Comments on Born-Infeld Theory or (more precisely) World-Volume Theories of D-Branes

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CONTENTS

Selected References

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In a closed-string boudary state formalism, the Dirac–Born–Infeld term originates from the NS-NS sector and the Chern–Simons term from the R-R sector.

Let us begin by considering a single D-brane ($N = 1$). In this case, ignoring fermi fields and taking a flat 10d background, the action for a D9-brane is

$$S_{DBI} = T_9 \int d^{10}\sigma \sqrt{-\det(g_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta})}.$$

Here T_9 is the D9-brane tension and $g_{\alpha\beta}$ is the pullback of the (flat) spacetime metric $\eta_{\mu\nu}$:

$$g_{\alpha\beta} = \eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$$

σ^α ($\alpha = 0, 1, \ldots, p$) are the world-volume coordinates

$X^\mu(\sigma)$ ($\mu = 0, 1, \ldots, 9$) are the embedding functions

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This formula was derived first for the bosonic string (Fradkin and Tseytlin, 1985) by computing the path integral corresponding to the disk partition function. It was subsequently generalized to the superstring.

The actions for Dp-branes with $p < 9$ can be deduced from the D9-brane case using T duality. The result, which agrees with dimensional reduction, is

$$S_{DBI} = T_p \int d^{p+1}\sigma \sqrt{-\det(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i + 2\pi\alpha' F_{\alpha\beta})}.$$

The index $i = p + 1, \ldots, 9$ labels the $9 - p$ directions transverse to the Dp-brane. The world-volume scalars X^i can be regarded as Goldstone bosons associated to broken translational symmetries. In particular, as observed by Bachas, for a D0-brane this gives

$$T_0 \int d\sigma^0 \sqrt{1 - \partial_0 X^i \partial_0 X^i},$$

which is the standard action for a relativistic particle of mass T_0.

The formula for S_{CS} (due to Douglas) in the presence of RR background fields is

$$\int \left(C e^{2\pi\alpha' F} \right)_{p+1}$$

where $C = \Sigma C^{(n)}$ is a formal sum of RR n-form fields. (n is odd for IIA and even for IIB.)

Is there an analogous low energy effective action for closed strings in terms of the gravity supermultiplet? No such formula is known. If one could be found, it could be useful for exploring whether curvatures are bounded and whether some spacetime singularities are thereby evaded.

1.1 Supersymmetrization

In the cases that are BPS, the inclusion of world volume fermions results in a supersymmetric D-brane action. The formulas were worked out by Aganagic, Popescu, and JHS in 1996 (and independently by Cederwall et al. and by Bergshoeff and Townsend). The idea is to embed the D-brane in superspace $(X^\mu,\, \theta_1^a,\, \theta_2^a)$, where $(\theta_1,\, \theta_2)$ are MW spinors.

The global $\mathcal{N} = 2$, $D = 10$ susy is realized on superspace in the usual way ($\delta\theta = \epsilon$, $\delta X^\mu = \bar{\epsilon}\,\Gamma^\mu\theta$). The D-brane action is constructed out of the susy invariants

$$\Pi^\mu_\alpha = \partial_\alpha X^\mu - \bar{\theta}\Gamma^\mu\partial_\alpha\theta$$

and $\partial_\alpha\theta$. In order that the θ's give the desired number of fermions, half must be compensated by a local fermionic symmetry: called *kappa symmetry.*

The requirements of global susy and local kappa symmetry determine the action. One finds

$$S_{DBI} = T_p \int d^{p+1}\sigma\sqrt{-\det(G_{\alpha\beta} + 2\pi\alpha'\mathcal{F}_{\alpha\beta})}$$

$$S_{CS} = \pm T_p \int \Omega_{p+1}$$

where

$$G_{\alpha\beta} = \eta_{\mu\nu}\Pi^\mu_\alpha\Pi^\nu_\beta$$

$$\mathcal{F}_{\alpha\beta} = F_{\alpha\beta} - B_{\alpha\beta} - b_{\alpha\beta}.$$

Here B is the pullback of the NS-NS 2-form background field and b is a two-form involving the fermi fields, which in the IIA case is

$$b = -\bar{\theta}\Gamma_{11}\Gamma_\mu d\theta(dX^\mu + \frac{1}{2}\bar{\theta}\Gamma^\mu d\theta).$$

For constant RR background fields Ω_{p+1} is closed, but there is an additional piece involving the θ's that contributes to

$$I_{p+2} = d\Omega_{p+1}.$$

It has the structure

$$\left\{e^{2\pi\alpha'\mathcal{F}} f(\Pi^\mu, d\theta)\right\}_{p+2}$$

1.2 Static Gauge

Let us focus on the $p = 9$ case, since the formulas for $p < 9$ can be inferred by dimensional reduction. As before, the local diffeomorphism symmetry is used to identify the embedding functions X^μ with the world volume coordinates σ^α. In addition, the local kappa symmetry is used to eliminate half of the θ coordinates. A simple choice that preserves the manifest 10d covariance is to simply set $\theta_2 = 0$. This has the remarkable consequence of completely eliminating the Chern–Simons term.

Renaming $\theta_1 = \lambda$ and setting $2\pi\alpha' = 1$ leaves

$$\int d^{10}\sigma\sqrt{-\det(\eta_{\alpha\beta} + F_{\alpha\beta} - 2\bar{\lambda}\Gamma_\alpha\partial_\beta\lambda + \bar{\lambda}\Gamma^\rho\partial_\alpha\lambda\bar{\lambda}\Gamma_\rho\partial_\beta\lambda)}.$$

This is the $\mathcal{N} = 1$, $D = 10$ super-Maxwell theory supplemented by higher-dimension interaction terms.

In addition to the 16 linearly realized supersymmetries of the free theory there are 16 additional nonlinearly realized supersymmetries. That is why this formula is reminiscent of the Volkov-Akulov action – λ can be interpreted as the Goldstone field for the broken supersymmetries.

When this D9-brane action is dimensionally reduced to give the Dp-brane action:

- 16 supersymmetries and $p+1$ translation symmetries are linearly realized
- 16 supersymmetries and $9-p$ translation [redacted]symmetries are non-linearly realized and correspond to Goldstone modes on the world volume

2 Non-Abelian Generalizations

The world-volume theory of N coincident Dp-branes is a $U(N)$ gauge theory. As such, it must be a non-Abelian generalization of the formulas of the preceding section. The explicit construction of such an action is a difficult problem that has been studied extensively, but is not yet completely settled.

Tseytlin (1997) proposed a specific recipe for generalizing Abelian formulas to non-Abelian ones. His proposal – referred to as the *symmetrized trace prescription* – works as follows. An expression in the Abelian theory, such as

$$\sqrt{-\det(\eta_{\alpha\beta} + F_{\alpha\beta})},$$

has an expansion of the form

$$1 + \frac{1}{4}F^2 - \frac{1}{8}(F^4 - (F^2)^2) + \ldots,$$

where $F^2 = F_{\alpha\beta}F^{\beta\alpha}$, etc. In the non-Abelian case, F is also a hermitian $N \times N$ matrix. Tseytlin's proposal is to take the trace of each term in the expansion, and to resolve the ordering ambiguities by averaging over all possible choices. Studies by Hashimoto and Taylor (1997) and others suggest that this is a correct rule through terms of order F^4, but that it fails at higher orders.

2.1 The Bosonic Chern–Simons Term

Myers (1999) discovered an interesting part of the answer by exploring consistency with T duality. He worked in the static gauge and focused on the dependence on the bosonic fields A_α and X^i, each of which are now $N \times N$ matrices. He included the dependence on B and C background fields. For the Chern–Simons term he obtained the result

$$S_{CS} = T_p \int \mathrm{Str}\left(P[e^{iI_X I_X} C e^B] e^F\right).$$

This is a subtle formula that requires some explanation. First of all $C = \Sigma C^{(n)}$, as before. $P[\ldots]$ means the pullback to the world volume, since B and C are bulk fields. X refers to the $9-p$ scalars X^i, which are now $N \times N$ matrices. The operation $I_X I_X$ acting on an n-form gives an $(n-2)$-form. For example,

$$I_X I_X C^{(2)} = X^j X^i C^{(2)}_{ij} = \frac{1}{2} C^{(2)}_{ij} [X^j, X^i]$$

Moreover, in the pullback of a function $f(x^\alpha, x^i)$, the matrices X^i need to substituted for the bulk coordinates x^i. This requires an ordering prescription, since $[X^i, X^j] \neq 0$. The proposed formula is

$$P[f] = \exp(X^i \frac{\partial}{\partial x^i}) f(\sigma^\alpha, x^i)|_{x^i=0}.$$

A crucial feature of this formula is that multi D-brane systems can be sources of higher D-brane charge as well as lower D-brane charge, since all the RR fields appear. This is to be contrasted with the Abelian case where $(Ce^F)_{p+1}$ only depends on $C^{(p+1)}, C^{(p-1)}, \ldots$.

Myers discovered a *dielectric effect* in which a background RR field strength can cause the brane to expand into new dimensions. For example, a system of N D0-branes in the presence of an electric $F^{(4)} = dC^{(3)}$ becomes a fuzzy two-sphere with $[X^i, X^j] \sim N\epsilon^{ijk}X^k$. For large N this describes an ordinary S^2 with radius proportional to N. This can be interpreted as a spherical D2-

brane with N D0-branes bound to it. The Myers effect is relevant to the appearance of "giant gravitons" on the AdS side of the AdS/CFT correspondence (McGreevy, Susskind, and Toumbas).

2.2 Supersymmetrization

Part of the rationale for Tseytlin's symmetrized trace prescription is that a field strength commutator

$$[F_{ij}, F_{kl}] \sim [D_i, D_j]F_{kl}$$

can be regarded as being higher-order in derivatives. This reflects an inherent ambiguity in the meaning of "slowly varying fields" in the non-Abelian case. This might be resolved by requiring that the action have all the desired symmetries: supersymmetry, kappa symmetry, etc.

In the case of N coincident D-branes the supersymmetric $U(N)$ world-volume theory should again have as its physical field content gauge fields A_α, transverse scalars

X^i, and fermi fields λ – this time all in the adjoint of the $U(N)$ Lie algebra.

In the case of D9-branes there are no transverse directions X^i. Thus, in static gauge, the only world-volume fields are the gauge fields A_α and the fermi fields θ. Still, this is quite general, because the results for $p < 9$ can be deduced by dimensional reduction. One still needs kappa transformations in the adjoint of $U(N)$ so that a gauge choice can reduce θ to λ.

This kind of a set-up has been explored recently by Bergshoeff, de Roo, and Sevrin. They carried out an iterative analysis that allowed them to deduce the action up to a certain order. Specifically, they determined terms in S_{CS} with the structures

$$\theta D\theta, \quad \theta D\theta F, \quad \theta D\theta F^2$$

and in S_{DBI} with the structures

$$1, \quad F^2, \quad \theta D\theta, \quad \theta D\theta F, \quad \theta D\theta F^2.$$

Up to this order they succeeded in obtaining unique results (modulo field redefinitions) with all the desired properties. They also gave the formulas in the gauge $\theta_2 = 0$, $\theta_1 = \lambda$. As in the Abelian case, S_{CS} does not contribute in this gauge. They observed, in particular, that the $\bar{\lambda} D \lambda F^2$ terms cannot be be expressed in terms of symmetrized traces.

The iterative analysis of Bergshoeff, de Roo, and Sevrin is technically difficult and cannot be pushed much further. It would be desirable to have explicit exact results so that one could explore the non-Abelian generalization of various effects that have been studied in the Abelian case. These include classical solutions that describe various sorts of solitons and brane configurations, as well as physical effects associated with electric fields approaching limiting values.

3 Conclusion

D-brane world volume actions are always given as the sum of a Dirac–Born–Infeld term and a Chern–Simons term. Each term contains a lot of important information.

A powerful approach that has received a great deal of attention lately is BSFT: boundary string field theory or background independent string field theory (Witten 1992). This provides the logical basis for deriving D-brane effective actions in terms of disk partition functions with appropriate boundary interactions.

The BSFT approach also allows one to formulate effective action descriptions of unstable D-brane systems (with tachyons) and to study the formation of various sorts of solitons. It also provides a promising approach to formulating the non-Abelian Born–Infeld problem that warrants further study. Unfortunately, the relevant path integrals might not be amenable to analytic evaluation.