Ramond-Ramond Couplings of Noncommutative Branes

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Strings 2001

Mumbai

January 7, 2001

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Based on:

S. Mukhi, N.V.S

hep-th/0009101 & Work in progress.

(JHEP 06(2000) 011)

1. Introduction

- A distinguished feature of D - branes in Superstring theories is that they couple to RR - fields given by the Chern-Simons terms.

- With the recent interest in the noncommutative descriptions of D - branes, a natural question to ask is: How are these couplings described in the noncommutative language? This question has received very little attention so far.

- We will address this question here.

- We will obtain the Chern-Simons terms on noncommutative branes. We will also see that these terms contain all the information about Myers terms on multiple branes.

- In the presence of constant NS-NS B-field the dynamics of a (Euclidean) Dp-brane can be described by the following DBI action: $(2\pi\alpha'=1)$

[SEIBERG WITTEN]

$$\hat{S}_{DBI} = \hat{T}_p \int d^{p+1}x \sqrt{\det(G_{ij} + \hat{F}_{ij} - \Phi_{ij})}$$
$$\hat{T}_p = \frac{(2\pi)^{\frac{1-p}{2}}}{G_s}$$
$$\hat{F}_{ij} = \partial_i \hat{A}_j - \partial_j \hat{A}_i - i[\hat{A}_i, \hat{A}_j]_*$$

and the fields are multiplied using $*$ - product.

$$f(x) * g(x) \equiv e^{\frac{i}{2}\theta^{ij}\partial_i\partial'_j} f(x) g(x')|_{x=x'}$$

- The parameters G_{ij}, Φ_{ij}, G_s and θ^{ij} are given in terms of the commutative variables g, B, g_s by:

$$\frac{1}{G+\Phi} = -\theta + \frac{1}{g+B}$$
$$G_s = g_s \left(\frac{\det(G+\Phi)}{\det(g+B)}\right)^{\frac{1}{2}}$$

- Φ denotes the freedom in the description.

- If we choose $\Phi_{ij} = -B_{ij}$, then

$$\theta^{ij} = (B^{-1})^{ij} \qquad G_{ij} = -B_{ik} g^{kl} B_{lj}$$
$$G_s = g_s \sqrt{\frac{\det\ B}{\det\ g}}$$

- Working in $\Phi = -B$ description $\hat{S}_{DBI}$ can be put in the form:

$$\hat{S}_{DBI} = T_p \int d^{p+1}x \, \frac{\mathrm{Pf}Q}{\mathrm{Pf}\theta} \sqrt{\det(g_{ij} + (Q^{-1})_{ij})}$$

Where $Q^{ij} = \theta^{ij} - \theta^{ik} \, \hat{F}_{kl} \, \theta^{lj} = -i[X^i, X^j]$ with $X^i = x^i + \theta^{ik} \hat{A}_k$ and $T_p = (2\pi)^{\frac{1-p}{2}} / g_s$

- Using:

$$\int d^{p+1}x \to \mathrm{Tr}(2\pi)^{\frac{p+1}{2}} \mathrm{Pf}\theta$$

$$\hat{S}_{DBI} = \frac{2\pi}{g_s} \mathrm{Tr}\left[\mathrm{Pf}Q \sqrt{\det(g_{ij} + (Q^{-1})_{ij})}\right]$$

- This action can be thought of as the action for infinitely many D(-1) - branes in a classical configuration.

CORNALBA
ISHIBASHI
SEIBERG

- Alternatively, we can start with the DBI action for N D(-1) - branes (with N $\to$ ∞):

[RAAMSDONK TAYLOR] [MYERS]

$$\hat{S}_{DBI} = \frac{2\pi}{g_s}\mathrm{Tr}\left[\sqrt{\det(\delta_i^j - i g_{ik}[X^k, X^j])}\right]$$

- Now consider the solution corresponding to a Dp-brane:

$$\begin{aligned} X^i &= x^i \qquad \text{for } \ i = 1, 2, ..., p+1 \\ X^j &= 0 \qquad \text{for } \ j = p+2, ..., 10. \\ \text{such that } \ & [x^i, x^j] = i\theta^{ij} \equiv (B^{-1})^{ij} \end{aligned}$$

- Consider fluctuations around this classical solution:

$$\begin{aligned} X^i &= x^i + \theta^{ik}\hat{A}_k \\ X^j &= \phi^j \end{aligned}$$

- Substituting these in the action for the D(-1) - branes gives the noncommutative DBI action of a Dp - brane in $\Phi = -B$ description.

- This prescription of obtaining noncommutative DBI action in $\Phi = -B$ description can be used to obtain the Chern-Simons terms as well.

2.1 Chern-Simons terms on BPS branes:

- The CS action for N D(-1) - branes: (N $\to \infty$)

[MYERS]

$$S_{cs} = \frac{2\pi}{g_s} \mathrm{Tr}\left[e^{i(\mathrm{i}_X \mathrm{i}_X)} \sum_n C^{(n)} \right]$$

- For example let us take the coupling to the RR 10-form $C^{(10)}$.

$$\begin{aligned} S_{cs} &= \frac{2\pi}{g_s} \mathrm{Tr}\left[\frac{i^5}{5!} X^{i_{10}} X^{i_9} ... X^{i_1} C^{(10)}_{i_1 i_2 ... i_{10}} \right] \\ &= -\frac{2\pi}{g_s} \mathrm{Tr}\left[\frac{1}{5! 2^5} (i[X^{i_1}, X^{i_2}]) ... (i[X^{i_9}, X^{i_{10}}]) C^{(10)}_{i_1 i_2 ... i_{10}} \right] \end{aligned}$$

- To obtain the noncommutative CS terms for a Dp - brane, we substitute the solution of X^i 's corresponding to a Dp - brane along with the fluctuations into the action above.

- In the case of a D9 - brane this gives rise to:

$$\rightarrow \hat{S}_{cs}^{(D9)} = \frac{2\pi}{g_s} \mathrm{Tr}[\frac{1}{5!2^5} \epsilon_{i_1 i_2 \ldots i_{10}} Q^{i_1 i_2} \ldots Q^{i_9 i_{10}}$$
$$\frac{1}{10!} \epsilon^{j_1 j_2 \cdots j_{10}} C^{(10)}_{j_1 j_2 \cdots j_{10}}]$$

$$= \frac{2\pi}{g_s} \mathrm{Tr}\left[\mathrm{Pf}\, Q\, C^{(10)}\right]$$

- Converting into an integral:

$$\hat{S}_{cs}^{(D9)} = \mu_9 \int d^{10}x \frac{\mathrm{Pf}\, Q}{\mathrm{Pf}\, \theta} \left[\frac{1}{10!} \epsilon^{j_1 j_2 \cdots j_{10}} C^{(10)}_{j_1 j_2 \cdots j_{10}}\right]$$

Recall: $Q = \theta - \theta\, \hat{F}\, \theta$.

- For obtaining lower form couplings, we repeat the above exercise starting with that form coupling to infinite D(-1) - branes. The result can be put in the following form:

$$\hat{S}_{cs}^{(D9)} = \mu_9 \int_x \frac{\mathrm{Pf} Q}{\mathrm{Pf} \theta} \sum_n C^{(n)}\ e^{Q^{-1}}$$

- For the other lower dimensional branes the result is:

$$\hat{S}_{cs}^{(Dp)} = \mu_p \int_x \frac{\mathrm{Pf}Q}{\mathrm{Pf}\theta} P\left[e^{i(\mathrm{i}_\Phi * \mathrm{i}_\Phi)} \sum_n C^{(n)}\right] e^{Q^{-1}}$$

- Notice that a single noncommutative brane can couple to all the forms!

2.2 Chern-Simons terms on non-BPS branes:

- Type II theories have unstable Dp- branes, where p is odd for type IIA and even for type IIB.
- Chern-Simons terms on these non-BPS branes play an important role in deducing the RR charges of the decay products.
- The CS terms on a single unstable brane in commutative description are given by:

$$S_{cs} = \frac{\mu_{p-2}}{2T_{min}} \int dT\ C^{(n)} e^{F+B}$$ [SEN]

Where T_{min} is the value of the tachyon at the minimum of the tachyon potential $V(T)$.

- We propose that the Chern-Simons action on the Euclidean D9 brane of type IIA in the noncommutative description is:

$$\hat{S}_{cs} = \frac{\mu_8}{2T_{min}} \int_x \frac{\mathrm{Pf}Q}{\mathrm{Pf}\theta}\, \mathcal{D}T\, C^{(n)}\, e^{Q^{-1}}$$

where $\mathcal{D}_i T = -i(Q^{-1})_{ij}[X^j, T]$

- A non-trivial check of this action is the following. Consider a noncommutative soliton which represents the decay of this D9 brane into N coincident D(-1) - branes.

- Condensing the tachyon over that soliton we will get N coincident D(-1) - branes. Substituting the solitonic solution along with fluctuations around that should give us the CS action on them.

- It is easy to verify that the proposed action does pass this check.

- The action for other unstable branes can be obtained by taking appropriate noncommutative soliton representing lower branes and substituting the solution along with fluctuations.

- For illustration let us do the tachyon condensation over the following soliton, which is supposed to represent the decay of a D9 - brane into N D7 - branes.

[GMS DMR HKLM]

$$T_{cl} = T_{max}P_N + T_{min}(1 - P_N)$$
$$X^i_{cl} = P_N x^i \quad \text{for} \quad i = 1, 2, \ldots, 8$$
$$X^i_{cl} = 0 \quad \text{for} \quad i = 9, 10$$

- This solution has the property that $[T_{cl}, X^i_{cl}] = 0$.

- Substituting this solution along with the fluctuations into the $C^{(9)}$ coupling in the proposed action for the unstable D9 - brane, we get

$$\hat{S}^{UD7}_{CS} = \frac{\mu_6}{2T_{min}} \mathrm{tr}_N \int_x (-i)\,[\delta X^9, \delta X^{10}]\,(-i)(Q^{-1}_{cl})_{12}\,[X^2_{cl}, \delta T]\, C^{(9)}_{23\ldots 10}$$

And also

$$\hat{S}^{UD7}_{CS} = \frac{\mu_6}{2T_{min}} \mathrm{tr}_N \int_x (-i)[\delta X^{10}, \delta T]\, C^{(9)}_{12\ldots 8,10}$$

- These are actually the two kinds of couplings that exist on a stack unstable branes. [JANSSEN . MEESSEN]

3 General Φ

- Here we want to know the CS couplings in other descriptions.

- As we have seen the RR - form $C^{(p+1)}$ coupling to a NC Dp - brane is given by:

$$\hat{S}(\Phi)_{CS} = \mu_p \int_x \sqrt{\det(1-\theta\hat{F})} C^{(p+1)}$$

- We propose that this is the answer in all the other descriptions, with θ given in that description.

- To verify this we can consider the variations of the action w.r.t θ and show that they vanish up to total derivatives and of $\mathcal{O}(\partial\hat{F})$ terms.

- The variation of the $\hat{F}$ w.r.t θ is

$$\delta\hat{F}_{ij}(\theta) = \delta\theta^{kl}\left[\hat{F}_{ik}\hat{F}_{jl} - \frac{1}{2}\hat{A}_k(\partial_l\hat{F}_{ij} + \hat{D}_l\hat{F}_{ij})\right] + \mathcal{O}(\partial\hat{F})$$

[S & W]

- Since we are going to ignore derivatives of $\hat{F}$, we drop the $*$ products between $\hat{F}$'s but leave them in the definitions of $\hat{F}$ and in $\hat{D}_l\hat{F}_{ij}$.

- Keeping the closed string RR field constant the variation of the action is given by:

$$
\begin{aligned}
&\delta\left[\sqrt{\det(1-\theta\hat{F})}\right] \\
&= -\frac{1}{2}\sqrt{\det(1-\theta\hat{F})}\ \mathrm{Tr}\left[\frac{1}{1-\theta\hat{F}}\delta\theta\hat{F} + \frac{1}{1-\theta\hat{F}}\theta\delta\hat{F}\right] \\
&= -\frac{1}{2}\sqrt{\det(1-\theta\hat{F})}\ \mathrm{Tr}[\frac{1}{1-\theta\hat{F}}\delta\theta\hat{F} + \left(\frac{1}{1-\theta\hat{F}}\right)^{j}{}_{m} \\
&\qquad \theta^{mi}\delta\theta^{kl}(\hat{F}_{ik}\hat{F}_{jl} - \frac{1}{2}\hat{A}_k(\partial_l\hat{F}_{ij} + \hat{D}_l\hat{F}_{ij}))] + \mathcal{O}(\partial\hat{F}) \\
&= -\frac{1}{2}\sqrt{\det(1-\theta\hat{F})}\ \mathrm{Tr}[\frac{1}{1-\theta\hat{F}}\delta\theta\hat{F} - \frac{1}{1-\theta\hat{F}}\theta\hat{F}\delta\theta\hat{F} \\
&\qquad - \delta\theta\hat{F}] + \mathcal{O}(\partial\hat{F}) + \text{total derivatives} \\
&= \mathcal{O}(\partial\hat{F}) + \text{total derivatives}
\end{aligned}
$$

- In the last step we have used the following identities:

$$
\begin{aligned}
&\partial_l\ \sqrt{\det(1-\theta\hat{F})} \\
&\quad = -\frac{1}{2}\sqrt{\det(1-\theta\hat{F})}\left(\frac{1}{1-\theta\hat{F}}\right)^{j}{}_{m}\ \theta^{mi}\partial_l\hat{F}_{ij} \\
&\hat{D}_l\ \sqrt{\det(1-\theta\hat{F})} \\
&\quad = -\frac{1}{2}\sqrt{\det(1-\theta\hat{F})}\left(\frac{1}{1-\theta\hat{F}}\right)^{j}{}_{m}\ \theta^{mi}\hat{D}_l\hat{F}_{ij} \\
&\qquad\qquad + \mathcal{O}(\partial_l\hat{F}\hat{D}_l\hat{F}),
\end{aligned}
$$

and

$$
\delta\theta^{kl}(\partial_l\hat{A}_k + \hat{D}_l\hat{A}_k) = \delta\theta^{kl}\hat{F}_{lk}
$$

4 Conclusions

- We have obtained the RR couplings of noncommutative branes in $\Phi = -B$ description for both BPS and non-BPS branes.

- On the non-BPS branes, by condensing the NC tachyon over solitonic solutions, we obtained the Myers terms on coincident non-BPS branes.

- We proposed the generalization of some of the couplings we found to other descriptions.

- One of the related interesting question is about the generalizations of these couplings to non-constant RR fields. For this, following the proposals of Das and Trivedi, one will have to attach "Wilson tails" to the operators that we found to restore gauge invariance.
(work in progress)

- Also one can carry out string amplitude computations to check the existence of these couplings. By that one can gain insight into the form of Wilson tails away form $\Phi = -B$ description. (work in progress)

- Consistency with T-dualities.
(Radu Tatar)