NON-COMMUTATIVE SOLITONS II

SHIRAZ MINWALLA

STRINGS 2001

BASED ON

hep-th 0009142

(AGANAGIC, GOPAKUMAR, S.M., STROMINGER)

NON-COMMUTATIVE GAUGE THEORY

$$S = \frac{1}{4 g_{YM}^2} \int F_{\mu\nu} F^{\mu\nu} d^d x$$

$$F_{\mu\nu} = [D_\mu, D_\nu] \quad ; \quad D_\mu = \partial_\mu + i[A_\mu, \ ]$$

$$[A, B]_{ik} = A_{ij} * B_{jk} - B_{ij} * A_{ik} .$$

(COMMUTATOR = MATRIX AND STAR)

U(1) THEORY NONTRIVIAL ; FOCUS

USEFUL OBSERVATION

$$\partial_\mu \phi = [-i (\Theta^{-1})_{\mu\nu} x^\nu, \phi]$$

ADJOINT

$$\Rightarrow \quad D_\mu \phi = [i C_\mu, \phi] \quad ; \quad C_\mu = A_\mu - \Theta^{-1}_{\mu\nu} x^\nu$$

$$F_{\mu\nu} = [C_\mu, C_\nu] + (\theta^{-1})_{\mu\nu}$$

$$S = \frac{1}{4g^2_{ym}} \int \left([C_\mu, C_\nu] + \theta^{-1}_{\mu\nu} \right)^2$$

PURELY ALGEBRAIC ; NO KINETIC TERM

THUS NON-COMMUTATIVE GAUGE THEORIES 'SIMPLER' THAN SCALAR TH.

IN PARTICULAR ; EXACT SOLUTIONS AT FINITE θ.

TO FIND SOLUTIONS

OPERATOR FORM

SWITCH TO COMPLEX COORDINATES

$$-\mathcal{L} = \frac{2\pi\theta}{4g^2_{ym}} \, \mathrm{Tr} \left[[C, \bar{C}] + \theta^{-1} \right]^2$$

$(c = c_x + i c_y \, , \bar{c} = c_x - i c_y)$

2+1 DIM

$A_0 = 0$ GAUGE

TIME INDEP CONFIGURATIONS

Vacuum

$$A = 0 \quad \Rightarrow \quad C = a^{+} \qquad \left([a, a^{+}] = \frac{1}{\theta} \right)$$

New Nontrivial Solution

Polychronacos, Bak, AGMS.

$$C = (S^{+})^{m}\, a^{+}\, S^{m}$$

$$S = \sum_{i=0}^{\infty} |i\rangle\langle i+1|$$

S almost unitary

$$S S^{+} = 1 \quad ; \quad S^{+} S = 1 - P_0 \qquad \left(P_0 = |0\rangle\langle 0| \right)$$

Flux

$$F = [C, \bar{C}] + \frac{1}{\theta}$$

$$= S^{+m} [a^{+}, a] S^{m} + \frac{1}{\theta}$$

$$= \frac{1}{\theta}\left[-S^{+m} S^{m} + 1 \right] = \frac{P_m}{\theta} \qquad \left(P_m = |0\rangle\langle 0| + \dots + |m-1\rangle\langle m-1| \right)$$

$$\left(\text{E.g. } m = 1, \quad F = \frac{2e^{-r^2/\theta}}{\theta} \right)$$

$$C_1 = \frac{1}{2\pi} \int F = \frac{2\pi\theta}{2\pi} \,\mathrm{Tr}\, F = \frac{\theta}{\theta} \,\mathrm{Tr}(P_m) = m$$

SUMMARY

SOLUTION = LUMP OF 'm' UNITS OF FLUX OVER AREA $m\theta$

(e.g. $m=1$, $F = \frac{2e^{-r^2/\theta}}{\theta}$)

APPLICATION?

RECALL D_2 BRANE IN LARGE B FIELD. WORLD VOLUME THEORY = NCSYM + α' CORRECTIONS

(THE SEIBERG WITTEN LIMIT) WHEN $\alpha' B \gg g$ ($g_{\mu\nu}$ SPATIAL $= g\delta_{\mu\nu}$)

$$\theta = \frac{1}{B} \quad ; \quad G_{\mu\nu} = \frac{(\alpha' B)^2}{g}$$

$$\Rightarrow \theta^2 = \theta^{\mu\nu}\theta^{\alpha\beta} G_{\mu\alpha} G_{\nu\beta} = (\alpha')^2 \left[\frac{\alpha' B}{g}\right]^2 \gg \alpha'^2$$

THUS SOLUTIONS ON SCALE NONCOMMUTATIVITY (LIKE THE FLUX LUMP DESC ABOVE) ARE WELL DESCRIBED BY NCYM.

(STRINGY α' CORRECTIONS NEGLIGIBLE)

THUS OUR FLUX LUMP ACCURATELY DESCRIBES A SOLUTION OF (CLASSICAL) STRING FIELD THEORY

WHAT SOLUTION?

LUMP = 'm' UNITS OF D0-BRANE CHARGE LOCALIZED AT THE ORIGIN.

NATURAL TO GUESS IT IS 'm' D0 BRANES, COINCIDENT, SITTING ON TOP OF D2 BRANE.

CHECKS

a) ENERGY OF SOLUTION MATCHES 0-2 BINDING ENERGY

b) SPECTRUM OF FLUCTUATIONS ABOUT SOLUTION EXACTLY MATCHES SPECTRUM OF 0-2 CFT, IN THE SEIBERG WITTEN LIMIT $m^2 = \frac{(2k-1)}{\theta}$

(NONTRIVIAL SPECTRUM; H.O. TOWER OF STATES)

ZERO BRANES OFF 2-BRANES!

THEORY ON D2 BRANE NCSYM.

· 16 SUPERCHARGES, 7 SCALAR FIELDS ϕ^a.

GENERALIZATION OF BEHIND SOLUTION EASY

$$C = (S^+)^m a^+ (S)^m + \sum_{n=0}^{m-1} C^n |n\rangle\langle n|$$

$$\bar{C} = (S^+)^m a (S)^m + \sum_{n=0}^{m-1} \bar{C}^n |n\rangle\langle n|$$

$$\phi^{\mu} = \sum_{n=0}^{m-1} (\phi^{\mu})^n |n\rangle\langle n|$$

INTERPRETATION

$C^n, \bar{C}^n$ — REPRESENT POSITIONS OF m D0 BRANES ON 2-BRANE

$(\phi^{\mu})^n$ REPRESENTS POSITIONS OF D0 BRANES IN TRANSVERSE DIRECTIONS

EVIDENCE: SPECTRUM MATCHES

THUS WE CAN MOVE THE 0-BRANES OFF THE 2-BRANE!

TACHYONS

0 BRANE WANTS TO DISSOLVE INTO 2 BRANE. A TACHYON IN THE SPECTRUM SIGNALS THIS INSTABILITY.

THIS TACHYON SURVIVES THE SEIBERG-WITTEN LIMIT (MASS $\frac{1}{\theta}$, NOT $\frac{1}{\alpha'}$).

HENCE CAN BE STUDIED IN FULL DETAIL IN NCYM.

WE CAN

a) INTEGRATE OUT ALL OTHER FLUCTUATIONS TO FIND (ONE BRANCH) OF THE TACHYON POTENTIAL, EXACTLY

ANS: $$V = \frac{\pi\theta}{g_{ym}^2}\left(|T|^2 - \frac{1}{\theta}\right)^2$$

b) FOLLOW TACHYON CONDENSATION TO ITS END POINT; SHOW IT CORRESPONDS TO SMEARED FLUX

c) ANSWER "WHERE DO 0-0 + 0-2 STRINGS GO"

ANS: THEY DISAPPEAR INTO THE 2-2 CONTINUUM

ZERO BRANES ON FOUR BRANES

CONSIDER $4+1$ NCSYM, $A_0 = 0$ GAUGE

FIELDS $A_1, \overline{A_1}, A_2, \overline{A_2}$

AFTER SHIFTING

$C_1, \overline{C_1}, C_2, \overline{C_2}$

FIELDS MAP TO TWO PARTICLE HILBERT SPACE.

VACUUM

$C_1 = a_1^+$ $\qquad$ $C_2 = a_2^+$

USE BEHIND TECHNIQUE

NEW SOLUTION

$C_1 = T^+ a_1^+ T \; ; \quad C_2 = T^+ a_2^+ T$

$\left(T = \sum_{k=0}^{\infty} |k\rangle\langle k+1| \right.$ (WE HAVE IMPOSED ARBIT ORDERING ON BASIS) $\left.\right)$

CHARGE OF SOLUTION

$$F_{\mu\nu} = \theta^{-1}_{\mu\nu} |0\rangle\langle 0|$$

SO

$$\mu = \frac{1}{8\pi^2}\int F\wedge F = \frac{(2\pi)^2 \sqrt{\det\theta}}{8\pi^2} \, \mathrm{Tr}\left(|0\rangle\langle 0|\right) \, \theta^{-1}\wedge\theta^{-1}$$

$$= \pm 1$$

THUS, THE SOLUTION REPRESENTS A ZERO BRANE ON A 4 BRANE IN THE SEIBERG WITTEN LIMIT.

CHECKS: ENERGIES MATCH.

NOTE: F IS SELFDUAL / ANTISELFDUAL IF AND ONLY IF θ IS

THUS OUR SOLUTION (IN NCSYM) IS BPS FOR θ SELFDUAL / ANTI SELF DUAL.

IT IS UNSTABLE (TO DECAY INTO A BPS SOLUTION OF SAME CHARGE) OTHERWISE

NONCOMMUTATIVE INSTANTONS
(COMPLETING THE PICTURE)

CONSIDER $U(2)$ NCYM.

SEIBERG WITTEN.

MODULI SPACE OF SELF DUAL $(F^- = 0)$
INSTANTONS IS FUNCTION OF θ^- ONLY.

SPECIAL CASE, $\theta^+ = 0$, $\theta^- \neq 0$
SELF DUAL INSTANTON MODULI SPACE
SMOOTENED OUT

POSSIBLE TO EXPLICITLY FIND SMOOTH &
SOLUTIONS EVERYWHERE. (NEKRASOV, SCHWARZ)

HOW ABOUT ANTI SELF DUAL
INSTANTONS IN THIS SPECIAL CASE?

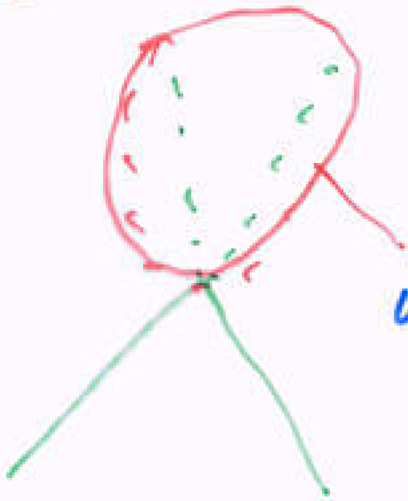
MODULI SPACE OF SUCH INSTANTONS UNAFFECTED BY NONCOMMUTATIVITY.

HOWEVER THE SOLUTIONS THEMSELVES ARE CERTAINLY AFFECTED.

PROPOSE

$$F = \theta^{-1} |0\rangle\langle 0| \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\phi^a = c^a |0\rangle\langle 0| \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$



REPRESENTS SOLUTIONS ON UPPER BRANCH

IN PARTICULAR THE SOLUTION AT THE SINGULAR POINT IN MODULI SPACE IS SMOOTH !!!

$$\left(F = \theta^{-1} |0\rangle\langle 0| \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \; ; \; \phi^a = 0 \right)$$

PROPOSAL VERIFIED BY ADHM CONSTRUCTION OF SOLUTIONS ON LOWER BRANCH (FURUUCHI)

SINGULAR MODULI SPACE IMPLEMENTED BY EVERYWHERE SMOOTH SOLUTIONS!