Brane Dynamics in CFT-Backgrounds

Strings 2001

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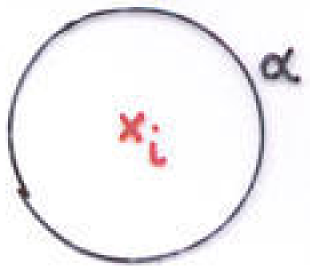
I. Introduction

Study of branes in small/strongly curved backgrounds requires full pert. string theory (→ boundary CFT)

(A) Construction of branes, i.e. of couplings brane – closed string



$\sim \langle \phi_i \rangle_\alpha$ [Cardy]....

(B) Dynamics of branes from RG-fixed boundary interactions

↔ sol. of non-commutative LEEA.

↑ in appropriate regime

Plan: I Introduction

II. Branes on $SU(2)_k^*$ curved, $H \neq 0$ symmetric

1 Decoupling limit ($k \to \infty$)

nc LEEA: YM + CS on fuzzy S^2

2 Constant condensates (fin k)

conserved charges in $K_H^*(S^3)$

III Other backgrounds

1. Group manifolds ($\neq SU(2)$)

2. Coset models ($N=2$ SMM)

* see also [Felder et al.] [Stanciu] [Bachas et al.] [Pawelczyk] .....

$\alpha = \frac{k}{2}$

$\vdots$

$\alpha = \frac{1}{2}$

$\alpha = 0$

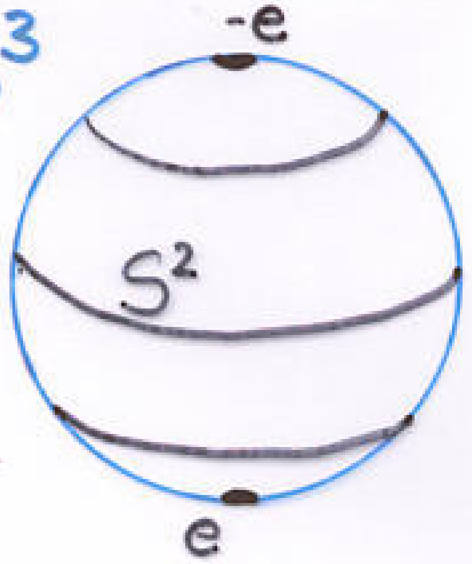
<u>Solution</u>: [Cardy; Runkel] There exist $k+1$ boundary ths [part. fct. OS-OPE] labeled by $\alpha = 0, \frac{1}{2}, 1, \dots, \frac{k}{2}$.

II Branes on $SU(2)_k \simeq S^3$

Problem: Constr. $SU(2)_k$-WZW on uhp w. boundary cond.

$$-k g^{-1}\partial g =: J(z) \underset{z=\bar{z}}{=} \bar{J}(\bar{z}) := k \bar{\partial} g g^{-1}$$

S^3



Branes carry B-field $dB = H$

WZW 3-form

[hep-th/9812193]

n-pt fcts of tach VO $\longrightarrow$ integral on fuzzy S^2

$J_i \leq \min(2\alpha, k-2\alpha) \longrightarrow J_i \leq 2\alpha$

$$\langle V^{J_1}_{m_1}(x_1) \cdots V^{J_\nu}_{m_\nu}(x_\nu) \rangle_\alpha \xrightarrow{k\to\infty} \mathrm{tr}_{2\alpha+1}\left(Y^{J_1}_{m_1} \cdots Y^{J_\nu}_{m_\nu} \right)$$

$$Y^{J_1}_{m_1} \in \mathrm{Mat}_{2\alpha+1} {}^{*}$$

$$^{*}\ \mathrm{Mat}_{2\alpha+1} \simeq \underset{\substack{\Downarrow \\ Y^0}}{[0]} + \underset{\substack{\Downarrow \\ Y^1_m}}{[1]} + \dots + \underset{\substack{\Downarrow \\ Y^{2\alpha}_m}}{[2\alpha]}$$

II.1 Decoupling Limit ($k \to \infty$)

Correlators for $k \to \infty$ ($S^3 \rightrightarrows \mathbb{R}^3$) determined by fuzzy S^2 (= $Mat_{2\alpha+1}$)

Effective FT: [0003187] $A_{\underset{\overset{\shortparallel}{1,2,3}}{a}} \in Mat_{\underset{\uparrow \text{CP dof}}{M}}(Mat_{2\alpha+1})$

$$S_{M[\alpha]}(A) \sim \frac{1}{4} \mathrm{Tr}\, F_{ab} F^{ab} - \frac{1}{2} \mathrm{Tr}\, \epsilon^{abc} CS_{abc}$$

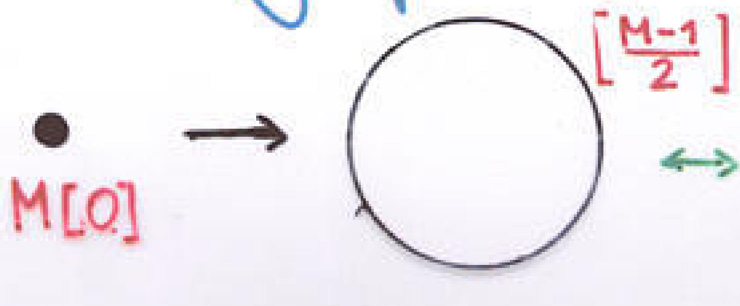
w. $F_{ab} = L_a A_b - L_b A_a + i[A_a, A_b] + \epsilon_{ab}{}^{d} A_d$

$$CS_{abc} = L_a A_b A_c + \frac{2i}{3} A_a A_b A_c + \frac{\epsilon_{bc}{}^{d}}{2} A_a A_d$$

inf. rotation: $L_a A = i[Y^1_a, A]$

EOM solved by reps of su(2)

i.p.



$\leftrightarrow$ [Myers]
(RR-flds)

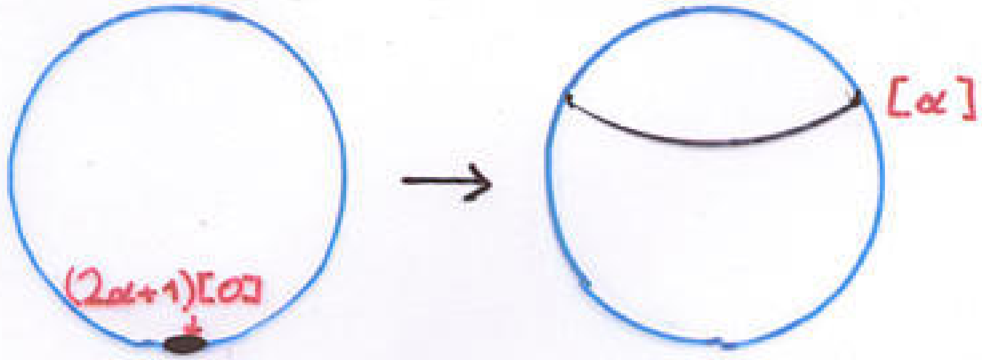
II.2 Constant condensates (k fin)

Const. flds $A_a = \Lambda_a$, Λ_a SU(2) rep.
induce flow into known RG-FP

$$\mathcal{S}^{2D}_{pert} \sim \int dx\, \Lambda_a J^a(x)$$

Kondo mod. impurity spin spin current

[Wilson]
⋮
⇒
[Affleck
Ludwig]



Charge conservation gives [Alekseev, VS]

$$Q[\alpha] = 2\alpha + 1 \in \mathbb{Z}_n =: \mathcal{C}_n(SU(2))$$

$$\overset{\text{twisted K-group}}{} \quad = \quad K^*_{n\mathcal{S}_3}(SU(2)) \quad //*$$

$$\underbrace{n\mathcal{S}_3}_{= H \in H^3(S^3, \mathbb{Z})}$$

* predicted by [Bouwknegt, Mathai]

III Other backgrounds

III.1. Group manifolds ($\neq SU(2)$)

(a) Branes wrapping conj. classes are bound states of D0 branes.

[Fuchs, Schweigert]

(b) Branes on "twined" conj. classes:

$[\ \{hg\omega(h^{-1}), h \in G\} \subset G \ , \ \omega \in Out(G)\ ]$

Constant condensates can be analysed

For charge group analysis implies

$$\mathcal{C}_n(SU(N)) = \overset{(a)}{\mathbb{Z}_n} + \overset{(b)}{\mathbb{Z}_{n'}} + \ldots \overset{?}{=} K^*_n(SU(N))$$

$n = (k+N)/\gcd(k+N, \mathrm{lcm}(1,\ldots,N))$

$n' | n$ [Fredenhagen, V.S]

III.2 Branes in coset models

e.g. $\dfrac{SU(N+1)_k}{SU(N)\times U(1)}$ $N=2$ SCFT (KS-quotient)

For $N=1$: SUSY MM with branes

$$(L , M , S)$$
$$L = 0,1,\dots,k \quad M = -k-1,\dots,k+2 \quad S = -1,0,1,2$$

$EA_{(k\to\infty)}$ obtained from YM + CS on fuzzy S^2 by reduction with:

$$L_3 A_a = \frac{i}{k}\,\epsilon_{3a}{}^{b} A_b \;\; ; \;\; A_3 = 0$$

⇒ any (L,M,S) bound state of branes $(0,M',S')$ [Fredenhagen, V.S]

... further backgrounds analysed ...