D1/D5 SYSTEMS IN THEORIES WITH 16 SUSY

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hep-th/

(AdS/CFT & D1/D5 systems
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(in Preparation))

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→ D1/D5 system in $\mathrm{II}B / T^4$ or $K3$ is reasonably well understood.

→ But for Type I (16 susy) has not been studied yet.

→ In general U-duality would map

$$
\begin{tikzcd}
\mathrm{II}B / K_3 \times S^1 \arrow[r] \arrow[d] & \text{Type I like models} \arrow[d] \\
(4,4) & (4,0)
\end{tikzcd}
$$

Effective world volume theory

→ As a starting point we study here Z_2 orbifolds of $\mathrm{II}B$ theory, with Z_2 being I_4, Ω, ΩI_4 together with a shift.

→ Applying Adiabatic argument → deduce CFT from Parent CFT.

Outline

1) U-duality chain between Ω, I_4, ΩI_4 models
2) Effective field theory of D1/D5 system
3) Test : ground state of 2).
   $\longrightarrow$ perturbative spectra in a U-dual theory
4) Discussion on D1/D5/KK 3-charge system.
5) AdS/CFT correspondence.

IIB on $S^1 \times T^4$

$\underset{x^1}{S^1} \times \underbrace{T^4}_{x^2,\, x^3,\, x^4,\, x^5}$

	$\xrightarrow{S}$	$\xrightarrow{T_{15}}$	$\xrightarrow{S}$	$\xrightarrow{T_{2345}}$	$\xrightarrow{S}$
D_1	F_1	P_1	P_1	P_1	P_1
D_{12345}	NS_{12345}	NS_{12345}	D_{12345}	D_1	F_1
P_1	P_1	F_1	D_1	D_{12345}	NS_{12345}
Ω	$(-1)^{F_L}$	$(-)^{F_L}$	Ω	ΩI_4	$(-1)^{F_L} I_4$
I_4	I_4	$(-1)^F I_4$	ΩI_4	Ω	$(-1)^{F_L}$
ΩI_4	$(-1)^F I_4$	I_4	I_4	I_4	I_4
x	x	C_{15}	B_{15}	G_{15}	G_{15}
C_{2345}	C_{12345}	C_{1234}	C_{1234}	C_{15}	B_{15}

IIB on $S^1 \times T^4 \times S^1$

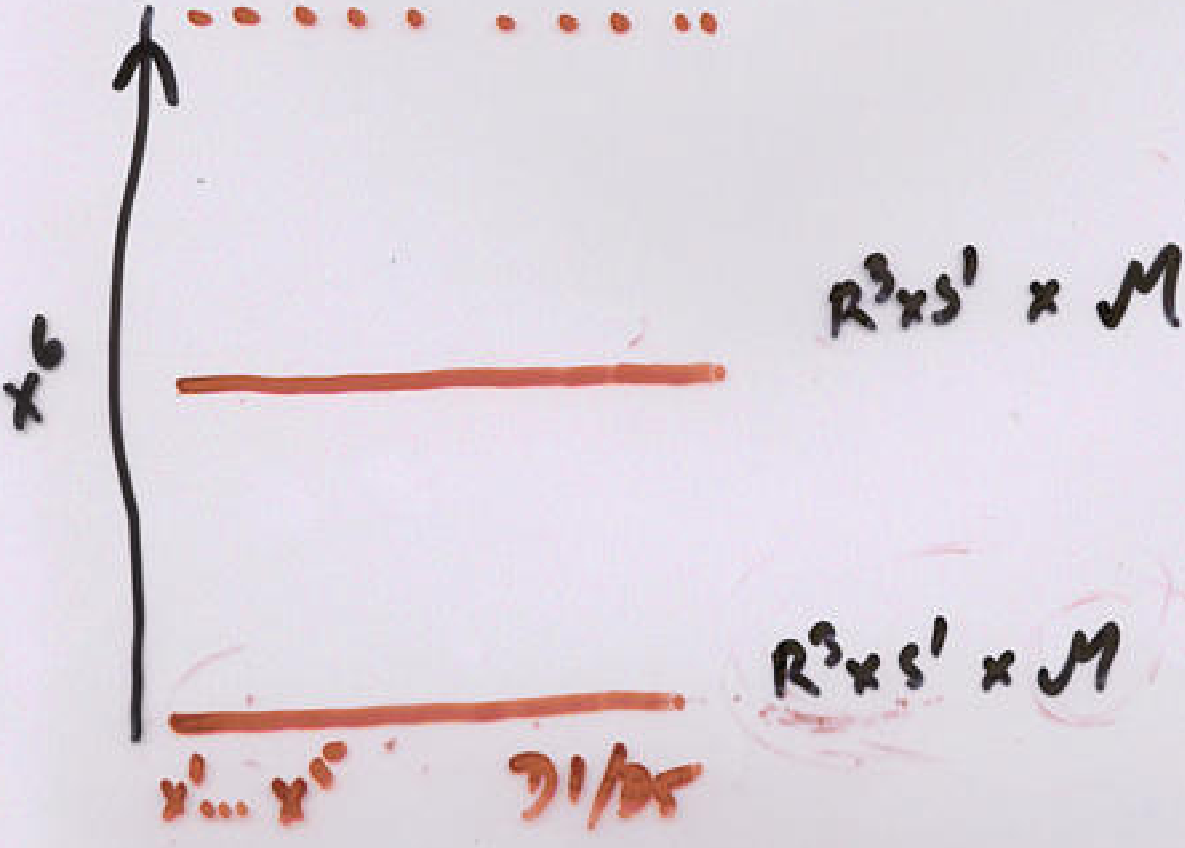
$\quad\quad\quad\;\; x^1 \quad x^2 \ldots x^5 \quad x^6$

$\mathbb{Z}_2$ ORBIFOLD $\quad\quad g \cdot \sigma_6$

$g = \Omega,\ \Omega I_{2345},\ J_{2345}$

σ_6 = HALF SHIFT ON x^6

EFFECTIVE THEORY



g will induce an action $\hat{g}$ on $\mathcal{M}$

$\hat{g}$ identification

$\Rightarrow$ ONLY ONE SET

$$R^3 \times S^1 \times \mathcal{M}$$

DESCRIBES THE SYSTEM

$\Rightarrow$ IF ONE SHIFTS THE ENTIRE SYSTEM $x^6 \to x^6 + \pi R^6$

$$\mathcal{M} \longrightarrow \hat{g}\mathcal{M}$$

THEN IT DESCRIBES THE ORIGINAL SYSTEM.

EFFECTIVE FIELD THEORY

$$(R^3 \times S^1 \times \mathcal{M})/\mathbb{Z}_2$$

$$\downarrow$$

$$\hat{g}\cdot\sigma$$

↳ SHIFT ALONG x^6

STARTING POINT $\text{II}B/T^4$

$$\mathcal{M} = T^4 \times (T^4)^N/S_N$$

(Maldacena, Moore, Strominger; TATA GROUP)

$Q_5 = 1 \qquad Q_1 = N$

$x = \frac{1}{2} \quad \& \quad C^{(4)} = \frac{N}{2}$

(Witten, Seiberg, Witten, Larsen, Martinec, TATA GROUP)

(4,4) Supermultiplets

	Boson	Left fermion	Right Fermion
$R^3\times S^1$	X_{AY}	ψ_A	$\tilde\psi_Y$
T^4	A_i	ψ_Y	$\tilde\psi_A$
$(T^4)^N$	$X_i^{(l)}$	$\psi_Y^{(l)}$	$\tilde\psi_A^{(l)}$

Little group $SO(4) \simeq SU(2)_A \times SU(2)_Y$

Tangent space of $R^3\times S^1$

$\hat g$	I_4	Ω	ΩI_4
X_{AY}	$+$	$+$	$+$
A_i	$-$	$-$	$+$
$X_i^{(l)}$	$-$	$+$	$-$

$U(1)$ Wilson line

Position of D1 inside D5

Action on the fermions is obtained by (4,4) SUSY and the fact that

$\hat g$ commutes with (4,4) SUSY for I_4

and " " (4,0) SUSY for Ω, ΩI_4

	I_4	Ω	ΩI_4
$\hat{g} =$	$J_4^{cm} \cdot I_4^{\text{sym Prod.}}$	$(-1)^{F_L} J_4^{c.m.}$	$(-1)^{F_L} J_4^{\text{sym}\cdot\text{Prod}}$
	$(4,4)$	$(4,0)$	$(4,0)$

Longitudinal Shift case

σ along X^1 (common world volume)

$\longrightarrow$ Periodic or anti-periodic B.C. along X^1 depending on the eigenvalues of $\hat{g}$.

GENERAL PROBLEM

$$\left((CFT)^N / S_N \right) / \mathbb{Z}_2$$

$$Z = \sum_{\mathcal{H}_{CFT}} q^{L_0} \bar{q}^{\bar{L}_0} y^{J_3} \tilde{y}^{\bar{J}_3}$$

$J_3, \bar{J}_3$ $\quad SU(2)_A \times SU(2)_Y$ Quantum #'s

$$Z_{ab} = \sum C_{ab}(\Delta, \bar{\Delta}, \ell, \tilde{\ell})\, q^{\Delta} \bar{q}^{\bar{\Delta}} y^{\ell} \tilde{y}^{\tilde{\ell}}$$

$a, b = \pm$ twists along σ, t directions

Put $\bar{\Delta} = 0$ $\quad$ ($\bar{L}_0$ sector is supersymm in (4,4) or (4,0) case)

~~Elliptic~~ The Four Sectors of the symmetric Product CFT give

$$E_{ab} = \sum_N p^N E_{N;ab}(q, y, \tilde{y}) \; ; \quad \tilde{\Delta} = 0$$

$$E_{++} = \left(1 - p^n q^m y^\ell \tilde{y}^{\tilde{\ell}}\right)^{-c_{++}(nm, \ell, \tilde{\ell})}$$

$$E_{+-} = \left(1 - p^n q^m y^\ell \tilde{y}^{\tilde{\ell}}\right)^{-\frac{1}{2}(c_{++} + c_{+-})(nm, \ell, \tilde{\ell})} \cdot \left(1 + p^n q^m y^\ell \tilde{y}^{\tilde{\ell}}\right)^{-\frac{1}{2}(c_{++} - c_{+-})(nm, \ell, \tilde{\ell})}$$

$$E_{-\pm} = \left(1 - p^{2n} q^m y^\ell \tilde{y}^{\tilde{\ell}}\right)^{-\frac{1}{2}(c_{++} + c_{+-})(2nm, \ell, \tilde{\ell})} \cdot \left(1 \mp p^{2n} q^{m-\frac{1}{2}} y^\ell \tilde{y}^{\tilde{\ell}}\right)^{-\frac{1}{2}(c_{++} - c_{+-})(2n(m-\frac{1}{2}), \ell, \tilde{\ell})} \cdot \left(1 - p^{2n-1} q^m y^\ell \tilde{y}^{\tilde{\ell}}\right)^{-\frac{1}{2}(c_{-+} + c_{--})((2n-1)m, \ell, \tilde{\ell})} \cdot \left(1 \mp p^{2n-1} q^{m-\frac{1}{2}} y^\ell \tilde{y}^{\tilde{\ell}}\right)^{-\frac{1}{2}(c_{-+} - c_{--})((2n-1)(m-\frac{1}{2}), \ell, \tilde{\ell})}$$

FOR $\Delta = 0$ (i.e. pure D1/D5 system)

E_{++} & E_{+-} agree with perturbative results in the U-dual theory.

3-CHARGE SYSTEM & a "PUZZLE"

Suppose $\exists$ U-duality

$$\begin{matrix} D_1 & & P_1 \\ D_{12345} & \xrightarrow{\text{U-duality}} & D_{12345} \\ P_1 & & D_1 \end{matrix}$$

$$E(p, q, y) \longrightarrow E(q, p, y)$$

(Recall $p^{Q_1} q^{KK}$) $\qquad$ ($y = 1$)

(A) $\qquad$ (B)

$$I_4 \longrightarrow \Omega I_4$$

D1/D5 system is (4, 4) $\qquad$ D1/D5 system is (4, 0)

$\downarrow$

Elliptic genus

$$E_{cm}(q, y)\, E_{Int}(p, q y)$$

$$E_{Int.}(p,q,y) = E_F(p,y) \cdot \hat{E}_{Int}(p,q,y)$$

(where $E_F(p,y) = E_{Int}(p,0,y)$)

$\Rightarrow$ U-duality implies:

$$E^A_{Ch} = E^B_F$$
$$E^A_F = E^B_{Ch}$$

TRUE FOR CFT's PROPOSED HERE

$$\hat{E}^A_{Int} = \hat{E}^B_{Int.} \rightarrow \text{Not true}$$

WHAT IS GOING WRONG !

THIS IS A GENERIC PHENOMENON WHEN MODEL (A) is (4,4) & sym. Prod CFT and MODEL (B) is (4,0)

(ALSO FOR IIB / K3)

Indeed take model (A) which is (4,4)
Sym. Prod. CFT.

$$E^A = E^A_{CM}(q,y)\, E^A_F(p,y)\, \hat{E}^A_{\text{Sym.prod.}}(p,q,y)$$

$\hat{E}^A_{\text{Sym.prod.}}$: Sym in $p \leftrightarrow q$

1) for q^0; E^A has second order zero in $(y-1)^2$ ($\frac{1}{2}$ BPS)

2) for q^m; $m>0$ E^A has fourth order zero $(y-1)^4$ ($\frac{1}{4}$ BPS)

$\downarrow$ U-duality

$$E^A_{CM}(p,y)\, E^A_F(q,y)\, \hat{E}^A_{\text{Sym.Prod.}}(p,q,y)$$

expand p^1 from the U-dual

Internal Contribution

$$E^A_{CM}(p,y)\, \hat{E}^A_{\text{Sym.prod.}}(p,q,y)$$

$$\rightarrow \boxed{E^A_1(q,y) - E^A_1(0,y) + f^A_1(y)}$$

where

→ $E_1(q, y)$ is the elliptic genus for single copy of $(CFT)^A$ $(4,4)$

→ $f_1(y)$ is the coeff of p^1 in $E^A_{CM}(p, y)$

This expression has no well defined modular property under some subgroup of $SL(2\mathbb{Z})$

$$\tau = \frac{\ln q}{2\pi i} \longrightarrow -\frac{1}{\tau}$$

$$z = \frac{\ln y}{2\pi i} \longrightarrow -z/\tau$$

UNLESS $E^A_1(0, y) = f^A_1(y)$

BUT THIS IS IN GENERAL NOT TRUE

$E^A_1(0, y)$ counts the # of D1/D5 BOUND STATES IN MODEL (A) ((4,4) Theory)

In GENERAL $E_1^A(\circ 1) \neq 0$

But $f_1^A(y) \to 0$ at $y \to 1$

since it counts the 1/4 BPS

states D1/D5 in MODEL (B)

(4,0) Theory

What is the possible resolution ?

1) Perhaps 1/4 BPS states are not stable. (cf. 3-string Junction a la Ashoke Sen in N=4 SYM Theory)

But in model A (4,4) theory the singularity (threshold) appear in a subspace of co-dimension > 1 in the moduli space.

$\Rightarrow$ Elliptic genus does not jump !!

$\Rightarrow$ 3-charge (D1, D5, KK)

must be stable.

2) Sym. Prod. theory is defined at

$x = 1/2$

In model (A) $x = 1/2 \longrightarrow x = 0$

In model (B) $x = 1/2$ $\Delta x = 0$

are ~~distinct~~ models.

U-duality on model (A) ~~at~~ $x = 1/2$

$\longrightarrow$ model (B) at $x = 0$

This is entirely singular

On the other hand $x = 1/2$ in model (B) at which Sym. Prod. CFT exists is mapped to Model (A) with non-zero G_{15} which breaks Lorentz invariance of the world sheet.

3. AdS / CFT Correspondence

If R_6 is very large the near horizon geometry of D1/D5 system

$\sim AdS_3 \times S^3 \times T^4$

Fluctuation analysis

$h_{r,s}$ $(m+r, m+s)_{(4,4)}$ super multiplets

h_{rs} are the hodge #'s of T^4

$h_{00}^{(1)}$

$h_{01}^{(2)}$ $h_{10}^{(2)}$

$h_{02}^{(1)}$ $h_{11}^{(4)}$ $h_{02}^{(1)}$

$h_{12}^{(2)}$ $h_{21}^{(2)}$

$h_{22}^{(1)}$

$\mathbb{Z}_2$ action on (c.c) primary

$\hat{g} = (-1)^{r+s}$ for I_4 model

$= (-1)^{s}$ for ΩI_4 model

Furthermore

$\hat{g}$ commutes with $\underset{(\text{Left})}{G_{-1/2}}$ & $\underset{(\text{Right})}{\tilde{G}_{-1/2}}$

for I_4 model (4,4)

But $\hat{g}$ commutes with $\tilde{G}_{-1/2}$ &

anti commutes with $G_{-1/2}$ for

ΩI_4 model. (4,0)

By Following Jan De Boer

associate "degree"

Poincare Polynomials give Correct

D1/D5 Bound states

Elliptic genus for multiparticle SUGRA states

for model I_4 $\xrightarrow{\text{agrees}}$ CFT for I_4 model

for model ΩI_4 $\xrightarrow{\text{agrees}}$ CFT for I_4 model with $p \leftrightarrow q$

$p^N q^\Delta$

$$\left.\begin{array}{l} \Delta < N/4 \\ \\ N < \Delta/4 \end{array}\right\}$$

This result seems to be consistent with

→ ~~for~~ (D1, D5, KK) system is stable in I_4 model

→ ΩI_4 model at $\chi=0$ is indeed U-dual to I_4 model

Question: Can one do SUGRA analysis for $\chi = 1/2$ for $\mathcal{N} I_4$ model?

AND

DOES This agree with the (4,0) CFT we have proposed here.?