More on Singularity Resolution

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Outline

- Introduction, review of n=2 enhançon. Brane probe: enhançon locus, where tension→0, yields singularity resolution.

- Remarks on finite-T SUGRA for Dp+4/K3.

- **Pilch-Warner n=2* flow geometry. Understanding physics via probes: D3-branes, n=2 structure, linear enhançon, brane distribution. Non-commutativity for n=2*.**

- Remarks on non-commutative version of no-hair/CMW duality.

Motivation

Spacetime singularities in classical string/M theory should be either [Horowitz-Myers]
(a) resolved by quantum effects or
(b) disallowed. (resolution → vacuum instability)

Encounter clothed and naked singularities:
- null, timelike, spacelike.

Geometries built from fundamental objects will have resolution: string/M theory "knows what to do" when cycles on which branes are wrapped become small.

Resolution mechanisms for specific systems:

- null singularities of n=4 BPS Dp≠3 brane geometries, in decoupling limit, resolved by SYM etc. [IMSY] (D6 naked)
- timelike naked singularities of n=2 brane geometries resolved by "enhançon" brane expansion mechanism. [JPP]

Related works

- $n=1^*$ dielectric-brane expansion
  [Polchinski-Strassler]
- "cascade", "transgression", and others:
  [Dasgupta-Mukhi],
  [Klebanov-Tseytlin],
  [Klebanov-Strassler],
  [Maldacena-Nunez],
  [Evans-Johnson-Petrini],
  [Grana-Polchinski],
  [Gubser],
  [Zayas-Tseytlin],
  [Kallosh-Mohaupt-Shmakova],
  [Buchel],
  [Cvetic-Lu-Pope],
  [Bertolini-DiVecchia-Frau-Lerda-Marotta-Pesando],
  [Johnson-Lovis-Page],
  [Polchinski],
  [Rajaraman],
  [Maldacena-Maoz],
  [Frau-Liccardo-Musto],
  [Gauntlett-Kim-Waldram],
  [Papadopoulos-Tseytlin],
  [Bain], AND OTHERS.

Here: concentrate on $n=2$ and $n=2^*$ systems.

Enhançon reminder [JPP]

N Dp+4/K3 realisation of n=2: ($d=p+1$)
K3 wrappage → induced Dp charge (-N).
[$p\leq 1$: IR wild; $p\geq 3$: geometry not A.F. → p=2]

Classical geometry as if built from H's of D6's plus K3-smeared D2's:

$$ds^2 = \frac{1}{\sqrt{H_2}\sqrt{H_6}} dx_{\|}^2 + \frac{\sqrt{H_2}}{\sqrt{H_6}} ds_{K3}^2 + \sqrt{H_2}\sqrt{H_6}\left(dr^2 + r^2 d\Omega^2\right)$$

$H_6 = \frac{g_s N}{2 l_s U}$ as usual, **but** $H_2 = 1 - \frac{V_*}{V}\frac{g_s N}{2 l_s U}$ has –

2+1 SYM 't Hooft coupling: (V finite)

$$g_{YM}^2 N = \frac{V_*}{V}\frac{g_s N}{l_s} \qquad V_* = (2\pi l_s)^4$$

SUGRA solution singular at $U_R = g_{YM}^2 N/2$
"repulson" [Kallosh-Linde]

Would-be horizon at U=0 → naked singularity.
$g_{\Omega\Omega}(U_R) = 0$: point singularity, can't go inside.

Probe physics

K3 curved; extra terms in brane action give

$$L = const. + \frac{1}{2}\vec{v}^2 \frac{1}{g_s l_s^3 (2\pi)^2}\left(\frac{V}{V_*} - 2\frac{g_s N}{2 l_s U}\right)$$

Moduli space metric → 0 at enhançon radius

$$U_E = 2U_R$$

Effective D6/K3 tension→0 at enhançon.

Cannot move brane inside enhançon locus.

→ Dyson sphere of branes;
resolution of singularity.

On gauge theory side, see brane expansion non-perturbatively, via S-W curve.

Finite temperature SUGRA

$$H \to 1 + \alpha_p \left(\frac{r_p}{r}\right)^{7-p}$$ for Dp ,

multiply $-g_{tt}, g^{rr}$ by $K(r) = 1 - \left(\frac{r_0}{r}\right)^{7-p}$,

and divide R-R form by

$$\alpha_p = -\left(\frac{r_0}{2r_p}\right)^{7-p} + \sqrt{1 + \left(\frac{r_0}{2r_p}\right)^{2(7-p)}} .$$

For $r_0 \to 0, \alpha_p \to 1$ while for $r_0 / r_p \uparrow\uparrow, \alpha_p \to 0$.

For enhançon, funny minus sign and $\delta M \geq 0$ give via e.o.m. $0 \leq \alpha_6 \leq 1$ ✓ but

$$\alpha_2 = \oplus \left(\frac{r_0}{2r_2}\right) + \sqrt{1 + \left(\frac{r_0}{2r_2}\right)^2} \quad (r_2 = \frac{V_*}{V} r_6)$$

Result: $1 \leq \alpha_2 < \infty$, locus $T \to 0$ > BPS case (!)

and $u_R > u_0$ (Corrects typo in original paper.)

(See also [Buchel]: T>0 Klebanov-Tseytlin solution.)

Finite temperature SUGRA, II

In decoupling limit, α_6 unity.

Non-BPS → potential energy

$$\boldsymbol{V} = \frac{\mu_2}{g_s}\left[\frac{1}{(1-\alpha_2\lambda/2U)}\left(\frac{1}{\alpha_2}-\sqrt{K}\right)-\frac{2U}{\lambda}\left(1-\sqrt{K}\right)\right].$$

Attractive because of weird sign ($1 \le \alpha_2 < \infty$) and

$$\alpha_2 = \oplus\left(\frac{U_0}{\lambda}\right)+\sqrt{1+\left(\frac{U_0}{\lambda}\right)^2}$$

Kinetic energy

$$T = \frac{\mu_2 V}{2g_s}\left[1-\frac{\lambda}{2U}(1+\alpha_2)\right]\frac{1}{\sqrt{K}}\left\{\frac{\left(\upsilon^U\right)^2}{K}+U^2\left(\upsilon^\Omega\right)^2\right\}$$

Hence, enhançon always outside horizon. (!)

V monotic all the way to enhançon.
Precise configuration of finite-*T* branes?
Schw.BH inside? → Gregory-Laflamme?

Pilch-Warner n=2*

Turn on

$$O_b = \sum_{j=1}^{4} Tr\left(X^j X^j\right) - 2\sum_{j=5}^{6} Tr\left(X^j X^j\right),$$

$$O_f = Tr\left(\lambda^3\lambda^3 + \lambda^4\lambda^4\right), \quad \overline{O_f}.$$

Consistent truncation Ansätz for SUGRA:

- finite # integration constants, c.f. gauge theory moduli space *M* has *O(N)* param's. PW: 1-parameter subspace of *M*.
- **Singularity structure manifest in *d*=10 .**

Tour-de-force calculations!

New features:

- *d*=10 dilaton varies with radius, and depends on two angles! Squashed S^5.
- Complicated R-R, NS-NS forms turned on.

Parameters: γ , n=4-breaking $k \propto mL$.

(+ [Brandhuber-Sfetsos], [Gubser])

Symmetries:

$\mathcal{N}$=2* has SU(2)xU(1) R-symmetry.

See via four Weyl fermions:

- #3,4 get mass, U(1)=SO(2) mixes them
- #1,2 massless, SU(2) mixes them.

6 scalars: transform as $6 = (4\times 4)_A$,
two invariant under R-sym
⇔ fixed plane for SUGRA solution where
radius of transverse squashed sphere →0.

R-sym does not act on azimuthal angle φ of PW; fields can (and do) depend on it in complicated fashion.

Accidental SUGRA symmetry: U(1)'
(Einstein metric only; *H* covariant.)
= combo of U(1) ⊂ SO(6) and U(1) ⊂ SL(2,R).

Leaves mass perturbation (fermion bilinears) invariant.

PW n=2* solution

d=10 Einstein frame metric

$$ds_E^2 = \frac{(cX_1X_2)}{\rho^3}\left\{\frac{k^2\rho^6}{c^2-1}dx_{\|}^2 - \frac{L^2}{\rho^6\left(c^2-1\right)^2}dc^2\right\} - \frac{(cX_1X_2)}{\rho^3}L^2 \times$$

$$\times\left[\frac{1}{c}d\vartheta^2 + \frac{\sin^2\vartheta}{X_2}d\varphi^2 + \rho^6\cos^2\vartheta\left(\frac{1}{cX_2}\sigma_3^2 + \frac{1}{X_1}\left(\sigma_1^2+\sigma_2^2\right)\right)\right]$$

R-R 5-form field strength:

$$F_{(5)} = f + *f, \quad f = 4dx^0\wedge dx^1\wedge dx^2\wedge dx^3\wedge dw(c,\vartheta)$$

and $H_{(3)}^{[i]}$ turned on in $\perp$ directions.

Dilaton runs

$$\tau = \frac{\tau_0 - \overline{\tau_0}B}{1-B}, \quad B = e^{2i\varphi}\frac{\sqrt{cX_1}-\sqrt{X_2}}{\sqrt{cX_1}+\sqrt{X_2}}$$

where $\boxed{\tau_0 = i/g_s + \theta_s/2\pi}$ is asymptotic coupling and $X_1 = \cos^2\vartheta + c\rho^6\sin^2\vartheta, \quad X_2 = c\cos^2\vartheta + \rho^6\sin^2\vartheta$.

Relation to usual isotropic coord r for n=4:

$$\rho^6 = c + \left(c^2-1\right)\left[\gamma + \frac{1}{2}\ln\left(\frac{c-1}{c+1}\right)\right], \quad c = \cosh\left(\frac{kL}{r}\right)$$

At $r\to\infty, \; c\to 1^+, \; \rho^6, X_1, X_2 \to 1$.

Understanding n=2*

Source-branes are simply D3-branes!

$$S_{probe} = -\tau_p \int d^{p+1}\xi\, e^{-\Phi} \sqrt{\det\left(\mathrm{P}[G+B]_{ab} + 2\pi l_s^2 F_{ab}\right)} + \tau_p \int \mathrm{P}\exp\left(2\pi l_s^2 F_{(2)} + B_{(2)}\right) \wedge \bigoplus_n \left(g_s C_{(n)}\right)$$

Probe analysis: must do in d=10.

Analysis simpler than appears because
- $B_{(2),\|}\ C_{(2),\|}\ F_{(2)}$ vanish,
- cross-terms in $F_{(5)}$ absent.

Find zeroes of potential energy at
- equator of 5-sphere, $\vartheta = \pi/2$.
- special radius ρ=0 dictated by $\gamma \le 0$.

Two coordinate patches required for fixed-plane required by R-symmetry:
- (ρ,ψ) plane
- (ϑ,φ) with identification $\vartheta \cong \pi - \vartheta$.

Moduli space is two-dimensional. Metric?

n=2 structure and enhançon

Want coords natural for *n*=2 structure:

demand $T(Y)=\frac{1}{2}\tau_3 e^{-\Phi}\upsilon^Y\upsilon^{\bar{Y}}$ for complex *Y*.

Complicated transformation yields

$$\tau(Y)=\frac{i}{g_s}\left(\frac{Y^2}{Y^2-k^2L^2}\right)^{1/2}+\frac{\theta_s}{2\pi}$$

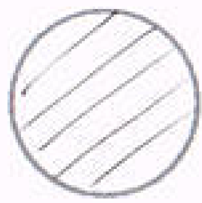
holomorphic fn of *Y* .

$\boxed{\gamma=0}$: circle in PW coords becomes
line segment of source-branes in *n*=2 coords
with branch cut from $Y=-kL$ to $Y=+kL$.

Kinetic energy *T*→0 on cut;
have linear enhançon.

(not *n*=2 limit of P-S story;
perturbation different here.)

$\boxed{\gamma\downarrow}$: line segment unsquashes;
as $\gamma\to-\infty$ system goes over to
n=4 Coulomb branch problem.



Gauge theory side

SUGRA non-singular in string frame; enhançon distribution as compact as possible.

Identify distribution of source D3-branes by demanding $\tau_{SUGRA} = \tau_{SYM}$

Seiberg-Witten: Coulomb vacua, QMically always $U(1)^{N-1}$. Let vev be $diag(\{a_i\}), \sum_i a_i = 0$

Prepotential for N-1 Vplets is

$$F = F_c + F_{pert} + F_{np}$$

F_{pert}: 1-loop exact – get via ∫ out charged ("W"s, Hplets)

F_{np}: hard to find at large-N. Important when have light BPS states e-val spacings < $O(1/N)$ Instanton corrections exponentially suppressed in SUGRA regime.

Brane distribution

Probe at *u*: vev is $diag(u, \{a_i - u/N\})$
F_{pert} gives

$$\tau(u) = \frac{i}{g_s} + \frac{\vartheta_s}{2\pi} + \frac{i}{2\pi}\sum_i \ln\left[\frac{(u-a_i-u/N)^2}{(u-a_i-u/N)^2 - m^2}\right]$$

Note: size of enhançon $a_0 \approx kL/l_s^2$ in *u* units ; this is >>*m* i.e. breaking small in PW *n*=2*.

Matching to SUGRA, find semicircle distribution

$$\rho(u) = \frac{2}{m^2 g_s}\sqrt{a_0^2 - u^2}$$

and precise relation $k = mL$.

Non-perturbative corrections turn on sharply at enhançon locus.

Gauge theory may provide clues to SUGRA dual for more general *n*=2* configuration.

Making $\mathcal{N}$=2* non-commutative

In gauge theory, phase for planar graphs is trivial for two-derivative terms giving metric on moduli space.

SUGRA: straightforward [Hashimoto-Itzhaki]

- T-dual on 2,3; S-dual; T-dual back.

Tedious algebra; checked solution in e.o.m.

B-field Δ gives new function $h = 1 + \frac{\Delta^4 G_{11}^4}{l_s^4}$ in

$$e^{\hat{\Phi}} = \frac{e^{\Phi}}{\sqrt{h}} \quad \hat{G}_{11} = G_{11} \quad \hat{G}_{22} = \hat{G}_{33} = \frac{G_{11}}{\sqrt{h}} \quad d\hat{s}^2_{E,other} = ds^2_{E,other}$$

$$\hat{B}_{(2)} = B_{(2)} + \frac{\Delta^4 G_{11}}{l_s^2 h} dx^2 \wedge dx^3 \quad \hat{C}_{(0)} = C_{(0)}$$

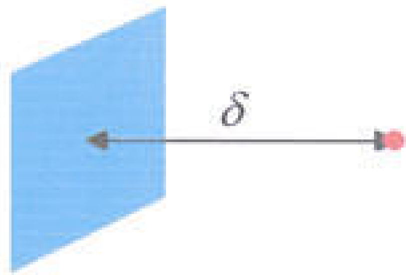
$$\hat{C}_{(2)} = C_{(2)} + \frac{\Delta^2}{l_s^2} \iota_2 \iota_3 C_{(4)} - \frac{\Delta^2 G_{11}^4}{l_s^2 h} C_{(0)} dx^2 \wedge dx^3$$

$$\hat{C}_{(4)} = \frac{1}{h} C_{(4)\parallel} + C_{(4)\perp} + \frac{\Delta^2}{l_s^2} \iota_2 \iota_3 C_{(6)} - \frac{\Delta^2 G_{11}^4}{l_s^2 h} C_{(2)} dx^2 \wedge dx^3$$

Probe moduli space metric unaffected √

NC version of no-hair/CMW duality

Strings'99 talk: [MP]



Dp+4 and Dp in static equilibrium at any separation δ

As $\delta \to 0$, ask about

localisation of wee-brane inside big-brane.

Found duality: baldness for $p \leq 1$
hair for $p \geq 2$

Recently: [Furuuchi] constructed gauge theory configuration relevant to NC (Dp+4,Dp) , including separation parameter.

Preliminary investigation of SUGRA yields similar results as before √ . Locality issues; probe physics using open Wilson loops.