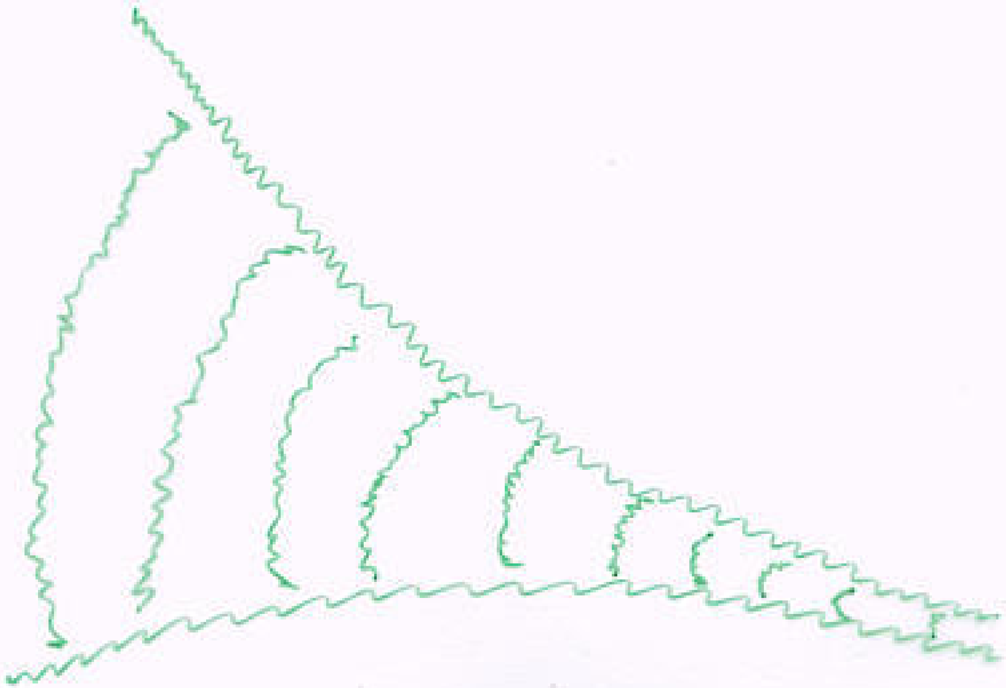
FUZZY FUNNELS:

Nonabelian Brane Intersections



Neil Constable
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(hep-th/9911136
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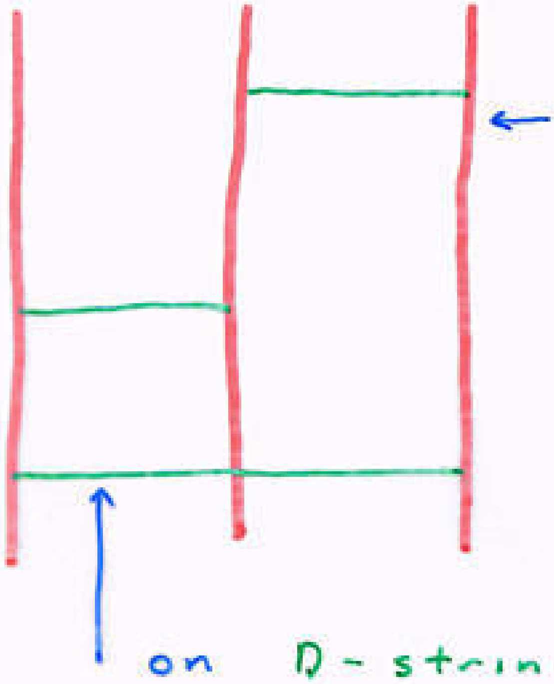
D-branes: when are configurations distinct or different manifestations of the same objects?



← on D3-branes, D-strings arise as magnetic monopoles of 3+1 $U(N_3)$ gauge theory

Callan, Maldacena;
Gibbons;
Hashimoto; ....

on D-strings, D3-branes arise as noncommutative geometries in 1+1 $U(N_1)$ scalar field theory

Diaconescu;
Constable, RCM, Tafjord;
.....

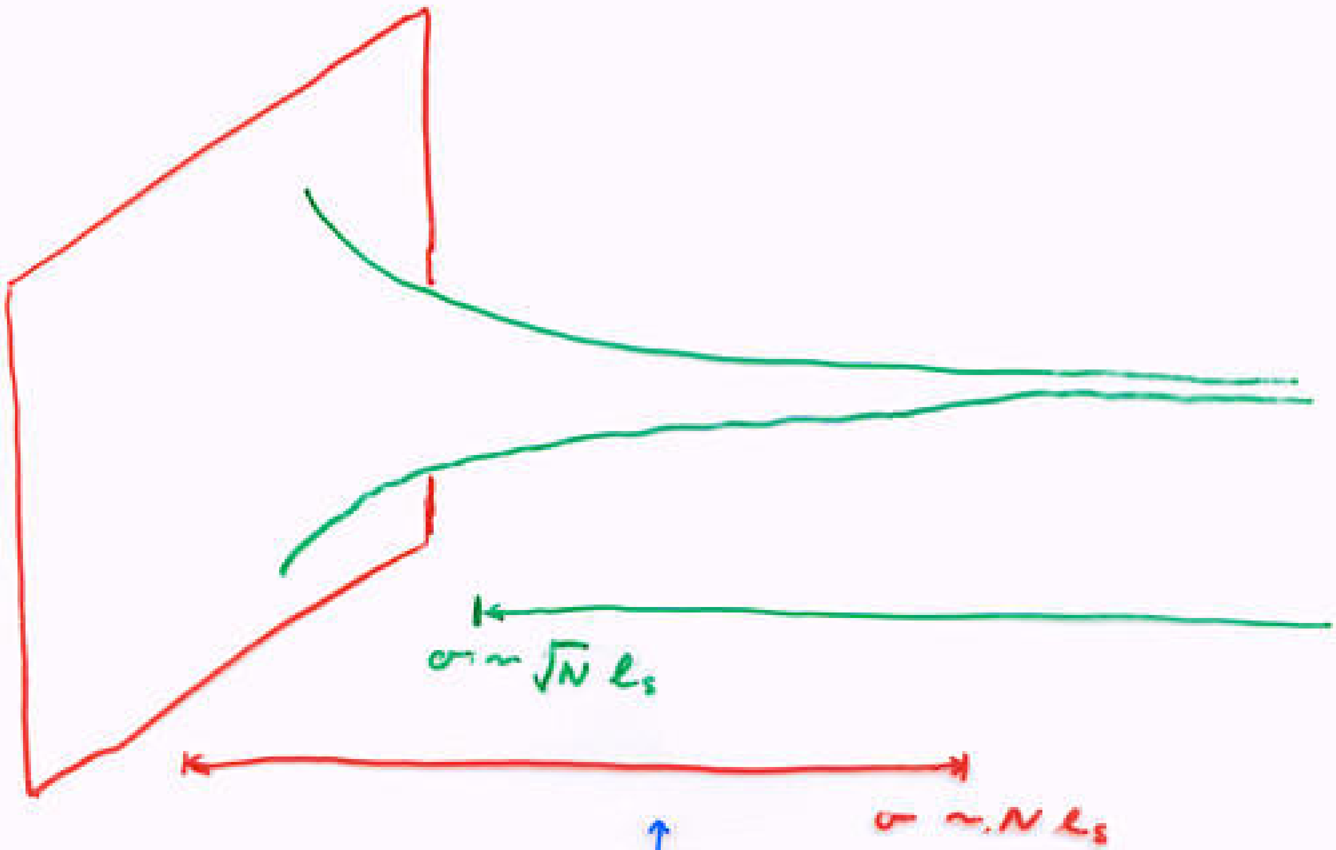
⟶ consider D1 ⊥ D5 system
– find 'similar' story

BI - action is NOT full low energy action

Why do things agree so well?

D3 - brane theory: avoid higher derivative corrections with 'large R or small σ'

D1 - brane theory: avoid commutator corrections with 'small R or large σ'



large overlap for large N

$R \gg \ell_s$

$\sigma \ll N\ell_s$

ensures

$\ell_s \partial_r^2 \sigma \ll \partial_r \sigma$

$R \ll \sqrt{N}\,\ell_s$

$\sigma \gg \sqrt{N}\,\ell_s$

ensures expansion of $\mathcal{L}_{NBI}$ converges rapidly

Lee, Peet, Thorlacius; Kastor, Traschen; Constable, RCM, Tafjord

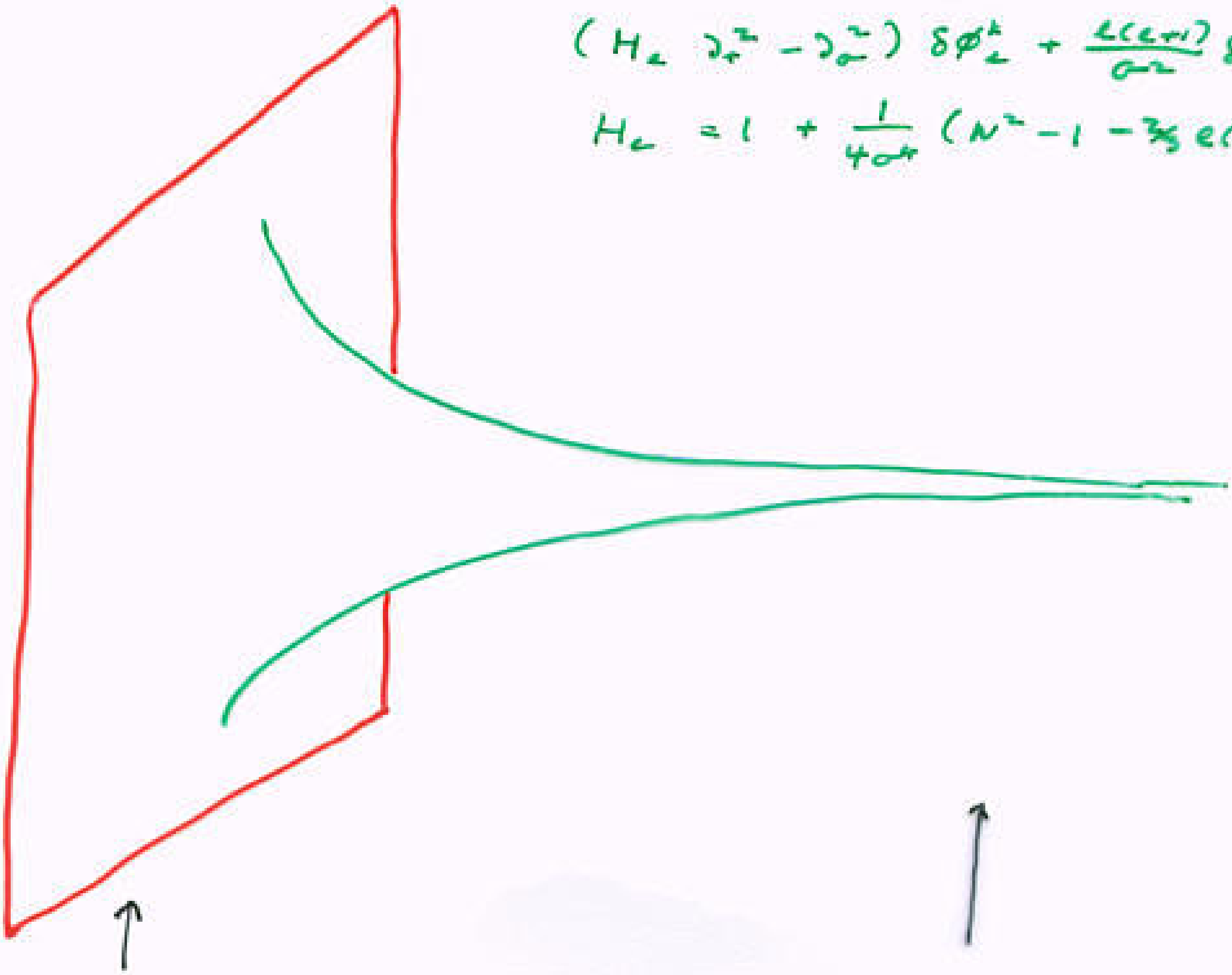
DYNAMICS:

Neither world-volume theory tells whole story!

linearized fluctuations: $\delta\phi_\ell^k = \sum_{\ell=0}^{N-1} \psi^k_{i_1\ldots i_\ell}\, \alpha^{i_1}\cdots\alpha^{i_\ell}$

$$(H_\ell\, \partial_t^2 - \partial_\sigma^2)\,\delta\phi_\ell^k + \frac{\ell(\ell+1)}{\sigma^2}\,\delta\phi_\ell^k = 0$$

$$H_\ell = 1 + \frac{1}{4\sigma^4}\left(N^2 - 1 - \tfrac{3}{5}\,\ell(\ell+1)\right)$$



modes of arbitrarily large ℓ propagate down spike

$\sigma \ll \frac{N}{\ell}\, \ell_s$

ℓ cut-off at $\ell_{max} = N-1$

$\sigma \gg \ell\, \ell_s$

overlap for $\ell \lesssim \sqrt{N}$

PUZZLE: how do we interface higher ℓ modes between two theories

More 'FUZZY FUNNELS':

Puzzle: leading order e.o.m <u>unchanged</u>

$$\partial_\sigma^2 \phi^i = [\phi^j, [\phi^j, \phi^i]]$$

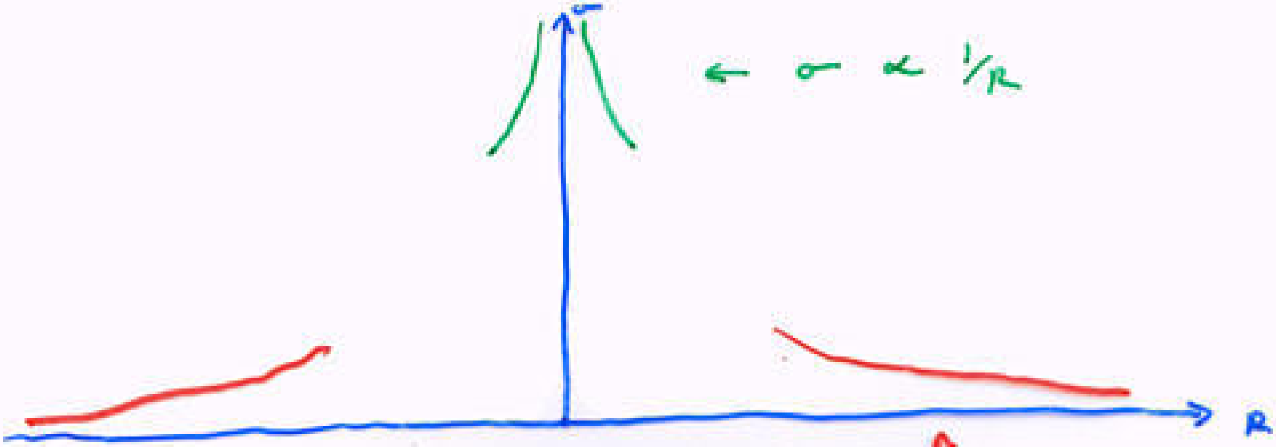
Ansatz: $\phi^i = R(\sigma)\, T^i$

radial profile — $R(\sigma)$

fixed matrices determine geometry — T^i

$$R'' = k R^3 \longrightarrow \begin{array}{l} R \propto 1/\sigma \\ \sigma \propto 1/R \end{array}$$

<u>too slow</u>!



$\sigma \propto \frac{1}{R^{p-2}}$

harmonic function on Dp-brane

universal behavior for small R $\longrightarrow$

full B.I. e.o.m. reproduce desired fall-off $\longrightarrow$

WARNING : leading order e.o.m. are not enough this time !!

D-strings growing into D5-branes

cross-section of funnel should be fuzzy four-sphere

Ansatz: $\phi^i = \frac{R(\sigma)}{c^2} G^i \quad i = 1,2,3,4,5$

Castelino, Lee, Taylor; Grosse, Klimcik, Presnajder

$$G^i = \Big(\underbrace{\Gamma^i \otimes 1 \otimes \cdots \otimes 1}_{n \text{ elements}} + 1 \otimes \Gamma^i \otimes \cdots \otimes 1 + \cdots + 1 \otimes \cdots \otimes \Gamma^i\Big)_{\mathrm{Sym}}$$

4x4 Dirac matrices $i = 1 \cdots 5$

totally symmetrized product

Dimension of matrices = # D strings

$$N_1 = \frac{1}{6}(n+1)(n+2)(n+3)$$

$$\Sigma (G^i)^2 = c^2 \, \mathbf{1} \longrightarrow c^2 = n(n+4)$$

$$R^2(\sigma) = \frac{1}{N} \sum_i \mathrm{Tr}\, \phi^{i\,2} \quad \leftarrow \text{physical radius}$$

G^i have appropriate physical properties:

I) spherical locus

II) SO(5) rotationally invariant

III) spectrum matches density of D1's along x-axis $\propto R^2 - x^2$

IV) funnel carries appropriate D5 charge (later)

$$\mathcal{L}_{NBI} = -T_1 \; STr \sqrt{-\det\left(\eta_{ab} + \partial_a \phi^i \partial_b \phi^j Q^{-1}_{ij}\right) \det Q}$$

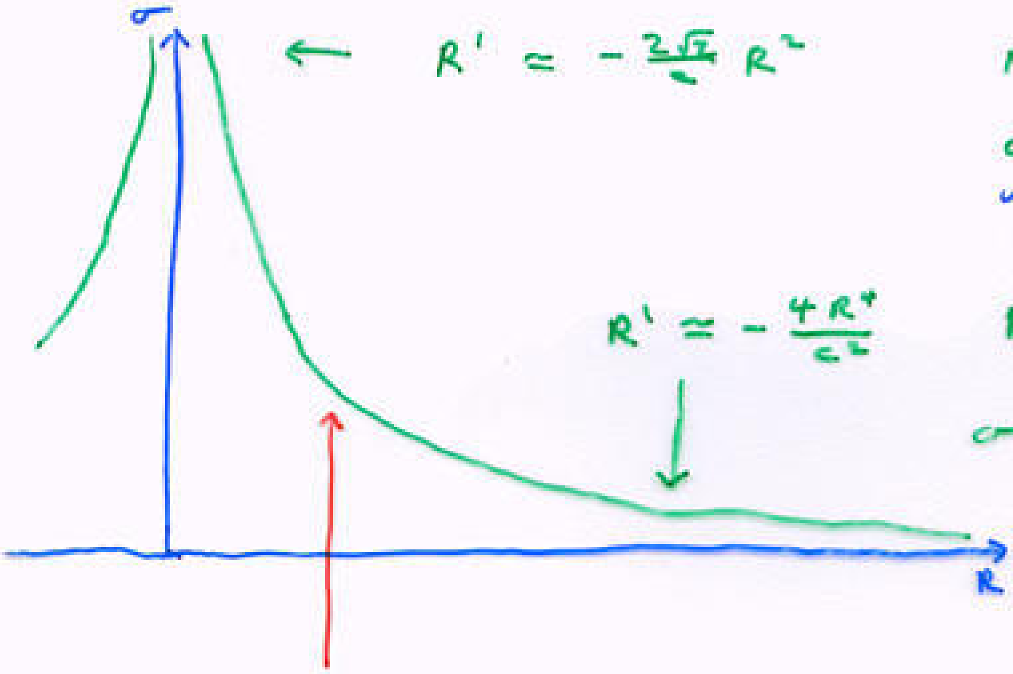
equation of motion is REALLLY BIG!

plug in ansatz:

$$\frac{1}{R'} \frac{d}{d\sigma} \left(\underbrace{\frac{1 + 4 R^4/c^2}{\sqrt{1 + (R')^2}}}_{\text{constant}} \right) = 0$$

$$\frac{dR}{d\sigma} = - \sqrt{8 R^4/c^2 + 16 R^8/c^4}$$ ← choose integration constant to give <u>infinite</u> funnel

→ solve for $\sigma(R)$ as elliptic integral



← $R' \simeq -\frac{2\sqrt{2}}{c} R^2$

$R \simeq \frac{c}{2\sqrt{2}} \frac{1}{\sigma}$

$\sigma \simeq \frac{c}{2\sqrt{2}} \frac{1}{R}$

universal

$R' \simeq -\frac{4R^4}{c^2}$

$R \simeq \left(\frac{c^2}{12}\right)^{1/3} \frac{1}{\sigma^{1/3}}$

$\sigma \simeq \frac{c^2}{12} \frac{1}{R^3}$

harmonic function

transition! $R^2 \simeq \sqrt{2}\,\pi\, c\, \ell_s^2$

D5 - charge:

$$-\tfrac{1}{2}\mu_1 \int \mathrm{STr}\ P\left[(i_\phi i_\phi)^2 C^{(6)} \right]$$

$$= \underbrace{\frac{6N_1(n+2)}{c^3}}_{N_5 \text{ D5-branes}} \mu_5 \int dt\, \Omega_4 R^4 dR \quad C^{(6)}_{t12345}$$

for large n, $\boxed{N_5 \simeq n}$

$N_5 \simeq (6N_1)^{1/3}$?

Energy :

$$E = -\int d\sigma\, \mathcal{L}_{NBI} = N_1 T_1 \int d\sigma\, (1 + 4R^4/c^2)^2$$

$$= \underbrace{N_1 T_1 \int d\sigma}_{N_1 \text{ D1-branes}} + \underbrace{\frac{6N_1}{c^2} T_5 \int \Omega_4 R^4 dR}_{n \text{ D5-branes}}$$

$$+ \underbrace{N_1 T_1 \int dR}_{\substack{N \text{ 'radial'} \\ \text{D1-branes}}} - \underbrace{\Delta E}_{\substack{\text{binding} \\ \text{energy}}}$$

$$\Delta E \simeq 1.01\ N_1 c^{1/2}\ T_1 \ell_s$$

D5 - branes growing D - strings

recall D - string would be source of 2^{nd} Chern class

$$\frac{1}{8\pi^2} \int_{S^4} \mathrm{Tr}\, F \wedge F = N_1$$

ansatz will only use $U(1)$ scalar and nonabelian gauge field

$$\mathcal{L}_{NBI} = - T_5 \;\; \mathrm{STr} \sqrt{-\det \left(\eta_{ab} + \partial_a \phi^i \partial_b \phi^i + F_{ab} \right)}$$

with spherical polar coordinates

$$ds^2 = -dt^2 + dr^2 + r^2 g_{ij}\, d\theta^i d\theta^j$$

take ansatz

$$\phi^9 = \sigma(r) \qquad\qquad A_i = A_i(\theta^k)$$

E.O.M. : gauge field

$$\partial_i \left[\sqrt{g} \; \frac{r^4 F^{ij} + \frac{1}{4} F^{ij} F_{kl} \tilde{F}^{kl}}{\left(r^8 + \frac{1}{2} r^4 F_{ij} F^{ij} + \frac{1}{16} (F_{ij} \tilde{F}^{ij})^2 \right)^{1/2}} \right] = 0$$

$\tilde{F}^{ij} = \frac{1}{2} \varepsilon^{ijkl} F_{kl}$ on S^4

reduces to ordinary YM e.o.m. if $F_{ij} = \pm \tilde{F}_{ij}$ and then automatically satisfied by Bianchi identity

Scalar field e.o.m.: $\frac{\partial}{\partial r} \underbrace{\frac{\delta \mathcal{L}}{\delta \sigma'}}_{\text{independent of } r} = 0$

$$\frac{\sigma'}{\sqrt{1+(\sigma')^2}} = \frac{k}{\mathrm{Tr}\,(r^4 + \frac{1}{4} F_{ij} \tilde{F}^{ij})}$$

$\leftarrow$ integration constant

$\leftarrow$ used $F_{ij} = \tilde{F}_{ij}$

for consistency, must assume we have homogenous instanton field configuration

$$\mathrm{Tr}\; F_{ij} \tilde{F}^{ij} = 6 N_1$$

with appropriate choice of k (recall $\mathrm{Tr}\,\mathbb{1} = N_5$)

$$\frac{d\sigma}{dr} = \frac{1}{\frac{4}{3}\frac{N_5}{N_1} r^4 + \frac{4}{9}\frac{N_5}{N_1^2} r^8}$$

$\longrightarrow$ matches FUZZY FUNNEL profile for large N (with $N_5 \equiv n$)

Energy:

$$E = \underbrace{N_1 T_1 \int d\sigma}_{N_1 \text{ D-strings}} + \underbrace{N_5 T_5 \int \Omega_4 r^4 dr}_{N_5 \text{ D5-branes}}$$

$$+ \underbrace{N_1 T_1 \int dr}_{\substack{N_1 \text{ (radial)} \\ \text{D-strings}}} - \underbrace{\Delta E}_{\substack{\text{Binding} \\ \text{Energy}}}$$

$$\Delta E = 1.01 \left(\frac{6 N_1}{N_5}\right)^{1/4} T_1 \ell_s$$

$\longrightarrow$ matches Fuzzy Funnel for large N

Comments / Questions:

1) D1 and D5 descriptions agree well.

is there region of overlap? YES

D1 : $R^2 \ll N_5 \ell_s^2$

D5 : $R^2 \gg N_5^{2/3} \ell_s^2$

2) (N_+, N_1) strings – add $U(1)$ electric field along tunnel / spike

solution scaled:

$$\sigma_{(N_+, N_1)}(R) = \sqrt{1 + \left(\frac{g_s N_1}{N_5}\right)^2}\ \sigma_{(0, N_1)}(R)$$

profile behaves more like harmonic solution further out from D5

$\longrightarrow$ as expected for pure fundamental strings (electric point charge)

3) not SUSY ; is solution stable? YES

minimum energy solution for fixed boundary conditions

4) energy of 'radial'? D-strings :

D1 $\parallel$ D5 is SUSY so <u>almost</u> simply add energy of horizontal strings

? compare to D-string in D5 supergravity bkgd ?

$$E = N_1 T_1 \int_0^\infty d\sigma\ e^{-\phi} (g_{tt} g_{\sigma\sigma})^{1/2}$$

$$= N_1 T_1 \int_0^\infty d\sigma \sqrt{1 + L^2/\sigma^2}$$

$\longleftarrow$ diverges at $\sigma \to \infty$ but <u>also</u> <u>at</u> <u>$\sigma \to 0$</u>

5) D-string story gave $N_1 \sim \frac{1}{6} N_5^3$

where does this arise in D5 story ?

assumed spherically symmetric instanton configuration - should not be expected in general (compare to magnetic monopoles with higher charge in SU(2) gauge theory)

6) restore SUSY with B-field ?

(Balasubramanian + Leigh)

- <u>not</u> spherically symmetric

7) funnels with other Fuzzy geometries

e.g., Trivedi + Vaidya

$SU(2) \times SU(2)$ (D3, D3')

$SU(3)/U(2) \sim CP_2$ D5

$SU(3)/U(1) \times U(1)$ (d=6) D7, D5

8) D1 ⊥ D7 may give more control with SUSY configuration (?)

- need fuzzy S^6

9) fluctuations - symmetrized polynomials of G's do not form closed algebra

(Castelino, Lee, Taylor ; Grosse, Klimcik, Presnajder)

$\longrightarrow$ extra internal / instanton degrees of freedom (?)

FUZZY SURFACES (OR NONCOMMUTATIVE GEOMETRIES) GIVE INTERESTING TOOL TO STUDY INTERSECTING D-BRANE CONFIGURATIONS

HOW MUCH MORE MAGIC HIDDEN IN (NONABELIAN) BORN-INFELD THEORY ?