(R.1)

LUMP SOLUTIONS IN STRING FIELD THEORY AROUND THE TACHYON VACUUM

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DYNAMICS OF A D-BRANE IN BOSONIC STRING THEORY IS DESCRIBED BY OPEN BOSONIC STRING FIELD THEORY (SFT)

ACTION OF CUBIC OPEN SFT:

$$\boxed{\frac{1}{2} \langle \Phi, Q_B \Phi \rangle + \frac{1}{3} \langle \Phi, \Phi * \Phi \rangle}$$

Witten

$|\Phi\rangle$: STRING FIELD (A STATE OF GHOST NO. 1 IN THE FIRST QUANTIZED THEORY)

$\langle \, , \, \rangle$: BPZ INNER PRODUCT

$*$: NON-COMMUTATIVE PRODUCT

SPECTRUM OF SFT CONTAINS A FIELD WITH -VE $(\text{MASS})^2$

→ THE TACHYON FIELD

CONJECTURE: THE TACHYON POTENTIAL HAS A MINIMUM DESCRIBING THE CLOSED STRING VACUUM.

AROUND THIS VACUUM THERE ARE NO PERTURBATIVE OPEN STRING EXCITATIONS.

BUT THERE ARE LUMP SOLUTIONS DESCRIBING LOWER DIMENSIONAL D-BRANES

NUMERICAL EVIDENCE IN CUBIC SFT

A.S., Zwiebach
Moeller, Taylor
Moeller, A.S., Zwiebach

GOAL: CONSTRUCT A STRING FIELD THEORY ACTION WHICH DESCRIBES THE DYNAMICS AROUND THE TACHYON VACUUM

$|\Psi\rangle$: SHIFTED STRING FIELD VARIABLE SUCH THAT:

$|\Psi\rangle = 0$ AT THE TACHYON VACUUM

THE ACTION $S(|\Psi\rangle)$ MUST

① HAVE THE USUAL ∞-DIMENSIONAL GAUGE INVARIANCE

② REFLECT ABSENCE OF PHYSICAL OPEN STRING STATES

③ CONTAIN LUMP SOLUTIONS REPRESENTING LOWER DIMENSIONAL D-BRANES

PROPOSED ACTION FOR SFT AROUND THE TACHYON VACUUM:

$$\boxed{S = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi|Q|\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)}$$

Q : AN OPERATOR MADE PURELY OF GHOST FIELDS AND SATISFYING CERTAIN PROPERTIES e.g.

NILPOTENCE, TRIVIAL COHOMOLOGY ETC.

GAUGE FIXING: (Horowitz, Morrow-Jones, Martin, Woodard)

$\mathcal{B}|\Psi\rangle = 0$, $\{Q, \mathcal{B}\}^{-1}$ FINITE

AN OPERATOR MADE OF GHOST FIELDS

⇒ STRING FIELD PROPAGATOR:

$$\boxed{\text{GHOST OPERATOR} \times (\text{IDENTITY})_{\text{MATTER}}}$$

EXAMPLE: $Q = c_0$, $\mathcal{B} = b_0 + \frac{1}{2}(b_2 + b_{-2})$

⇒ PROPAGATOR: $b_0 + \frac{1}{2}(b_2 + b_{-2})$

IN ORDER TO TEST THIS PROPOSAL WE NEED TO SHOW THAT THIS SFT CONTAINS ALL THE D-BRANES AS CLASSICAL LUMP SOLUTIONS

STRATEGY :

$$\boxed{|\Phi\rangle = \int d^{26}k \, [\underbrace{\phi(k)}_{\text{"TACHYON"}} \, c_1 |k\rangle + \cdots\cdots]}$$

① CONSTRUCT THE GENERATING FUNCTIONAL OF TACHYON GREENS FUNCTION $\longrightarrow W[J]$

② EFFECTIVE ACTION $\Gamma[\phi]$ IS LEGENDRE TRANSFORM OF $W[J]$

③ SOLVE EQS. OF MOTION DERIVED FROM $\Gamma[\phi]$

$$\boxed{W[J] = \sum_{n=2}^{\infty} \frac{1}{n!} \int d^{26}p_1 \cdots d^{26}p_n \; g^{(n)}(p_1, \cdots p_n) \; J(p_1) \cdots J(p_n) \, \delta\left(\sum_{i=1}^{n} p_i\right)}$$

n- TACHYON GREENS FUNCTION

CONSTRUCTED USING FEYNMAN RULES

VERTICES: SAME AS IN ORDINARY CUBIC SFT

PROPAGATOR:

$$\boxed{\text{GHOST OPERATOR} \times (\text{IDENTITY})_{\text{matter}}}$$

INSTEAD OF $b_0 L_0^{-1}$

IN CONVENTIONAL CUBIC SFT WE WRITE:

$$\boxed{b_0 L_0^{-1} = b_0 \int_0^\infty dt\, e^{-tL_0}}$$

$t \to$ LENGTH OF THE STRIP REPRESENTING THE PROPAGATOR

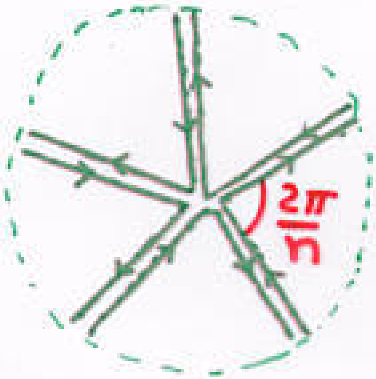
HERE PROPAGATOR = GHOST OP.

NET RESULT:

$b_0 \to$ GHOST OPERATOR
$t = 0$

$\Rightarrow$ WE GLUE VERTICES WITH ZERO LENGTH PROPAGATORS.

WORLD SHEET PICTURE OF n-TACHYON GREENS FUNCTION:



IN CFT LANGUAGE:

$$\boxed{g^{(n)}(p_1, \cdots p_n)\ \delta\left(\sum_{i=1}^{n} p_i\right) = C_n \left\langle \prod_{k=1}^{n} \underbrace{f_k^{(n)} \circ e^{i p_k \cdot X(0)}}_{\left(f_k^{(n)\prime}(0)\right)^{p_k^2} e^{i p_k \cdot X(f_k^{(n)}(0))}} \right\rangle_{\text{matter}}}$$

C_n → GHOST CORRELATOR

$$\boxed{f_k^{(n)}(z) = e^{2\pi i \frac{k-1}{n}} \left(\frac{1+iz}{1-iz}\right)^{\frac{2}{n}}}$$

FINAL RESULT:

$$\boxed{\begin{aligned} & g^{(n)}(p_1, \cdots p_n) \\ & = c_n \exp\Big[\ln\tfrac{4}{n}\sum_{k=1}^{n} p_k^2 \\ & + \sum_{\substack{k,\ell=1 \\ k\neq\ell}}^{n} p_k \cdot p_\ell \ln\big(2\sin\tfrac{\pi}{n}|k-\ell|\big)\Big] \end{aligned}}$$

ghost
Contr.

THIS DETERMINES $W[J]$

$$\phi(p) \equiv \delta W[J] / \delta J(-p)$$

$$\Gamma[\phi] \equiv \int d^{26}p \; J(-p)\,\phi(p) - W[J]$$

$$\Rightarrow \delta\Gamma[\phi]/\delta\phi(p) = J(-p)$$

CLASSICAL EQS. OF MOTION:

$$\boxed{\delta\Gamma/\delta\phi(p) = 0 \quad \Rightarrow \quad J(p) = 0}$$

$\Rightarrow$ A CLASSICAL SOLN. IS GIVEN BY:

$$\boxed{\phi_{cl}(p) = \delta W[J]/\delta J(-p)\Big|_{J(q)=0}}$$

$$\boxed{W = \sum_{n=2}^{\infty} \frac{1}{n!} \int d^{26}p_1 \cdots d^{26}p_n \; g^{(n)}(p_1, \cdots p_n)\, J(p_1) \cdots J(p_n)\, \delta\left(\sum_{i=1}^{n} p_i\right)}$$

THUS NAIVELY $\delta W/\delta J(p)\big|_{J=0} = 0$

$$\Rightarrow \boxed{\phi_{cl}(p) = 0}$$

Q. HOW DO WE GET $\phi_{cl} \neq 0$?

A. TYPICALLY $W[J]$ HAS BRANCH CUTS IN COMPLEX $J(p)$ SPACE.

ON A NON-TRIVIAL BRANCH:

$$\boxed{\delta W/\delta J(-p)\big|_{J=0} \neq 0 \;\Rightarrow\; \phi_{cl}(p) \neq 0}$$

EXAMPLE: TREE LEVEL φ^3 FIELD THEORY

(IN ZERO MOMENTUM SECTOR)

$$\boxed{\Gamma[\phi] = -\tfrac{1}{2} m^2 \phi^2 + \tfrac{1}{3} \lambda \phi^3}$$

$$J = \delta\Gamma/\delta\phi = -m^2 \phi + \lambda \phi^2$$

$$\Rightarrow \boxed{\phi = \frac{1}{2\lambda}\left(m^2 \pm \sqrt{m^4 + 4\lambda J}\right) \equiv \delta W/\delta J}$$

$$\Rightarrow W = \frac{1}{2\lambda}\left(m^2 J \pm \frac{1}{6\lambda}(m^4 + 4\lambda J)^{3/2} + m^6/6\lambda\right)$$

ON THE $-$ BRANCH:

$$\boxed{\phi = 0, \quad W = 0 \quad \text{AT} \quad J = 0}$$

ON THE $+$ BRANCH:

$$\boxed{\phi = m^2/\lambda, \quad W = m^6/6\lambda^2 \quad \text{AT } J = 0}$$

STRATEGY (IN GENERAL)

① FIND BRANCH POINTS OF $W[J]$ AND $\delta W/\delta J(q)$ IN THE COMPLEX $J(p)$ SPACE

② ANALYTICALLY CONTINUE $\delta W/\delta J(q)$ AROUND THESE BRANCH POINTS AND RETURN TO $J=0$

$\Rightarrow$ GENERATES NON-TRIVIAL SOLUTIONS OF THE EQS. OF MOTION

WE NOW RETURN TO OUR PROBLEM.

$$\boxed{\frac{\delta W}{\delta J(p_1)} = \sum_{n=2}^{\infty} \frac{1}{(n-1)!} \int d^{26}p_1 \cdots d^{26}p_n \; g^{(n)}(p_1, \cdots p_n)\, J(p_2) \cdots J(p_n)\, \delta\Big(\sum_{i=1}^{n} p_i\Big)}$$

$$\boxed{g^{(n)}(p_1, \cdots p_n) = C_n \exp\Big[\ln\frac{4}{n} \sum_{k=1}^{n} p_k^2 + \sum_{\substack{k,l=1 \\ k\neq l}}^{n} p_k \cdot p_l \ln\Big(2 \sin\frac{\pi}{n}|k-l|\Big)\Big]}$$

Q. DOES $\delta W/\delta J(p_1)$ HAVE BRANCH POINTS IN THE COMPLEX $J(p)$ SPACE?

A. DEPENDS ON THE RATE OF GROWTH OF C_n FOR LARGE n

THIS IS NOT KNOWN

STRATEGY

① WE SHALL ASSUME THAT $\delta W / \delta J(p)$ HAS A BRANCH POINT REPRESENTING A TRANSLATIONALLY INVARIANT SOLUTION

→ THE D-25 BRANE

② THEN WE SHALL TRY TO SHOW THAT GIVEN THIS ASSUMPTION, THERE ARE ALSO LUMP SOLUTIONS REPRESENTING LOWER DIMENSIONAL D-BRANES.

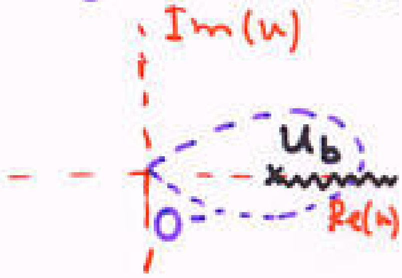
TRANSLATIONALLY INVARIANT CONFIGURATION:

$$\boxed{J(p) = u\,\delta(p)}$$

$$\boxed{\left.\frac{\delta W}{\delta J(p)}\right|_{J(q)=u\,\delta(q)} = \delta(p)\underbrace{\sum_{n=2}^{\infty}\frac{c_n}{(n-1)!}u^n}_{F(u)}}$$

ASSUME THAT $F(u)$ HAS A BRANCH POINT AT $u = \underline{u_b}$

0^- : RESULT OF GOING AROUND u_b AND RETURNING TO $u=0$.



$$\boxed{\phi_0 \equiv \lim_{u\to 0^-} F(u)}$$

$$\Rightarrow \lim_{u\to 0^-}\left.\frac{\delta W}{\delta J(p)}\right|_{J(q)=u\,\delta(q)} = \phi_0\,\delta(p)$$

$\rightarrow$ D-25 BRANE SOLUTION

WE SHALL ALSO ASSUME THAT u_b IS THE CLOSEST SINGULARITY TO THE ORIGIN. OF $F(u)$.

$\Rightarrow$ THE RADIUS OF CONVERGENCE OF THE TAYLOR SERIES EXPANSION OF $F(u)$ IS $|u_b|$

THUS THE BRANCH POINT AT u_b CONTROLS THE RATE OF GROWTH OF THE COEFFICIENTS c_n.

NOW CONSIDER A SOURCE OF THE FORM:

$$\boxed{J(p) = u\,\delta(p_{\parallel})\,K\,e^{-\frac{1}{2}\alpha p_{\perp}^{2}}}$$

K, α: CONSTANTS

$(p_{\parallel}, p_{\perp})$: $(N_{\parallel}+1, N_{\perp})$ DIM. VECTORS

p^{μ}

$$\boxed{N_{\parallel} + N_{\perp} = 25}$$

IN POSITION SPACE J IS INDEPENDENT OF $x_{\parallel}$ AND IS A GAUSSIAN ALONG $x_{\perp}$

WE WOULD LIKE:

$$\frac{\delta W}{\delta J(p)}\bigg|_{J(q) = u\,\delta(q_{\parallel})\,K\,e^{-\frac{1}{2}\alpha q_{\perp}^{2}}} = \underbrace{\gamma}_{\text{CONSTANTS}}\underbrace{K}_{}\,e^{-\frac{1}{2}\underbrace{\beta}_{} p_{\perp}^{2}}\,\delta(p_{\parallel})\,\underbrace{F(u)}_{\text{SAME FN AS } J(p) = u\,\delta(p) \text{ CASE}} + \delta(p_{\parallel})\,\underbrace{\mathcal{H}(u, p_{\perp}^{2})}_{\text{HAS MILDER SINGULARITY AT } u_b \text{ THAN } F(u)}$$

IN THIS CASE WE CAN GENERATE A NON-TRIVIAL CLASSICAL SOLUTION BY TAKING u AROUND u_b AND RETURNING TO THE ORIGIN.

$$\lim_{u\to 0^-} F(u) = \phi_0 .$$

DEFINE:

$$\boxed{h(p_\perp,\alpha) \equiv \lim_{u\to 0^-} \mathcal{H}(u, p_\perp, \alpha).}$$

ASSUMPTION: $h(p_\perp,\alpha)$ IS WELL-DEFINED.

$$\boxed{\begin{aligned}\phi_{cl}(-p) &\equiv \lim_{u\to 0^-} \frac{\delta W}{\delta J(p)}\Big|_{J(q)=u\,\delta(q_\parallel)\,K e^{-\frac{1}{2}\alpha q_\perp^2}} \\ &= \gamma K e^{-\frac{1}{2}\beta p_\perp^2}\,\delta(p_\parallel)\,\phi_0 + h(p_\perp,\alpha)\,\delta(p_\parallel)\end{aligned}}$$

$\to$ CODIMENSION $N_\perp$ LUMP SOLUTION

THE KEY REQUIREMENT:

$$\boxed{\left.\frac{\delta W}{\delta J(p)}\right|_{J(q)=u\,\delta(q_{\parallel})\,K\,e^{-\frac{1}{2}\alpha q_{\perp}^{2}}} = \gamma K\, e^{-\frac{1}{2}\beta p_{\perp}^{2}}\,\delta(p_{\parallel})\,F(u) + \text{sub-leading}}$$

FOR SOME CHOICE OF α, β, γ, K

IS THIS TRUE?

IF

$$\boxed{\text{l.h.s.} = \sum_n l_n u^n, \qquad \text{r.h.s.} = \sum_n r_n u^n}$$

THEN WE NEED:

$$\boxed{\lim_{n\to\infty} \frac{l_n}{r_n} = 1}$$

THE UNKNOWN COEFFICIENTS c_n DROP OUT IN THIS RATIO.

DEFINE :

$$b_s \equiv \ln\frac{n}{4} + \frac{\alpha}{2} - \sum_{k=1}^{n-1} \ln\left(2\sin\left(\frac{\pi k}{n}\right)\right) \cos\left(\frac{2\pi k s}{n}\right)$$

$$f(\alpha, n) \equiv \sum_{s=1}^{n-1} \ln\frac{\pi}{b_s} - \ln n$$

$$\tilde{f}(\alpha, n) \equiv \frac{1}{n}\sum_{s=1}^{n-1} \frac{1}{b_s}$$

FOR $\lim_{n\to\infty} \frac{l_n}{r_n} = 1$ WE NEED

$$\lim_{n\to\infty} \left(\beta - \frac{2}{\tilde{f}(\alpha, n)} + \alpha\right) = 0$$

$$\lim_{n\to\infty} \left[(n-2)\ln K + \frac{1}{2} N_1 f(\alpha, n) - \ln\gamma - \frac{1}{2} N_2 \ln\frac{2\pi}{\alpha+\beta}\right] = 0$$

Q. ARE THERE K, α, β, γ SATISFYING THESE EQUATIONS?

NUMERICAL RESULTS:

$$\boxed{\begin{aligned} &\frac{2}{\tilde{f}(\alpha,n)} - \alpha = C(\alpha) + \tilde{s}_n(\alpha) \\ &f(\alpha,n) = A(\alpha)\, n + B(\alpha) + s_n(\alpha) \end{aligned}}$$

$A(\alpha)$, $B(\alpha)$, $C(\alpha)$: FINITE

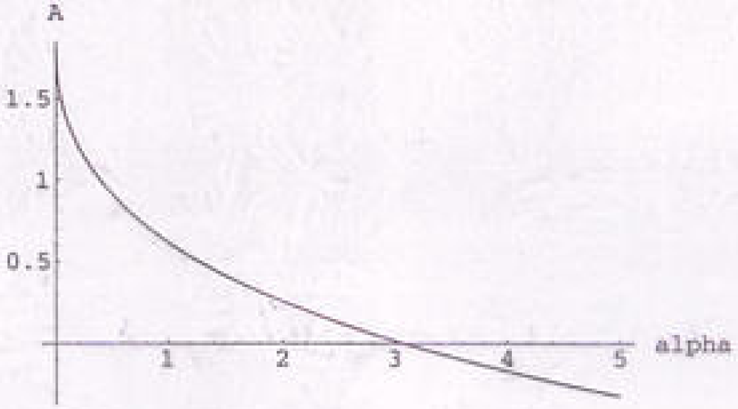
$s_n(\alpha)$, $\tilde{s}_n(\alpha) \to 0$ AS $n \to \infty$

FURTHERMORE: $\boxed{B(\alpha) = 0}$

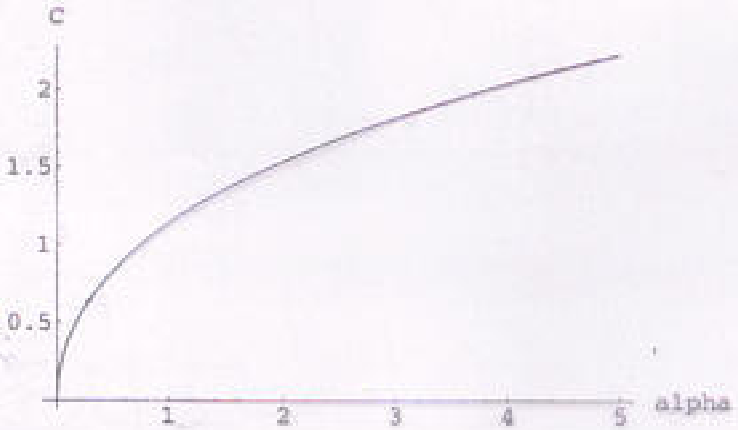
DEMANDING $\lim_{n\to\infty} \frac{\ell_n}{r_n} = 1$ GIVES:

$$\boxed{\begin{aligned} &\beta = C(\alpha) \\ &K = e^{-\frac{1}{2} N_\perp A(\alpha)} \\ &\gamma = e^{\frac{1}{2} N_\perp \left(2A(\alpha) + B(\alpha) - \ln \frac{2\pi}{\alpha+\beta}\right)} \end{aligned}}$$

$A(\alpha)$



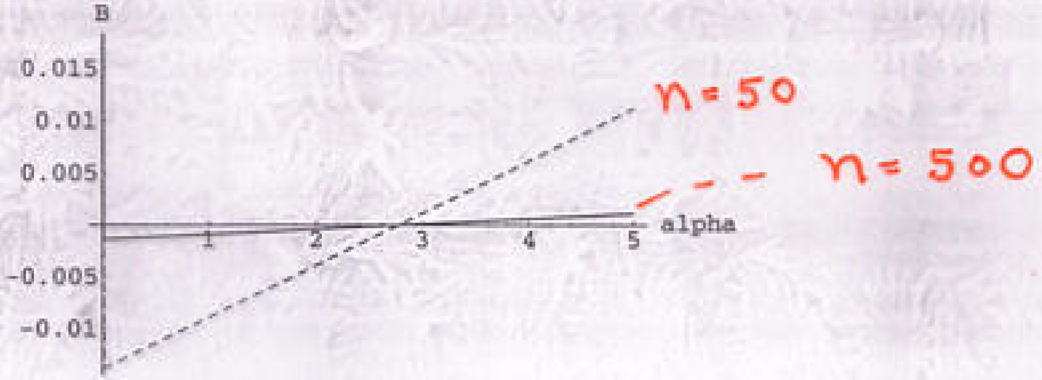
$C(\alpha)$



—— : $n = 500$

- - - - : $n = 50$

$$2f(\alpha,n) - f(\alpha,2n) = B(\alpha) + 2\,s_n(\alpha) - s_{2n}(\alpha)$$



THIS SHOWS THE EXISTENCE OF THE LUMP SOLUTION.

UNFORTUNATELY THE PRECISE FORM OF THE SOLUTION DEPENDS ON THE CONTRIBUTION FROM THE SUBLEADING TERMS.

NOTE: DIFFERENT VALUES OF $\alpha \rightarrow$ DIFFERENT PATHS IN THE COMPLEX $J(p)$ PLANE WHICH CAN BE CONTINUOUSLY DEFORMED TO EACH OTHER

$\Rightarrow$ FINAL SOLUTION $\left.\frac{\delta W}{\delta J(p)}\right|_{\alpha=0}$ SHOULD BE INDEPENDENT OF α.

CALCULATION OF THE TENSION OF THE LUMP SOLUTION

$$\Gamma[\phi] = \int d^{26}p \; J(p)\, \phi(-p) - W[J]$$

AT THE CLASSICAL SOLN $J=0$

$$\Rightarrow \boxed{\Gamma[\phi_{cl}] = - W[J]\big|_{J=0}}$$

STRATEGY:

CALCULATE $W[J = u K e^{-\frac{1}{2}\alpha p_\perp^2} \delta(p_\parallel)]$

AND TAKE THE $u \to 0$ LIMIT AFTER GOING AROUND u_b.

$$\boxed{\begin{aligned} & W[J(p) = u K e^{-\frac{1}{2}\alpha p_\perp^2}\, \delta(p_\parallel)] \\ & = \frac{V_\parallel}{(2\pi)^{N_\parallel+1}} \sum_{n=2}^{\infty} \frac{1}{n!} C_n u^n K^n \underbrace{\left\{ \frac{1}{n} \prod_{s=1}^{n-1} \frac{\pi}{b_s} \right\}^{N_\perp/2}}_{e^{\frac{N_\perp}{2} f(\alpha, n)}} \end{aligned}}$$

$\delta(p_\parallel = 0)$

= WORLD-VOLUME OF THE D-BRANE

FINAL RESULT FOR W:

$$\boxed{W = \frac{V_{11}}{(2\pi)^{N_{11}+1}} \sum_{n=2}^{\infty} \frac{1}{n!} c_n u^n e^{\frac{N_1}{2}(B(\alpha)+\delta_n(\alpha))}}$$

TENSION OF THE LUMP:

$$\boxed{\mathcal{T}_{N_{11}} = -\Gamma[\phi_{cl}]/V_{11} = \frac{1}{V_{11}} \lim_{u \to 0^-} W}$$

WE SHALL NOW MAKE THE <u>DRASTIC</u> ASSUMPTION THAT THE CONTRIBUTION TO $\mathcal{T}_{N_{11}}$ FROM THE SUBLEADING TERMS $\delta_n(\alpha)$ VANISHES

(THIS WOULD BE THE CASE IF $\delta_n(\alpha) \sim e^{-an}$ FOR LARGE n, BUT NUMERICAL ANALYSIS SHOWS THAT $\delta_n(\alpha) \sim \frac{1}{n^a}$ FOR SOME INTEGER a)

AFTER DROPPING THE $\delta_n(\alpha)$ TERMS WE GET:

$$\boxed{\mathcal{I}_{N_\parallel} = \frac{1}{(2\pi)^{N_\parallel+1}} e^{\frac{N_\perp}{2} B(\alpha)} \lim_{u \to 0-} \sum_{n=2}^{\infty} \frac{1}{n!} c_n u^n}$$

NOTE $N_\perp = 25 - N_\parallel$

$$\Rightarrow \boxed{\frac{\mathcal{I}_{N_\parallel - 1}}{\mathcal{I}_{N_\parallel}} = 2\pi\, e^{\frac{1}{2} B(\alpha)}}$$

RECALL THE NUMERICAL RESULT:

$$B(\alpha) = 0$$

$$\Rightarrow \boxed{\mathcal{I}_{N_\parallel - 1} / \mathcal{I}_{N_\parallel} = 2\pi}$$

THIS IS THE EXPECTED RESULT FOR THE D-BRANE.

α-INDEPENDENCE OF $\mathcal{I}_{N_\parallel}$ IS A CONSISTENCY CHECK.

CONCLUSION/SUMMARY

① WE PROPOSED A GENERAL ANSATZ FOR SFT ACTION AROUND THE TACHYON VACUUM

② ASSUMING THE EXISTENCE OF A D-25-BRANE SOLUTION AND SUITABLE ANALYTICITY PROPERTIES WE SHOWED THE EXISTENCE OF LUMP SOLUTIONS OF LOWER DIMENSIONS

③ WITH THE HELP OF SOME FURTHER ASSUMPTIONS WE COMPUTED THE RATIOS OF TENSIONS OF THE LUMPS

THE RESULT AGREES WITH THAT FOR THE D-BRANES