On Field Theory of Open Strings, Tachyon Condensation and Closed Strings

based on
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1. Introduction

2. Background Independent Open String Field Theory

3. Exact tachyon potential

4. New vacuum and closed strings

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An open bosonic string in 26 dim., $D25$, (one can also discuss any other Dp) contains tachyon T ($m^2 = -\frac{1}{2}$ in some units), massless gauge field A, and an infinite tower of massive fields.

Tachyon - the theory is unstable.

Conjecture 1 (A. Sen, '99): the tachyon potential has the form:

$$V(T) = Mf(T)$$

M - mass of D-brane and $f(T)$ - universal function.

$f(T)$ has the stationary point, $T_c; f(T_c) = -1$.

Conjecture 2 (A. Sen, '99): there are soliton configurations of tachyon field on unstable Dp-branes - lower-dimensional branes.

Related work

Cubic open string field theory or its extensions:
Kostelecky, Samuel, '90; Sen, Zwiebach, '99;
In '00 - Taylor; Moeller, Taylor; Harvey, Kraus; de Mello Koch, Jevicki, Mihailescu, Tatar; Moeller, Sen, Zwiebach; de Mello Koch, Rodrigues; Moeller; Sen, Zwiebach; Berkovits; Berkovits, Sen, Zwiebach; De Smet, Raeymakers; Iqbal, Naqvi; Sen; Kostelecky, Potting; Hata, Shinohara; Zwiebach; Minahan, Zwiebach; ...

p-adic - Ghoshal, Sen, '00

OS sigma model in the '80's: Fradkin, Tseytlin, '85; Abouelsaood, Callan, Nappi, Yost, 87, ...

Closed from Open (SFT) - Strominger, '87; S. Sh., '97 (unpublished); Kraus, Harvey, Larsen, Martinec, '00; Yi, '99; Bergman, Hori, Yi '00; Sen '00, Mandal, Wadia, '00; Kleban, Lawrence, Shenker, '00; ....

RG - Harvey, Kutasov, Martinec, '00, ...

Background independent open string theory:
Kutasov, Marino, Moore, '00; Ghoshal, Sen, '00; Cornalba, '00; Okuyama, '00; Tseytlin, '00 .....

Deform the world-sheet action:

$$I_{ws} = I_0 + \int_{C \to \partial D} \mathcal{V}(X, b, c)$$

$$< ... >= \int [dX][db][dc] e^{-I_{ws}} ...$$

Consider only the case when ghosts decouple:

$$\mathcal{O} = cV(X)$$

The boundary term in the action modifies the boundary condition on the map $X^\mu(z, \bar{z})$ from the Neumann boundary condition (this follows from I_0):

$$\partial_r X^\mu(\sigma) = 0$$

to "arbitrary" non-linear condition:

$$\partial_r X^\mu(\sigma) = \frac{\partial}{\partial X^\mu(\sigma)} \int_{\partial D} V(X)$$

I_{ws} defines the family of boundary 2d quantum field theories on the disk.

The action $S(\mathcal{O})$ is defined on this space and is formally independent of the choice of a particular open string background (Witten, '92):

$$dS = < d\int_{\partial D}\mathcal{O} \quad \{Q, \quad \int_{\partial D}\mathcal{O}\} >$$

Since $d\int\mathcal{O}$ is arbitrary - all solutions of the equation $dS = 0$ correspond to the boundary deformations with $\{Q, \int\mathcal{O}\} = 0$; $\Rightarrow$ 2d theory is conformal (scale invariant, $\beta = 0$) $\Rightarrow$ valid string background.

What is the space of deformations given by $V(X(\sigma))$?

Assumption (restriction?) - V can be expanded into "Taylor series" in the derivatives of $X(\sigma)$:

$$V(X) = T(X(\sigma)) + A_\mu(X(\sigma))\partial X^\mu(\sigma) +$$

$$C_{\mu\nu}(X(\sigma))\partial X^\mu(\sigma)\partial X^\nu(\sigma) + D_\mu(X(\sigma))\partial^2 X^\mu(\sigma) + \ldots$$

The action now becomes the functional of coefficients:

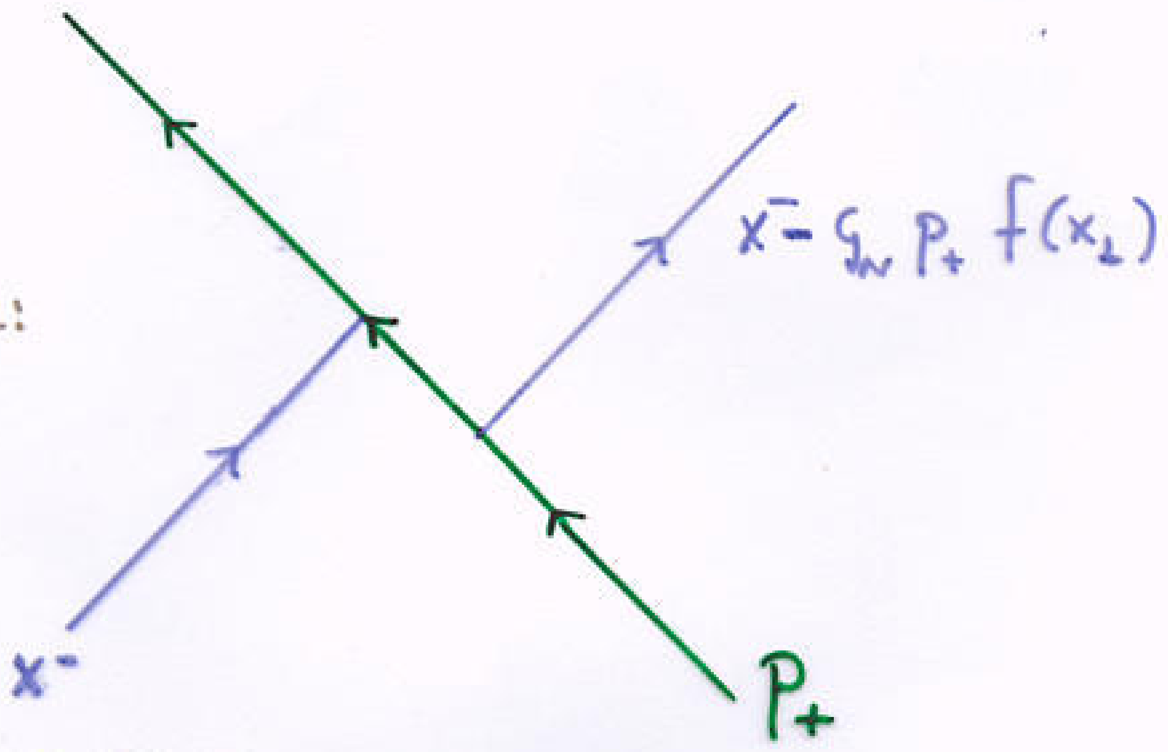
$$S = S(T(X(\sigma)), A_\mu(X(\sigma)), \ldots)$$

The goal: write S as an integral over the space-time (constant mode of $X(\sigma)$: $X(\sigma) = X + \phi(\sigma)$, $\int \phi(\sigma) = 0$) of some "local" functional of fields $T(X), A(X), \ldots$ and their derivatives.

GRAVITATIONAL SCATTERING

$s \to \infty$, t fixed

CLASSICAL:



QUANTUM:

$$\varphi_L(x^+,0)\,\varphi_R(x^-,x_\perp) = e^{iG_N f(x_\perp)\partial_+\partial_-}\,\varphi_R(x^-,x_\perp)\,\varphi_L(x^+,0)$$

$\Rightarrow$ SPACE-TIME NON-COMMUTATIVITY

1. It seems that the action is only formally background independent since in order to compute it from world-sheet one needs to use some expansion in world-sheet field theory.

2. The action seems to depend on the choice of coordinates in the space of boundary interactions (choice of contact terms).

Ignore contact terms $\Rightarrow$ the action is given by the same formula but only with the linear part in β (S. Sh. I, 1993).

Witten, II, 1992 $\Rightarrow$ zeros of vector field define equations of motion. Problem - linear EOM.

Exact Tachyon Potential

Turn on only tachyon - $V(X(\sigma)) = T(X(\sigma))$.

Find the action $S(T)$ exact in T and second order in ∂T.

We know (derivative expansion of β and Z):

$$\beta^T(X) = [2\Delta T + T] + a_0(T) +$$

$$a_1(T)(\partial T) + a_2(T)(\partial^2 T) + a_3(T)(\partial T)^2 + ...$$

$$Z(T) = \int dX e^{-T}(1 + b(T)(\partial T)^2 + ...)$$

In this specific case the basic relation becomes:

$$S(T) = -\int dX \beta^T(X) \frac{\partial}{\partial T(X)} Z(T) + Z(T)$$

The condition - $\beta = 0$ is the equation of motion for $S(T)$ in lowest order in T (around $T = 0 \to 2\Delta T + T + ... = 0$) fixes the two derivative action (and relevant unknows $a_i(T), b(T)$) (Gerasimov, S. Sh. '2000):

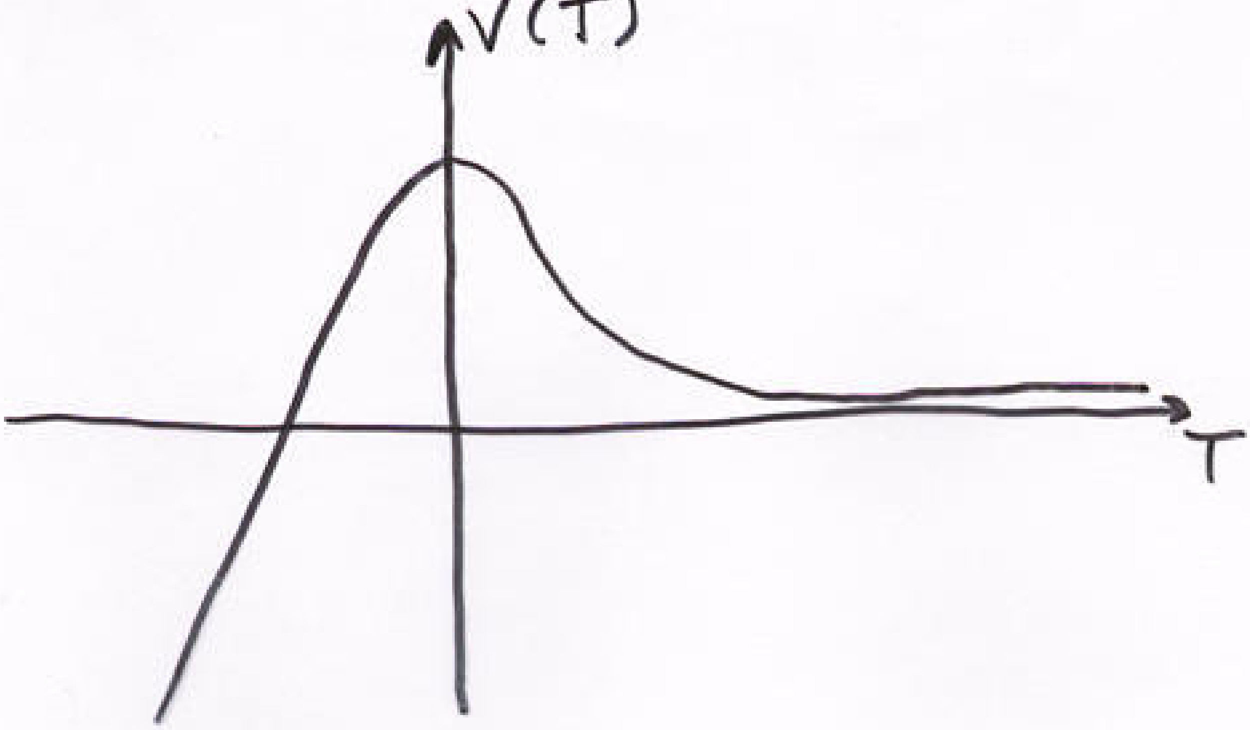
$$S(T) = \int dX[e^{-T}(\partial T)^2 + e^{-T}(1+T)]$$

$$\beta^T = 2\Delta T + T - (\partial T)^2$$

$$Z(T) = \int e^{-T}$$

$$\partial_T S = e^{-T}\beta^T = e^{-T}(2\Delta T + T - (\partial T)^2) = 0$$

$$S(T)_{on-shell} = Z(T) = \int e^{-T(X)}$$



$$V(T) = e^{-T}(1+T)$$

World-sheet action written in terms of only boundary data:

$$H(\theta,\theta') = \frac{1}{2}\sum_{k\in\mathbb{Z}} e^{ik(\theta-\theta')}|k|$$

$$X(\theta) = X + \phi(\theta); \int \phi(\theta) = 0; [dX(\theta)] = dX\,[d\phi]$$

$$I_{ws} = \int\int d\theta d\theta' X^{\mu}(\theta) H(\theta,\theta') X_{\mu}(\theta') + \int d\theta T(X(\theta)) =$$

$$T(X) + \int \phi^{\mu}(\theta)[H(\theta-\theta')\delta_{\mu\nu} + \delta(\theta-\theta')\partial_{\mu}\partial_{\nu}T(X)]\phi^{\nu}(\theta') +$$

$$+O(\partial^3 T(X))$$

In the two derivative approximation:

$$Z(T) = \int dX e^{-T(X)} det'[H + \partial^2 T]^{-\frac{1}{2}}$$

This can be computed exactly. For $\partial_\mu \partial_\nu T = \delta^{\mu\nu} \partial_\mu^2 T$ (Witten '92):

$$Z = \int dX e^{-T(X)} \prod_\mu \sqrt{\partial_\mu^2 T(X)} e^{\gamma \partial_\mu^2 T(X)} \Gamma(\partial_\mu^2 T(X))$$

In the two-derivative approximation this gives:

$$Z(T) = \int dX e^{-T(X)} (1 + b(T)(\partial T)^2)$$

$$b(T) = 0$$

$$\beta^T = 2\Delta T + T$$

Thus the action is:

$$S(T) = \int dX[e^{-T}2(\partial T)^2 + e^{-T}(1+T)]$$

with equations of motion:

$$e^{-T}(T + 4\Delta T - 2(\partial T)^2) = 0$$

The metric G in $\partial_i S = G_{ij}\beta^j = 0$:

$$G(\delta_1 T, \delta_2 T) = \int dX e^{-T}(\delta_1 T \delta_2 T - 2(\partial_\mu \delta_1 T)(\partial_\mu \delta_2 T))$$

$$e^{-T}(1 + 2\Delta - 2\partial_\mu T \partial_\mu + ...)(2\Delta T + T) = 0$$

The linear form of β for arbitrary $T(X)$ looks strange since we miss a possible higher order in T (but second order in ∂T) terms which shall come from a 3-point function.

This metric is rather complicated and is not an obvious expansion of some invertable metric in the space of fields $\Rightarrow$ we need to choose new coordinates (change the regularization). Note: linear part of β is invariant under scaling of the tachyon ($\beta^i \partial_i$ is a vector field), quadratic in ∂T part of the action is not:

$$T \to T - \partial^2 T + (\partial T)^2$$

In new coordinates:

$$S(T) = \int dX [e^{-T} (\partial T)^2 + e^{-T}(1+T)]$$

$$\partial_T S = e^{-T} \beta^T = e^{-T}(2\Delta T + T - (\partial T)^2) = 0$$

$$S(T)_{on-shell} = Z(T) = \int e^{-T(X)}$$

This potential has unstable extremum at $T = 0$ (tachyon) and stable at $T = \infty$. The difference between the values of this potential is 1 as predicted by A. Sen (Conjecture 1).

In a new variable with the canonical kinetic term (p-adic string for $p \to 1$): $\Phi = e^{-\frac{T}{2}}$

$$S(\phi) = \int [4(\partial\Phi)^2 - \Phi^2 log\frac{\Phi^2}{e}]$$

In the unstable vacuum $T = 0, \Phi = 1$; $m^2 = -\frac{1}{2}$; in new vacuum $T = \infty, \Phi = 0$ mass is ∞.

DN boundary conditions (Dp-brane, $p \leq 25$):

$$\partial_r X^a(\sigma) = 0, \quad a = 1, ..., p$$

$$X^i(\sigma) = 0, \quad i = p + 1, ..., 26$$

are obviously conformal $\Rightarrow$ critical points (Conjecture 2). Note: $S(t_c) = Z(t_c)$; e. g.:

$$T(X) = a + u_\mu (X^\mu)^2 \Rightarrow \partial_r X^\mu = u_\mu X^\mu$$

$$u_i \to \infty, \quad u_a \to 0$$

Add gauge field A. Construct the action using the same basic relations (Gerasimov, S. Sh., 2000).

Introduce the closed string fields G and B (for covariance):

$$S(G,B,A,T) = S_{closed}(G,B) + \int d^{26}X\sqrt{G}$$

$$(e^{-T}(1+T)+e^{-T}||dT||^2+\frac{1}{4}e^{-T}||B-dA||^2+\cdots)$$

One can choose a different regularization and obtain the action which is an expansion of BI action

$$\int V(T)\sqrt{det(G-B+dA)}$$

up to second order in dA, but it would lead to a complicated metric.

In $\Phi = e^{-\frac{T}{2}}$ coordinates:

$$S(G,B,A,\Phi) = S_{closed}(G,B) +$$

$$\int d^{26}X\sqrt{G}(\Phi^2(1-2\log\Phi) + 4||d\Phi||^2 +$$

$$+\frac{1}{4}\Phi^2||B-dA||^2 + \cdots)$$

Abelian Higgs model for complex scalar and gauge field in angular coordinates: $\Phi = He^{i\phi}, \mathcal{A}$:

$$S(H,\phi,\mathcal{A}) = \int dX(\frac{1}{g^2}F(\mathcal{A})^2 + |dH|^2 +$$

$$+H^2|\mathcal{A}-d\phi|^2 + \lambda(H^2-H_0^2)^2)$$

$$B \to \mathcal{A}, \qquad A \to \phi, \qquad \Phi = e^{-\frac{1}{2}T} \to H$$

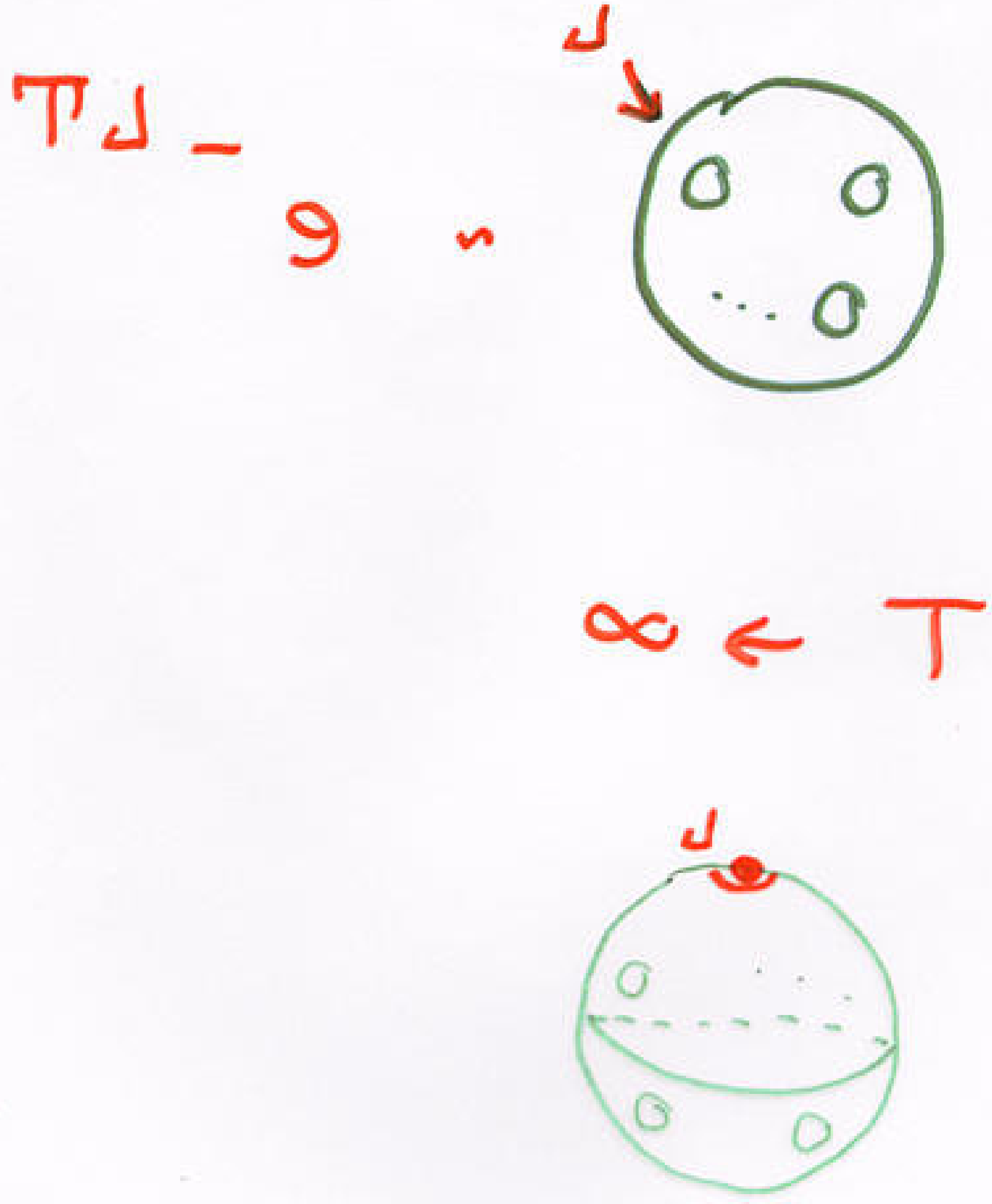
Exactly like in a symmetric point for an abelian Higgs model where phase ϕ is not a good coordinate - in string theory the gauge field A becomes ill-defined; the same is true for all open string modes (A. Sen; Gerasimov, S. Sh.).

Radial variable - T_0. Angular - all other.

What is analog of cartezian coordinates? Wilson loops? String field?

$$\Psi(X_*(\sigma)) = \int_{X_*} e^{-\int_0^\pi [T(X(\sigma)) + A_\mu(X(\sigma))dX^\mu + \ldots]}$$

"Open String degrees of freedom', coefficients in expansion of the boundary operator in derivatives of $X(\sigma)$, are bad coordinates in new, "closed string", vacuum. New coordinates must be used - "closed string degrees of freedom". Conjecture 3.


∞ ← T

New coordinates - $U(1)$ gauge invariant, functionals of closed strings (like $\phi \to \exp(i\phi)$ is invariant with respect to the shift $2\pi\mathbb{Z}$).

In the new vacuum we have only closed strings In this sense open string field coordinates are intermediate in this chain of relations.

What about cubic OS field theory coordinates?

$$S = (\Psi, Q\Psi) + \Psi \star \Psi \star \Psi = -\beta^i \partial_i Z + Z$$

with $\star$ defined through conformal map to "1/3 of pizza", thus $\Psi^3 = Z$?

If we keep the restricted class of boundary fields $T(X(\sigma)) + A_\mu(X(\sigma))dX^\mu + ...$ ⇒ gauge parameters for closed string gauge symmetry which has been restored in the new vacuum. *A new branch opens - closed strings.*

What is the space of boundary conditions? How does it contain CS degrees of freedom?

World-sheet picture: OS theory in the background of the constant OS tachyon mode $T = T_0$. The 2d sigma - not conformal. Boundary term $\Rightarrow$ factor for each boundary:

$$Z \sim e^{-RT_0}$$

Let us take some boundary to be of unit length, then we have the overall damping factor of the string amplitudes with the holes. Thus at the new vacuum $T_0 = \infty$ and there should not be any boundary at all.

"Hole cutting" operator - at the new string vacuum the coefficient in front of this operator in the 2D action is zero. Thus there are no holes and no open strings in this vacuum.