OPEN QUESTIONS:

- MATRIX MODEL

  $\Longrightarrow$ # D0 = N

  CAN ONE REPRODUCE THE COMMUTATIVE ($B=0$) RESULTS (OSFT, BSFT, ...)?

- COMPARE EXACT TACHYON POTENTIAL HERE WITH BSFT (H_{BFSS} VS g-FUNCTION)

- UNSTABLE D-BRANE

- INSIGHT ABOUT THE NATURE OF THE TACHYONIC VACUUM (OPEN STRING EXCITATIONS, ...)