MAGNETIC BRANES & GIANT GRAVITONS

hep-th/0008203

SUMIT DAS
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- WE ARE GETTING USED TO THE IDEA IN STRING THEORY, THAT MOVING OBJECTS OFTEN GROW IN TRANSVERSE SIZE.
- SUGGESTED BY: INFORMATION SPREADING NEAR A BLACK HOLE HORIZON
- SUPPORTED BY: STRING UNCERTAINITY PRINCIPLE
  NON-COMMUTATIVE GAUGE THEORIES
- IN THIS TALK WE EXAMINE THE IDEA FOR "GIANT GRAVITONS". MASSLESS PARTICLES IN SUPERGRAVITY, WHEN MOVING IN APPROPRIATE EXTERNAL FIELDS, BLOW UP INTO EXPANDED BRANE CONFIGURATIONS.
- IN SOME CASES PROVIDES A QUANTITATIVELY PRECISE CONTEXT FOR TRANSVERSE GROWTH.

OUTLINE:

1. MAGNETIC ANALOGUE OF MYERS' DIELECTRIC EFFECT
2. GIANT GRAVITONS: A QUICK REMINDER
3. D2-BRANES AS EXPANDED D0 BRANES (IN 4-BRANE BACKGROUND)
4. MORE EXAMPLES OF GIANT GRAVITONS WITH LESS OR EVEN NO SUPERSYMMETRY AND NO CONFORMAL INVARIANCE.

TWO REFERENCES IN PARTICULAR:

1. McGREEVY, SUSSKIND & TOUMBAS, "INVASION OF GIANT GRAVITONS IN Ad-S SPACE" hepth/0003075, JHEP 0006 (2000) 008

2. MYERS, "DIELECTRIC BRANES" JHEP 9912 022 (1999).

DIELECTRIC EFFECT:

- BRANES COUPLE TO HIGHER FORM RR- FIELDS

MYERS
RAAMSDONK & TAYLOR.

e.g. D0-BRANES

$$\Delta\mathcal{L} = -i\frac{T_0}{3\lambda}\int \mathrm{Tr}\left(C^{(3)}_{0ij}\, x^i x^j\right) dx^0$$

$F^{(4)}_{0ijk}$: CONSTANT

$$\Delta\mathcal{L} = -i\frac{T_0}{3\lambda}\int F^{(4)}_{0ijk}\,\mathrm{Tr}(x^i x^j x^k)\, dx^0$$

$$(\quad \lambda = 2\pi l_s^2 \quad)$$

- ENERGY FOR STATIC CONFIGURATION OF N BRANES :

$$E = T_0 N - \frac{T_0}{4\lambda^2}\sum_{ab}\mathrm{Tr}([x^a,x^b]^2) - i\frac{T_0}{3\lambda}\mathrm{Tr}(x^i x^j x^k)\,F^{(4)}_{0ijk}$$

(UPTO QUARTIC TERMS)

- SOLUTION: $F^{(4)}_{0ijk} = \begin{cases} -F\epsilon_{ijk} & \text{if } i,j,k \in \{1,2,3\} \\ 0 & \text{OTHERWISE} \end{cases}$

$$x^i = \frac{\lambda F}{2} J^i \qquad i = 1,2,3$$

N DIM. REPRESENTATION OF SU(2) ALGEBRA

- A "FUZZY" SPHERE

RADIUS $R^2 = \sum x^{i\,2}$

FOR N DIM. IRREP: $R \simeq \frac{\lambda}{4} F N \qquad (N \gg 1)$

- HAS DIPOLE MOMENT WITH RESPECT TO $F^{(4)}$.

- ANOTHER CONFIGURATION WITH SAME QUANTUM NUMBERS: D2 BRANE WRAPPED ON S^2 WITH N UNITS OF MAGNETIC FLUX $\left(\int F = 2\pi N\right)$ ON IT'S WORLD VOLUME.

$$E = \underbrace{4\pi T_2 \sqrt{r^4 + \frac{N^2}{4}\lambda^2}}_{\text{DBI}} - \underbrace{\frac{4\pi}{3} T_2 F r^3}_{\text{CHERNS SIMONS}}$$

r: RADIUS OF S^2

- FOR $N\lambda \gg r$

$$E \cong 4\pi T_2 \left[\frac{N\lambda}{2} + \frac{r^4}{\lambda N} - \frac{4\pi}{3} T_2 r^3 F \right]$$

- GIVES ENERGY AND RADIUS WHICH AGREE WITH FUZZY SPHERE.

- ONE CAN REGARD THE FUZZY S^2 D0-BRANE CONFIGURATION AS PROVIDING AN ALTERNATE (MORE MICROSCOPIC) DESCRIPTION OF THE SPHERICAL D2-BRANE.

MAGNETIC EFFECT:

- GENERAL COUPLING OF D0-BRANES TO RR 3-FORM IS:

$$\Delta \mathcal{L} = \int Tr\left[C^{(3)}_{ijk}\, x^i x^j \frac{dx^k}{d\tau}\right] d\tau$$

- IN STATIC GAUGE:

$$\Delta \mathcal{L} = \int Tr\left[C^{(3)}_{0ij}\, x^i x^j\right] dt \; +$$

$$\int Tr\left[C^{(3)}_{ijk}\, x^i x^j \frac{dx^k}{dt}\right] dt$$

- IF MAGNETIC FIELD F^{4}_{ijkl} : CONSTANT & NO ELECTRIC FIELD

$$\Delta \mathcal{L} = F^{(4)}_{ijkl} \int Tr\left[x^i x^j x^k \frac{dx^l}{dt}\right] dt$$

- CONSEQUENCES

CONSTANT MAGNETIC FIELD

$$F^{(4)}_{ijkl} = \begin{cases} -F\,\epsilon_{ijkl} & i,j,k,l \in \{1,2,3,4\} \\ 0 & \text{OTHERWISE} \end{cases}$$

$$\mathcal{L} = -NT_0 + \frac{T_0}{2}\sum_{i=1}^{9} \mathrm{Tr}\,\dot{x}^{i\,2} + \frac{T_0}{4\lambda^2}\sum_{ab}\mathrm{Tr}\,[x^a, x^b]^2$$

$$+ \frac{i T_0}{3\lambda} F^{(4)}_{ijkl}\,\mathrm{Tr}\left(x^i x^j x^k \dot{x}^l\right)$$

SOL$\underline{\underline{N}}$:

$$x^4 = v\, x^0\, \mathbb{1}$$

$$x^i = \frac{\lambda}{2} F v \; J^i \qquad i = 1,2,3$$

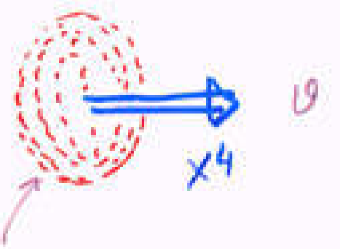
N DIM REP OF SU(2)

$$R \approx \frac{\lambda}{4} F v N$$

$$E = NT_0 + \frac{T_0}{2} N v^2 - \frac{T_0}{128}\lambda^2 F^4 v^4 N^3$$

- ONE PARAMETER FAMILY OF SOLUTIONS LABELLED BY VELOCITY 'v' (OR MOMENTUM).

- TRANSVERSE SIZE GROWS WITH INCREASING VELOCITY:



$R = \frac{\lambda}{4} F v N$

(OF COURSE NO VIOLATION OF LORENTZ INVARIACE)

- RESULTING CONFIGURATION HAS ELECTRIC & MAGNETIC DIPOLE MOMENTS.

- MAGNETIC INTERACTION LOWERS ENERGY (FOR FIXED MOMENTUM P_4). IN THIS SENSE A PARAMAGNET.

- THERE IS A SOLUTION FOR A D2-BRANE WITH THE SAME QUANTUM NUMBERS. IN APPROPRIATE LIMIT (RADIUS SMALL, NON-RELATIVISTIC MOTION) IT GIVES SAME ENERGY AND RADIUS AS "PUFFED" D0-BRANES. THE "PUFFED D0-BRANES PROVIDE AN ALTERNATE DESCRIPTION OF THE D2-BRANE.

. GIANT GRAVITONS:

MCGREEVY
TOUMBAS
SUSSKIND

• NEAR HORIZON REGION OF M5-BRANE.

$AdS_7 \times S^4$

$$ds^2 = -g_{tt}\,dt^2 + \sum_{i=1}^{5} g_{ii}\,dx^{i\,2} + g_{rr}\,dr^2 + R^2 d\Omega_4^2$$

$$= \underbrace{\frac{r^2}{R^2}\left(-dt^2 + \sum_{i=1}^{5} dx^{i\,2}\right) + \frac{R^2}{r^2}dr^2}_{AdS_7} + R^2 \underbrace{d\Omega_4^2}_{S^4}$$

MASSLESS PARTICLE:

P_φ : ANGULAR MOMENTUM IN S^4

$$\frac{P_0^2}{g_{00}} - \frac{P_r^2}{g_{rr}} - \frac{P_\varphi^2}{R^2} = 0 \qquad (p^\mu p_\mu = 0)$$

$$E = p_0 = \sqrt{g_{00}}\left[\frac{P_r^2}{g_{rr}} + \frac{P_\varphi^2}{R^2}\right]^{1/2}$$

• SAME ENERGY (FOR FIXED ANGULAR MOMENTUM) FOR A SOLUTION CONSISTING OF M2 BRANE WRAPPING AN $S^2 \subset S^4$ AND ROTATING.

- TO DESCRIBE S^2 WRAPPED BY 2-BRANE:

CHOOSE COORDINATES ON UNIT S^4:

$$\sum_{i=1}^{5} (y^i)^2 = 1$$

$$y^1 = \sqrt{1-\rho^2}\cos\phi, \qquad y^2 = \sqrt{1-\rho^2}\sin\phi$$

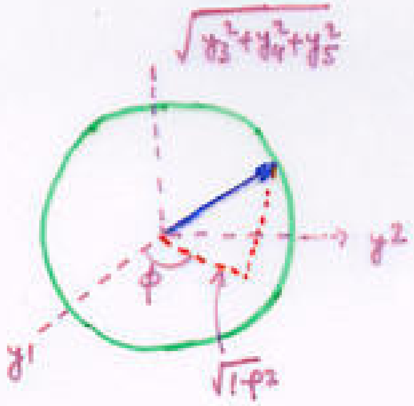
$$y^3 = \rho\cos\theta, \quad y^4 = \rho\sin\theta\cos\psi, \quad y^5 = \rho\sin\theta\sin\psi$$

- (θ,ψ) PARAMETRISE A TWO-SPHERE

$$(y^3)^2 + (y^4)^2 + (y^5)^2 = \rho^2$$

- M2-BRANE WRAPS THIS S^2

ALSO: $r(t)$, $\rho(t)$, $\phi(t)$



$$S_{DBI} = - T_2 4\pi \int dt (R^2 \rho^2) \sqrt{g_{tt} - g_{rr}\dot{r}^2 - g_{\rho\rho}\dot{\rho}^2 - g_{\varphi\varphi}\dot{\varphi}^2}$$

$$S_{CS} = \int C^{(3)} = \underset{\substack{\uparrow \\ \text{\# of M5 Branes}}}{N_5} \int \rho^3 \dot{\phi}$$

HAMILTONIAN:

$$H = \sqrt{g_{tt}} \left[\underbrace{(T_2 \, 4\pi R^2 \rho^2)^2}_{(\text{Mass})^2} + \frac{P_r^2}{g_{rr}} + \frac{P_\rho^2}{g_{\rho\rho}} + \frac{\overbrace{(P_\varphi - N_5 \rho^3)^2}^{}}{g_{\varphi\varphi}} \right]^{1/2}$$

$$(P_\varphi - N_5 \rho^3)^2 \rightarrow (P_\varphi - eA)^2$$

NOW SINCE :

$$\boxed{4\pi T_2 R^3 = N_5} \quad \text{IMP. CONDITION}$$

HAMILTONIAN CAN BE REWRITTEN AS:

$$H = \sqrt{g_{tt}}\left[\frac{P_r^2}{g_{rr}} + \frac{P_\varphi^2}{R^2} + \frac{P_\rho^2}{g_{\rho\rho}} + \frac{(\rho P_\varphi - N\rho^2)^2}{g_{\varphi\varphi}}\right]^{1/2}$$

- POTENTIAL IN "ρ" DIRECTION MINIMISED AT:

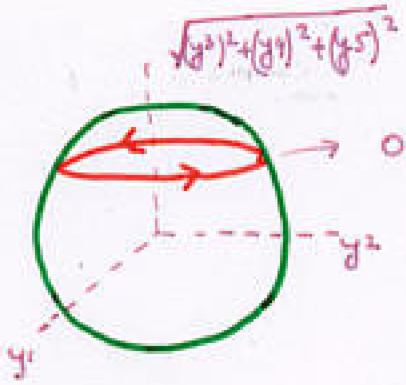
$$\rho = \frac{P_\varphi}{N}$$

ONE CAN ALSO THEN TAKE

$$P_\rho = 0$$

$$\boxed{H = \sqrt{g_{tt}}\left[\frac{P_r^2}{g_{rr}} + \frac{P_\varphi^2}{R^2}\right]^{1/2}}$$

- SAME AS FOR A MASSLESS PARTICLE
- THIS SOLUTION MINIMISES ENERGY (FOR FIXED P_φ)



- **BOUND:** $P_\varphi = N_5 \rho$

  $\rho_{max} = 1$

  $P_\varphi \leq N_5$

- AGREES WITH STRINGY EXCLUSION PRINCIPLE

  MALDACENA
  STROMINGER

- IN FACT THESE ARE BPS STATES

  HASHIMOTO
  HIRANO
  ITZHAKI

  GRISARU, MYERS
  TAFJORD

- PICTURE: FOR LARGE ENOUGH ANGULAR MOMENTUM GRAVITONS EXPAND INTO A TWO-BRANE.

- CAN WE UNDERSTAND THIS BLOWING UP OF THE GRAVITON IN MORE DETAIL?

- NOT DIRECTLY.
- BASIC LESSON OF MATRIX THEORY: EASIER TO LOOK AT PROCESSES AFTER LARGE BOOST.

- COMPACTIFY ONE DIRECTION PARALLEL TO M5-BRANE. ADD N_0 UNITS OF MOMENTUM FOR GRAVITON

IN IIA PICTURE: M5 $\longrightarrow$ D4

GRAVITON $\rightarrow$ N_0 D0 BRANES

M2 BRANE $\longrightarrow$ D2-BRANE

(WITH N_0 UNITS OF MOMENTUM) (WITH N_0 UNITS OF MAGNETIC FLUX IN WORLD VOLUME)

- CAN WE UNDERSTAND D2-BRANE AS PUFFED-UP D0-BRANES?

D2-BRANE: AFTER TURNING ON MAGNETIC FLUX, SPECIAL SOLUTIONS (WHICH MINIMIZE ENERGY):

$$H = \sqrt{g_{tt}} \left[\underbrace{\left(N_0 T_0 e^{-\phi}\right)^2}_{\substack{\text{EXTRA MASS TERM} \\ N_0 \text{ D0 BRANES}}} + \frac{P_\varphi^2}{g_{\varphi\varphi}} + \frac{P_r^2}{g_{rr}} \right]^{1/2}$$

- **D0-BRANES**: NEED NON-ABELIAN D0-BRANE ACTION IN CURVED SPACE.

- IN GENERAL LAGRANGIAN AMBIGUOUS DUE TO ORDERING PROBLEMS.

- $$L_{DBI} = -T_0 \, \mathrm{Tr}\left[e^{-\phi(x)} \sqrt{g_{tt}(x)} \left\{ 1 - \frac{1}{2} \frac{g_{ij}(x)}{g_{tt}(x)} \dot{x}^i \dot{x}^j - \frac{1}{4\lambda^2} \sum [x^a, x^b][x^c, x^d] \, g_{ac}(x) \, g_{bd}(x) \right\} \right]$$

$$L_{C.S.} = -i \frac{T_0}{\lambda} \mathrm{Tr}\left[C^{(3)}_{ijk}(x) \, x^i x^j \dot{x}^k \right]$$

- LUCKILY, ON PUTTING IN AN ANSATZ APPROPRIATE FOR THE SOLUTION WE SEEK, THE ORDERING AMBIGUITY DISAPPEARS.

4-BRANE METRIC:

- $$ds^2 = H^{-1/2}\left(-dt^2 + \sum_{i=1}^{4} dx_i^2\right) + H^{1/2} \sum_{i=5}^{9} (dx^i)^2$$

$$H = \frac{R^3}{r^3} \quad ; \quad r^2 = \sum_{i=5}^{9} (x^i)^2$$

ALTERNATIVELY,

$$ds^2 = H^{-1/2}\left(-dt^2 + \sum_{i=1}^{4} dx_i^2\right) + H^{1/2}\left[dr^2 + r^2 \frac{d\rho^2}{1-\rho^2} + r^2(1-\rho^2)d\varphi^2 + r^2\rho^2 d\Omega_2^2\right]$$

- D2 - BRANE WRAPS $S^{(2)}$ SPANNED BY (θ, ψ).

- SUGGESTS ANSATZ FOR D0-BRANES:

$$X^5 = r\sqrt{1-\rho^2}\,\cos\phi\,\mathbb{1}\,, \quad X^6 = r\sqrt{1-\rho^2}\,\sin\phi\,\mathbb{1}$$

$$X^7 = \frac{2}{N_0} r\rho\, J^1$$

$$X^8 = \frac{2}{N_0} r\rho\, J^2$$

$$X^9 = \frac{2}{N_0} r\rho\, J^3$$

N_0 DIM. REP OF SU(2)

- WITH THIS ANSATZ

$$(X^7)^2 + (X^8)^2 + (X^9)^2 = r^2\rho^2\,\mathbb{1}$$

$$\sum_{i=5}^{9} (X^i)^2 = r^2\,\mathbb{1}$$

- METRIC & DILATON DEPEND ONLY ON r & $r\rho$, THESE ARE MULTIPLES OF IDENTITY MATRIX AND CAN THEREFORE BE REPLACED BY "C" NUMBER.

- SIMILARLY C-S TERM CAN ALSO BE INTERPRETED UNAMBIGUOUSLY.

- LEADING TO LAGRANGIAN:

$$L = -N_0 T_0\, e^{-\phi} \sqrt{g_{tt}} \left[1 - \frac{1}{2} \frac{g_{rr}}{g_{tt}} \dot{r}^2 - \frac{1}{2} \frac{g_{\varphi\varphi}}{g_{tt}} \dot{\varphi}^2 + \frac{2}{N_0^2 \lambda^2} H r^4 \rho^4 \right] + N \dot{\varphi} \rho^3$$

OR HAMILTONIAN (FOR SPECIAL SOLUTIONS)

$$H = \sqrt{g_{tt}} \left[N_0 T_0\, e^{-\phi} + \frac{1}{2 N_0 T_0 e^{-\phi}} \left(\frac{P_\varphi^2}{g_{\phi\phi}} + \frac{P_r^2}{g_{rr}} \right) \right]$$

- THESE PRECISELY AGREE WITH THE NON-RELATIVISTIC LIMIT OF THE D2-BRANE HAMILTONIAN WE GOT PREVIOUSLY.

- THUS ENERGY, RADIUS, etc OBTAINED FROM D0-BRANE & D2-BRANE DESCRIPTIONS AGREE.

GIANT GRAVITONS IN BACKGROUNDS WITH LESS OR NO SUPERSYMMETRY

- CONSIDER $D(6-p)$ BRANE BACKGROUND

$$ds^2 = -g_{tt}\,dt^2 + \sum_{i=1}^{6-p} g_{ii}\,(dx^i)^2 + g_{rr}\,dr^2 + f(r)\,r^2\,d\Omega_{p+2}^2$$

HAS $SO(p+3)$ ROTATIONAL SYMMETRY.

- TAKE $d\Omega_{p+2}^2 = \frac{1}{1-\rho^2}\,d\rho^2 + (1-\rho^2)\,d\varphi^2 + \rho^2\,d\Omega_p^2$

- CONSIDER Dp BRANE WHICH WRAPS AN $S^p \subset S^{p+2}$ AND ALSO ROTATES ON THE S^{p+2}.

- SO FAR NO RESTRICTION TO NEAR-HORIZON OR EXTREMAL BLACK BRANE CASE.

. ANALYSIS OF EQ$^{\underline{N}}$ OF MOTION FOR DP BRANE SIMILAR TO M2 BRANE DISCUSSED EARLIER.

• MAKE ANSATZ : DYNAMICAL VARIABLES

$$r(t), \quad \rho(t), \quad \phi(t)$$

$$H = \sqrt{g_{tt}} \left[\Big\{ \overset{(\text{Mass})^2}{T_p e^{-\phi} V_p \left(f(r) r^2 \rho^2 \right)^{p/2}} \Big\}^2 + \frac{P_r^2}{g_{rr}} + \frac{P_\rho^2}{g_{\rho\rho}} + \frac{\left(P_\varphi - N\rho^{p+1}\right)^2}{g_{\varphi\varphi}} \right]^{1/2}$$

(V_p = VOL. OF UNIT S^p)

. IF AN IMPORTANT CONDITION

$$\boxed{T_p e^{-\phi} V_p \left(f(r) r^2 \right)^{\frac{p+1}{2}} = N(6-p)}$$

IS MET

OF (6-p) BRANES.

THEN HAMILTONIAN CAN BE REWRITTEN:

$$H = \sqrt{g_{tt}} \left[\frac{p_\varphi^2}{f(r) r^2} + \frac{p_r^2}{g_{rr}} + \frac{p_\rho^2}{g_{\rho\rho}} + \frac{(\rho p_\varphi - N\rho^p)^2}{g_{\varphi\varphi}} \right]^{1/2}$$

CHOOSING $\rho^{p-1} = \frac{p_\varphi}{N}$

ALLOWS US TO SET $p_\rho = 0$

GIVING:

$$H = \sqrt{g_{tt}} \left[\frac{p_\varphi^2}{f(r) r^2} + \frac{p_r^2}{g_{rr}} \right]^{1/2}$$

THIS IS EXACTLY THE SAME AS FOR A MASSLESS PARTICLE.

- $0 \leq \rho \leq 1 \Rightarrow$ MAX VAL. OF $p_\varphi = N$

BEYOND THIS EXPANDED BRANE ~~CONFIGURATN~~ SOLUTIONS WHICH BEHAVE LIKE MASSLESS PARTICLES DO NOT EXIST. THIS IS THE ANALOG OF THE STRINGY EXCLUSION PRINCIPLE.

CONDITION:

$$T_p \, e^{-\phi} \, V_p \left(f(r) \, r^2 \right)^{\frac{p+1}{2}} = N_{(6-p)}$$

WHEN IS THIS MET?

I NEAR HORIZON EXTREMAL GEOMETRY ✓

II NEAR HORIZON NON-EXTREMAL GEOMETRY ✓

III FULL (NOT NEAR-HORIZON) GEOMETRY ✗

- SIGNIFICANTLY CONDITION IS MET WHEN NO SUPERSYMMETRY IS PRESERVED BY BACKGROUND.

- SIMILAR STORY FOR M2 & M5 CASES.

- SIGNIFICANCE OF CONDITION UNCLEAR

BUT ONE OBSERVATION:

- DUE TO $e^{-\phi}$ FACTOR MULTIPLYING BI ACTION METRIC SEEN BY D-P BRANE DIFFERS FROM STRING METRIC BY CONFORMAL FACTOR:

$$ds_p^2 = \left(e^{-2\phi}\right)^{\frac{1}{p+1}} \; ds_{string}^2$$

- CONDITION SAYS RADIUS OF (P+2) SPHERE IN THIS METRIC EQUALS NUMBER OF (6-P) BRANES
- FOR EXTREMAL, NEAR HORIZON GEOMETRY IN FACT ds_p^2 IS $AdS \times S^{p+2}$!
- DEPARTING FROM EXTREMALITY IN NEAR HORIZON LIMIT LEAVES RADIUS OF S^{p+2} UNCHANGED.

DISCUSSION:

i) (EVEN) MORE EXAMPLES OF GIANT GRAVITONS: Dp - Dp' SYSTEMS

e.g.: D0-D4 SYSTEM

$$ds^2 = (H_0 H_4)^{-1/2}(-dt^2) + \left(\frac{H_0}{H_4}\right)^{1/2}\underbrace{(dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2)}_{T^4}$$

$$+ (H_4 H_0)^{1/2}(dr^2 + r^2 \underbrace{d\Omega_4^2}_{S^4})$$

$$e^{\phi} = H_0^{3/4} \; H_4^{-1/4}$$

IN NEAR HORIZON LIMIT:

i) D2 BRANE (DUAL TO D4 BRANE) WRAPPING $S^2 \subset S^4$ CAN BEHAVE LIKE MASSLESS PARTICLE

ii) D6-BRANE WRAPPING $T^4 \times S^2$ ALSO BEHAVES LIKE MASSLESS PARTICLE.

WHAT IS THE EXPLANATION FOR GRAVITONS BLOWING UP IN ALL THESE VARIOUS CASES?

ii) POLCHINSKI & STRASSLER FOUND SUGRA DUAL TO DIELECTRIC EFFECT. WHAT IS SUGRA DUAL FOR MAGNETIC EFFECT ?

CAN THE DUAL BE ~~INSTEAD~~ OBTAINED BY IMPROPER BOOST OF THE DIELECTRIC CASE ?

iii) OTHER FUZZY SURFACES ?

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FUZZY COSETS

e.g. $G = SU(3)$

$\frac{SU(3)}{U(2)}$: CP^2 : 4 DIM.

$\frac{SU(3)}{U(1)\times U(1)}$: 6 DIM.

CAN THESE BE OBTAINED CONSISTENTLY EITHER WITH APPROPRIATE SOURCE BRANES OR AS BOUNDARY HOLOGRAMS ?