FREE ENERGY AND CRITICAL TEMPERATURE IN ELEVEN DIMENSIONS

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- HAGEDORN TRANSITION: IN SUPERSTRING THEORY, IT IS A 1ST ORDER PHASE TRANSITION AT $T_{CR} < T_{HAG}$ WITH LARGE LATENT HEAT LEADING TO INSTABILITY OF THERMAL ENSEMBLE (ATICK AND WITTEN, 1988)
  (NCOS IN $D<6$: 2ND ORDER, FS ARE LIBERATED FROM D-BRANE (GUBSER ET AL, 0009140))
- WHAT IS THE FATE OF THE HAGEDORN TRANSITION IN $D=11$?

OR: WHAT HAPPENS TO T_{HAG} IN TYPE IIA AT $g_A \gg 1$?

- THE EXISTENCE OF T_{HAG} IS DUE TO THE STRING-LIKE DEGREES OF FREEDOM. UNDERSTANDING DETAILS OF THE THERMODYNAMICS IN $D=11$ WOULD TELL US SOMETHING ABOUT MICROSCOPIC DEGREES OF FREEDOM IN M-THEORY.
- THE PRESENCE OF IR DIVERGENCES IN FREE ENERGY INDICATES TACHYON INSTABILITIES LEADING TO PHASE TRANSITION.
- CAN WE COMPUTE FREE ENERGY IN M-THEORY?
  WE NEED TO KNOW MICROSCOPIC THEORY.
- "Zeroth ORDER" QUESTION: CAN WE COMPUTE A SENSIBLE FREE ENERGY IN $D=11$ SUPERGRAVITY?

  IS IT UV FINITE?

ONE-LOOP FREE ENERGY IN D = 11 SUPERGRAVITY WITH COMPACT X^{11}

$$\boxed{F_{\text{SUGRA}}(T, g_A) = -V\, T^{10}\, \frac{2^9\, \Gamma(\frac{11}{2})\, \zeta(11)}{\pi^{11/2}\; g_{eff}^{9/2}} \left(E_{\frac{11}{2}}(g_{eff}) - F_{\frac{11}{2}}(g_{eff}) \right)}$$

$$\boxed{g_{eff} \equiv \frac{R_{11}}{R_0} = 2\pi \sqrt{\alpha'}\; g_A T}$$

FINITE !!

$$F_{\text{SUGRA}}(T, g_A) = -V\, T^{10}\, \frac{2^8\, \Gamma(11/2)}{\pi^{\frac{11}{2}}\; g_{eff}^{9/2}} \sum_{m,n} \left(1 - (-1)^m\right) \left(\frac{g_{eff}}{(m^2 + n^2 g_{eff}^2)} \right)^{11/2}$$

WEAK COUPLING

$$\boxed{\frac{F_{SG}}{VT} = -\, 24\, \zeta(10)\, \pi^{-5}\, T^9 \,(2^{10}-1) + O\left(e^{-\frac{2\pi}{g_{eff}}}\right), \quad g_{eff} \ll 1.}$$

STRONG COUPLING

$$\frac{F_{SG}}{(2\pi R_{11}) V T} = -\, \frac{1.934.415\; \zeta(11)}{64\, \pi^5}\, T^{10} - \frac{3 \cdot 2^{12}}{\pi^5}\, \zeta(10)\, \frac{T^{10}}{g_{eff}^{10}} + O\left(e^{-2\pi g_{eff}}\right), \quad g_{eff} \gg 1$$

$945 \cdot (2^{11}-1) = 1.934.415$

CRITICAL TEMPERATURE IN M-THEORY

$$\boxed{T_{CR}^{\,2} = \frac{1}{8\alpha'}\,\frac{\sqrt{g_{eff}}}{2\zeta(3)\left(E_{3/2}(g_{eff}) - F_{3/2}(g_{eff})\right)}}$$

OR

$$\boxed{\frac{1}{T_{cr}^{\,2}} = \frac{8\alpha'}{\sqrt{g_{eff}}}\sum_{m,n}\left(1-(-1)^m\right)\left(\frac{g_{eff}}{(m^2+n^2 g_{eff}^2)}\right)^{3/2}}$$

$$\boxed{T_{cr} = T_{HAG} + O\left(e^{-\frac{2\pi}{g_{eff}}}\right), \quad g_{eff} \ll 1}$$

$$\left(T_{HAG} = \frac{1}{2\pi\sqrt{2\alpha'}}\right)$$

$$T_{cr} \simeq \frac{1}{a\sqrt{\alpha'}\,(2\pi g_A)^{1/3}} = \frac{1}{a}\,l_p^{-1}, \quad g_{eff} \gg 1$$

$$a \equiv \left(28\,\zeta(3)\right)^{1/3}$$

$$\Rightarrow \boxed{\begin{array}{c} T_{cr} = \frac{1}{a}\,l_p^{-1} \simeq 0.31\,l_p^{-1} \\ R_{11}\to\infty \end{array}}$$

Gravitational Instabilities

- STATISTICAL MECHANICS : VALID FOR INFINITE VOLUME OR SIZE $\sim R \gg \ell_P$ (1)

(SO THAT THE SYSTEM CONTAINS MANY DEGREES OF FREEDOM)

- FLAT THEORY DESCRIPTION :

VALID FOR $\frac{G_N E}{R^8} \ll 1$, $G_N \sim \ell_P^9$

OR

$$\ell_P^9 E \underbrace{\frac{R^2}{R^{10}}}_{=V} = \ell_P^9 \frac{E}{V} R^2 \ll 1 \quad (2)$$

CONSIDER THE THERMAL ENSEMBLE AT $T = O(\ell_P^{-1})$
IS THE METRIC FLAT ?

$\Rightarrow \quad \frac{F}{V} = \text{const.}\ \ell_P^{-11} \simeq \frac{E}{V}$ (SINCE THERE IS NO OTHER PARAMETER IN $D=11$)

$$(2) \Rightarrow \ell_P^9 \frac{R^2}{\ell_P^{11}} = \frac{R^2}{\ell_P^2} \ll 1$$

CONTRARY TO (1)

WE EXPECT GRAVITATIONAL COLLAPSE WHEN $T = O(\ell_P^{-1})$

FREE ENERGY - SIMPLIFIED MODEL

We SUM UP THE INDIVIDUAL FREE ENERGIES OF KK scalar fields

$$F(T) = T \sum_m \int \frac{d^{D-2}p}{(2\pi)^{D-1}} \log\left(1 - e^{-\frac{\omega_k}{T}}\right), \quad \omega_k^2 = \vec{k}^2 + \frac{m^2}{R_{11}^2}$$

Expanding the log and USING $e^{-2\sqrt{ab}} = \sqrt{\frac{b}{\pi}} \int_0^\infty \frac{dt}{t^{3/2}} e^{-at-\frac{b}{t}}$

$$\Rightarrow F(T) = -\sum_m \int_0^\infty \frac{dt}{t^{\frac{D+1}{2}}} \sum_{\omega=1}^\infty \exp\left[-\frac{\pi m^2 t}{R_{11}^2} - \frac{\pi \omega^2 R_0^2}{t}\right]$$

Including vacuum part $\omega = 0$ and setting $D = 11$:

Poisson Resummation in ω

$$\Rightarrow F(T) = -2\pi T \int_0^\infty \frac{dt}{t^{11/2}} \sum_{k,m} \exp\left[-\pi t \underbrace{\left(\frac{k^2}{R_0^2} + \frac{m^2}{R_{11}^2}\right)}_{\text{KK scalar mass}}\right]$$

Poisson Resummation in m

$$\Rightarrow F(T) = -R_{11} \int_0^\infty \frac{ds}{s} s^{11/2} \sum_{\omega,\omega'} \exp\left[-\pi s(\omega^2 R_0^2 + \omega'^2 R_{11}^2)\right]$$

UV DIVERGENT ! (0,0) TERM

$$\Rightarrow \boxed{F(T) = -R_{11} \sum_{(\omega,\omega')\neq(0,0)} \left(\omega^2 R_0^2 + \omega'^2 R_{11}^2\right)^{-\frac{11}{2}} = -\frac{2R_{11}\Gamma(11/2)}{(\pi R_{11} R_0)^{11/2}} \zeta(11) E_{\frac{11}{2}}(g_{eff}) \quad + \text{ DIV TERM}}$$

SYMMETRY :

$$F(g_{eff}, A) = g_{eff} F\left(\frac{1}{g_{eff}}, A\right)$$

$A = R_0 R_{11}$

$g_{eff} = \frac{R_{11}}{R_0}$

(FROM $Z(g_{eff}) = Z(1/g_{eff})$)

ONE LOOP FREE ENERGY IN D=11 SUPERGRAVITY

4- GRAVITON AMPLITUDE IN MAXIMAL SUPERGRAVITIES :

$$A_4^{\text{SUGRA}}\Big|_D = \sum_m e^{-\frac{\pi \tau m^2}{R_D^2}} A_4^{\text{SUGRA}}\Big|_{D-1}$$

[GREEN, Schwarz, BRINK ; GREEN, GUTPERLE, VANHOVE ; J.R. , TSEYTLIN]

FREE ENERGY : TAKE $\alpha' \to 0$ LIMIT OF STRING-THEORY EXPRESSION IN D-DIMENSIONS.

SIMILAR FORMULA APPLIES.

GENUS ONE F IN TYPE II SUPERSTRING [ATICK, WITTEN]

$$F_{\text{STRING}} = -\frac{1}{4} V (4\pi^2\alpha')^{-5} \int \frac{d^2\tau}{\tau_2^6} |\eta(\tau)|^{-24} \sum_{\omega',\omega} e^{-\frac{\pi r_0^2}{\tau_2}|\omega'+\omega\tau|^2}$$

$$\times \Big[(|\theta_2|^8 + |\theta_3|^8 + |\theta_4|^8)(0,\tau) + e^{i(\omega+\omega')} (\theta_2^4\bar\theta_4^4 + \theta_4^4\bar\theta_2^4)$$

$$- e^{i\pi\omega'} (\theta_2^4\bar\theta_3^4 + \theta_3^4\bar\theta_2^4)(0,\tau) - e^{i\pi\omega} (\theta_3^4\bar\theta_4^4 + \theta_4^4\bar\theta_3^4)(0,\tau)\Big]$$

$$r_0^2 = \frac{R_0^2}{\alpha'}$$

D= 10 SUGRA F_{SG} :

1) SEPARATE $\omega=0$ term, $F_{\text{STRING}} = F'_{\text{STRING}} + F_0$

2) TAKE $\alpha' \to 0$ LIMIT IN F_0 , USING LARGE τ_2 expansions

$$\eta(\tau) \cong q^{\frac{1}{12}}(1-q^2) , \quad \theta_2(0,\tau) \cong 2q^{1/4}(1+q^2) , \quad q \equiv e^{i\pi\tau}$$

$$\theta_3 \cong 1+2q , \quad \theta_4(0,\tau) \cong 1-2q$$

WE OBTAIN

$$F_0 = -256\, V (4\pi^2\alpha')^{-5} \int_0^\infty \frac{d\tau_2}{\tau_2^6} \sum_{\omega'} \left(1-(-1)^{\omega'}\right) e^{-\frac{\pi r_0^2}{\tau_2}\omega'^2}$$

By MAKING POISSON RESUMMATION

$$F_0 = -256\, \frac{V}{r_0} (4\pi^2\alpha')^{-5} \int_0^\infty \frac{d\tau_2}{\tau_2^{11/2}} \sum_k \left[e^{-\frac{\pi\tau_2}{r_0^2}k^2} - e^{-\frac{\pi\tau_2}{r_0^2}(k+\frac{1}{2})^2}\right]$$

NOW ADD KALUZA-KLEIN MODES OF D=11

$$\Rightarrow \quad F_{SG} = -256\frac{V}{r_0} (4\pi^2\alpha')^{-5} \int_0^\infty \frac{d\tau_2}{\tau_2^{11/2}} \sum_{k,m} e^{-\pi\tau_2\frac{m^2}{g_A^2}} \left[e^{-\frac{\pi\tau_2}{r_0^2}k^2} - e^{-\frac{\pi\tau_2}{r_0^2}(k+\frac{1}{2})^2}\right]$$

NOW MAKE POISSON RESUMMATION IN BOTH k, m

$$\Rightarrow \quad F_{SG} = -256\, V g_A (4\pi^2\alpha')^{-5} \int_0^\infty \frac{ds}{s}\, s^{11/2} \sum_{\omega', n} \left(1-(-1)^{\omega'}\right) e^{-\pi s(\omega'^2 r_0^2 + n^2 g_A^2)}$$

DIVERGENT TERM $\omega' = n = 0$ CANCELS OUT

THUS

$$\boxed{F_{SG} = -256\, V g_A (4\pi^2\alpha')^{-5} \frac{\Gamma(11/2)}{(\pi r_0 g_A)^{11/2}} \sum_{\omega', n} \left[1-(-1)^{\omega'}\right] \frac{g_{eff}^{11/2}}{\left(\omega'^2 + n^2 g_{eff}^2\right)^{11/2}}}$$

$$g_{eff} = \frac{R_{11}}{R_0} = \frac{g_A}{r_0} = 2\pi\sqrt{\alpha'}\, T g_A$$

HAGEDORN TEMPERATURE IN STRING THEORY

THERMAL ENSEMBLE: Euclidean space with $X^0 = X^0 + 2\pi R_0$

$$T = (2\pi R_0)^{-1}$$

$\Rightarrow$ WINDING STRING STATES, $X^0(\sigma,\tau) = x^0 + 2\alpha' p^0 \tau + 2R_0 w_0 \sigma + \underbrace{\tilde{X}^0(\sigma,\tau)}_{\text{SINGLE VALUED}}$

$$p_0 = \frac{m_0}{R_0}, \quad m_0, w_0 = 0, \pm 1, \pm 2, \ldots$$

HAMILTONIAN: $H = \alpha' p_i^2 + \frac{w_0^2 R_0^2}{\alpha'} + \frac{\alpha' m^2}{R_0^2} + 2(N_L + N_R - a_L - a_R) = 0$

BOSONIC STRING: $a_L = a_R = 1$, TYPE II NS-NS, $a_L = a_R = \frac{1}{2}$

T_{HAG}: TEMPERATURE AT WHICH H becomes NEGATIVE FOR SOME STATE (FREE ENERGY DIVERGES AT $T > T_{HAG}$)

THIS FIRST OCCURS FOR THE STATE $N_L = N_R = 0$, $m_0 = 0$, $w_0 = \pm 1$

$$\Rightarrow \frac{R_0^2}{\alpha'} - 2(a_L + a_R) = 0 \Rightarrow \boxed{T_H = \frac{1}{2\pi\sqrt{2\alpha'(a_L + a_R)}}}$$

GREEN-SCHWARZ FORMULATION:

FERMIONS MUST BE ANTIPERIODIC UNDER $X^0 \to X^0 + 2\pi R_0$.

FOR $w_0 = \pm 1$, MODE EXPANSIONS ARE AS IN NS-NS FORMULATION

$$N_L = \sum_{n=1}^{\infty} \left[\alpha^i_{-n} \alpha^i_n + \left(n - \tfrac{1}{2}\right) S^a_{-n} S^a_n \right], \quad N_R = \sum_{n=1}^{\infty} \left[\tilde{\alpha}^i_{-n} \tilde{\alpha}^i_n + \left(n - \tfrac{1}{2}\right) \tilde{S}^a_{-n} \tilde{S}^a_n \right]$$

$$i = 1, \ldots, 8, \quad a = 1, \ldots, 8$$

$$\Rightarrow a_L = a_R = \frac{1}{2} \Rightarrow \boxed{T_H = \frac{1}{2\pi\sqrt{2\alpha'}}}$$

M-THEORY AT FINITE TEMPERATURE

CONSIDER $D=11$ EUCLIDEAN SPACE WITH TOPOLOGY $\mathbb{R}^9 \times T^2$ and $(+,-)$ BOUNDARY CONDITIONS FOR FERMIONS.

$$X^0 = X^0 + 2\pi R_0 \,, \quad X^{11} = X^{11} + 2\pi R^{11} \,, \quad T = \frac{1}{2\pi R_0}$$

ANTIPERIODIC PERIODIC

HAVING THIS TOPOLOGY, MEMBRANES CAN WRAP ON T^2.

$$H \simeq (R_0 R_{11} T_2)^2 \omega_0^2 + \underbrace{\text{oscillations}}_{=0} + \text{VACUUM ENERGY}$$

$\mapsto$ $H < 0$ FOR SOME STATE BELOW R_0^{CRIT} (OR ABOVE T_{CR})

$$H = \frac{\ell_0^2}{R_0^2} + \frac{\ell_{11}^2}{R_{11}^2} + \frac{\omega_0^2 R_0^2}{\alpha'} + \frac{1}{\alpha'} H_0 + \frac{1}{\alpha'} \frac{1}{g_A^2} H_{INT}$$

$$\alpha' = (4\pi^2 R_{11} T_2)^{-1} \,, \quad g_A^2 = \frac{R_{11}^2}{\alpha'}$$

$$H_0 = \sum_{k,m} \left(\alpha^i_{(-k,-m)} \alpha^i_{(k,m)} + \tilde{\alpha}^i_{(-k,-m)} \tilde{\alpha}^i_{(k,m)} \right)$$

$$\omega_{km} = \sqrt{k^2 + \omega_0^2 m^2 \frac{R_0^2}{R_{11}^2}}$$

$$H \simeq H_{ZERO} + \sum_{km} \left(P_{-km} P_{km} + \omega_{km}^2 X_{-km} X_{km} \right) + \frac{1}{g_A^2} \sum_{k_i m_i} q_{k_i m_i} X_{km} X_{km} X_{km} X_{km}$$

$$X_{km} = \frac{i}{\omega_{km}} (\alpha_{km} + \tilde{\alpha}_{-k-m}) \qquad P_{km} = (\alpha_{km} - \tilde{\alpha}_{-k-m})$$

IN THE LIMIT $g_A \to \infty$, THE HAMILTONIAN DESCRIBES A COLLECTION OF HARMONIC OSCILLATORS.

$$g_A \to \infty \quad , \quad \frac{R_{11}}{R_0} = \text{FIXED} \quad , \quad \alpha' = \text{FIXED}$$

$(T_2 \to 0)$

VACUUM ENERGY: $H = :H: + 2\mathcal{E}$

- BOSONIC THEORY:

$$\mathcal{E}_B = \frac{1}{2}(D-3)\sum_{k,m} \omega_{km}$$

- SUPERSYMMETRIC THEORY: $\mathcal{E} = \mathcal{E}_B + \mathcal{E}_F = 0$

- SUPERSYMMETRIC THEORY WITH ANTIPERIODIC FERMIONS ON X^0 $(\omega_0 = 1)$

$$H = \sum_{k,m}\left(\alpha^i_{(-k,-m)}\alpha^i_{(k,m)} + \tilde{\alpha}^i_{(-k,-m)}\tilde{\alpha}^i_{(k,m)} + \omega_{k+\frac{1}{2},m}\left(S^a_{(-k,-m)}S^a_{(k,m)} + \tilde{S}^a_{(-k,-m)}\tilde{S}^a_{(k,m)}\right)\right)$$

$$\Longrightarrow \quad \mathcal{E} = \mathcal{E}_B + \mathcal{E}_F$$

$$\boxed{\mathcal{E}_B = \frac{1}{2}\,8\sum_{k,m}\omega_{km} \quad , \quad \mathcal{E}_F = -\frac{1}{2}\,8\sum_{k,m}\omega_{k+\frac{1}{2},m}}$$

THE BOSONIC VACUUM ENERGY IS RELATED TO NON-HOLOMORPHIC EISENSTEIN SERIES

$$\xi(2r)\, E_r(\Omega) = \frac{1}{2} \sum_{(k,m)\neq(0,0)} \frac{\Omega_2^r}{|k+m\Omega|^{2r}} \quad , \quad \Omega = \Omega_1 + i\Omega_2$$

FUNCTIONAL RELATION :

$$E_r(\Omega) = c\, E_{1-r}(\Omega) \quad , \quad c = \frac{\xi(2-2r)\,\Gamma(1-r)}{\sqrt{\pi}\,\Gamma(\frac{1}{2}-r)\,\xi(1-2r)} \quad , r < \tfrac{1}{2}$$

(ξ - function REG + POISSON RESUMMATION)

IN THE PRESENT CASE $\quad \Omega = \dfrac{i}{g_{eff}} \quad , \quad r = -\dfrac{1}{2}$

WHERE

$$\boxed{g_{eff} = \frac{R_{11}}{R_0} = 2\pi T g_A \sqrt{\alpha'}}$$

THUS

$$\boxed{\mathcal{E}_B = -\frac{8}{4\pi^2\sqrt{g_{eff}}}\, \xi(3)\, E_{3/2}(g_{eff})}$$

FERMION CONTRIBUTION : WRITE

$$\sum_{k,m} \frac{1}{(\omega_{k+\frac{1}{2},m})^{2\nu}} = \frac{\pi^\nu}{\Gamma(\nu)} \sum_{k,m} \int_0^\infty \frac{d\tau}{\tau}\, \tau^\nu\, e^{-\pi\tau(\omega^2_{k+\frac{1}{2},m})}$$

BY POISSON RESUMMATION

$$\boxed{\mathcal{E}_F = 8\,\frac{g_{eff}}{8\pi^2} \sum_{(w,w')\neq(0,0)} (-1)^{w} \left(w^2 + w'^2 g_{eff}^2\right)^{-3/2}}$$

DEFINE EISENSTEIN-TYPE SERIES AS

$$\boxed{\zeta(2r)\, F_r(\Omega) = \frac{1}{2}\sum_{(k,m)\neq(0,0)} (-1)^k \frac{\Omega_2^{\,r}}{|k+m\Omega|^{2r}}, \quad r > \frac{1}{2}}$$

IT IS <u>NOT</u> $SL(2,\mathbb{Z})$ INVARIANT.

WE FIND
$$\mathcal{E}_F = \frac{2}{\pi^2\sqrt{g_{eff}}}\,\zeta(3)\, F_{3/2}(g_{eff})$$

THUS
$$\mathcal{E} = \frac{2\,\zeta(3)}{\pi^2\sqrt{g_{eff}}}\left[-E_{\frac{3}{2}}(g_{eff}) + F_{\frac{3}{2}}(g_{eff})\right]$$

$g_{eff} \ll 1$

$$\mathcal{E}_B = 8\left(-\frac{1}{4\pi^2}\frac{\zeta(3)}{g^2_{eff}} - \frac{1}{12}\right), \quad \mathcal{E}_F = 8\left(\frac{1}{4\pi^2}\frac{\zeta(3)}{g^2_{eff}} - \frac{1}{24}\right)$$
$$+ O(e^{-2\pi/g_{eff}})$$

$g_{eff} \gg 1$

$$\mathcal{E}_B = 8\left(-\frac{1}{4\pi^2}\zeta(3)\, g_{eff} - \frac{1}{12\, g_{eff}}\right), \quad \mathcal{E}_F = 8\,\frac{(-3)}{4\cdot 4\pi^2}\zeta(3)\, g_{eff}$$
$$+ O(e^{-2\pi g_{eff}})$$

$$\longrightarrow \begin{cases} \mathcal{E} = -1 + O(e^{-2\pi/g_{eff}}), & g_{eff} \ll 1 \\ \mathcal{E} = -\frac{7}{16\pi^2}\zeta(3)\, g_{eff} - \frac{1}{12\, g_{eff}} + O(e^{-2\pi g_{eff}}), & g_{eff} \gg 1 \end{cases}$$

Critical Temperature

As T is increased from zero, there is some $T = T_{CR}$ at which a winding state becomes tachyonic, giving

$H < 0$ for $T > T_{CR}$. Consider $l_0 = l_{11} = 0$

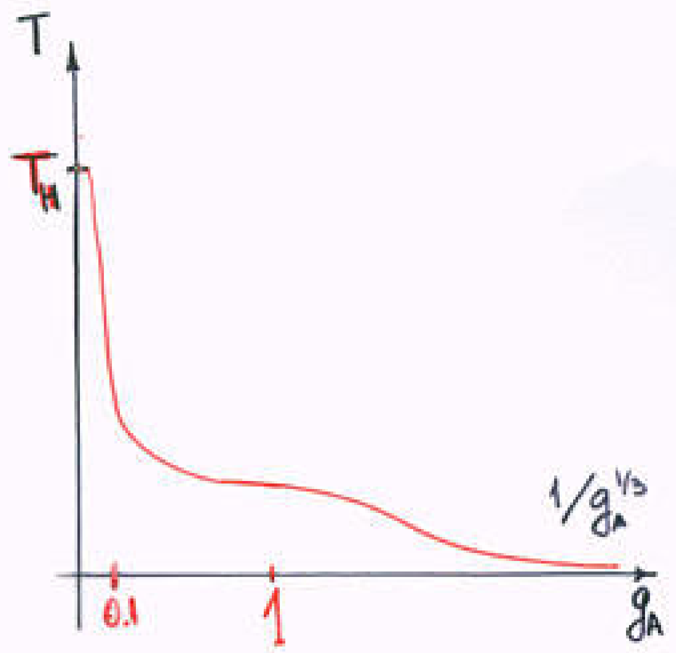
$$H = 0 = \frac{R_0^2}{\alpha'} + 2\mathcal{E} \qquad , \quad R_0 = \frac{1}{2\pi T_{CR}}$$

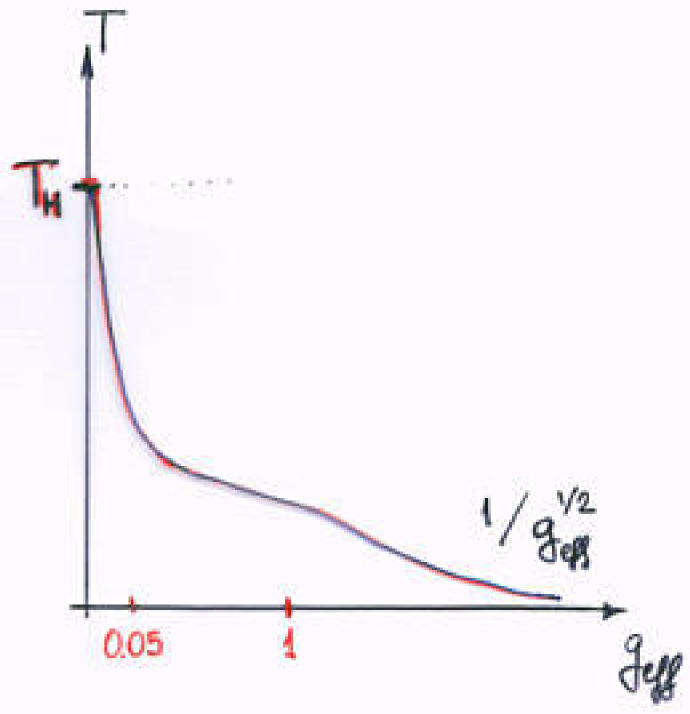
$$\Rightarrow \quad R_0^2 = 2\alpha' \frac{2\zeta(3)}{\pi^2 \sqrt{g_{eff}}} \left(E_{3/2}(g_{eff}) - F_{3/2}(g_{eff}) \right)$$

i.e.

$$\boxed{T_{CR}^2 = \frac{1}{8\alpha'} \frac{\sqrt{g_{eff}}}{2\zeta(3)\left(E_{3/2}(g_{eff}) - F_{3/2}(g_{eff}) \right)}} \quad , \; g_{eff} = 2\pi\sqrt{\alpha'}\, T_{cr}\, g_A$$

Numerical Plots





SUMMARY AND CONCLUSIONS

- EXACT, FINITE FORMULA FOR 1-LOOP FREE ENERGY IN $D=11$ SUPERGRAVITY.
- CASIMIR ENERGY FOR MEMBRANES AT FINITE TEMPERATURE FOR $g_A >> 1$ (QUADRATIC APPROXIMATION)
- TEMPERATURE AT WHICH TACHYON INSTABILITIES APPEAR:

$$T_{cr} = T_{cr}(g_A)$$

- $T_{cr} \cong 0.3\, \ell_P^{-1}$ COINCIDES WITH THE TEMPERATURE A THERMAL ENSEMBLE WITH $V = O(\ell_P)$ COLLAPSES INTO A BLACK HOLE.

- $\mathcal{E}(g_A)$ HAS THE RIGHT VALUE AT $g_A << 1$.
  EXACT INCLUDING NON-LINEARITIES ?

- T_{HAG} HAS A SUDDEN CHANGE NEAR $g_A \cong 0.1$
  $\Rightarrow$ THE RESUMMATION OF NON-PERTURBATIVE CORRECTIONS IS IMPORTANT ALREADY FOR $g_A \cong 0.1$.

- TYPE IIB: T_{HAG} AT $g_B >> 1$ CAN BE FOUND BY S-DUALITY.

$$\left\{ \begin{array}{l} FS \rightarrow D\text{-STRINGS} \\ g_B \rightarrow \frac{1}{g_B} \; , \; \alpha' \rightarrow \alpha'_D = g_B \alpha' \end{array} \right. \Longrightarrow T_H = \frac{1}{2\pi \sqrt{2\alpha'_D (a_L + a_R)}}$$

SAME PHYSICS.

WHAT ABOUT FINITE g_B? CALCULATION INVOLVES MEMBRANES ON T^3.