

QUANTUM GRAVITY IN deSITTER SPACE

(7)

deSITTER (dS) SPACE IS THE
MAXIMALLY SYMMETRIC SPACE WITH
 $\Lambda > 0$

$$ds^2 = -dt^2 + \cosh^2 t \, d\underbrace{\Omega^2}_{\text{sphere}}$$

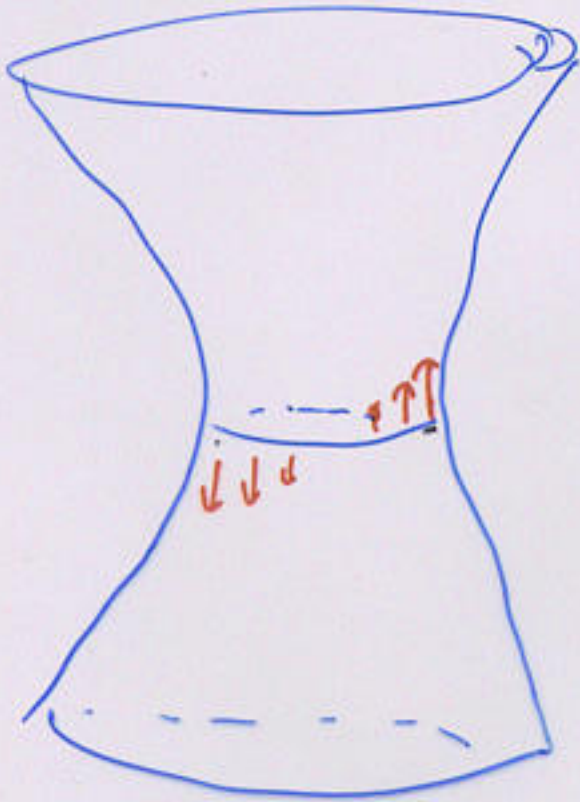
COMPACT SPATIAL SECTIONS, SO

WHEN WE SPEAK OF ASYMPTOTICALLY

dS. SPACE (as we should in the
presence of gravity - the metric fluctuates)

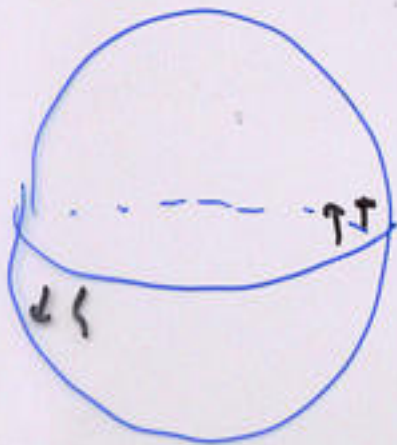
THE ASYMPTOTIA IN QUESTION IS IN
THE PAST AND FUTURE - IN CONTRAST
TO SPATIAL ASYMPTOTIA FOR $\Lambda \leq 0$.

IN dS SPACE THERE IS NO POSITIVE (2)
CONSERVED ENERGY



(WHICH IS WHY
dS SUPERSYMMETRY
IS IMPOSSIBLE)
 $Q^2 = ?$

IN THE EUCLIDEAN VERSION ALL
SYMMETRY GENERATORS ARE ROTATIONS



\Rightarrow THIS LEADS TO A dS
TEMPERATURE

(FIGARI, HOECH-KRON, NAPPI,
1973)

AND ASSOCIATED WITH THIS TEMPERATURE (3)

IS A dS ENTROPY (HAWKING)

LIKE THE BEKENSTEIN-HAWKING ENTROPY OF A BLACK HOLE, THE dS ENTROPY CAN BE WRITTEN

$$S = \frac{A}{4G_N} \quad \text{WHERE } A = \text{HORIZON ENTROPY}$$

HERE, HOWEVER, THIS IS AN OBSERVER-

DEPENDENT ^{HORIZON} ~~ENTROPY~~ AND

THE PHYSICAL MEANING

IS LESS CLEAR.



IT HAS BEEN ARGUED (BANKS AT STRINGS 2000) (4)

THAT IN ASYMPTOTICALLY Q5 SPACE
(with gravity)
THE HILBERT SPACE HAS A FINITE

DIMENSION N , WITH

$$S = \ln N$$

(N DEFINITELY DOES NOT APPEAR TO
BE FINITE IN PERTURBATION THEORY;

HOWEVER PERT. THEORY IS AN EXPANSION

IN $G_N \Lambda^{1/2}$ AND IF THE ABOVE

FORMULA FOR N IS CORRECT,

$$N \sim \exp \left(\frac{1}{G_N^2 \Lambda} \right)$$

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IF THIS IS SO THEN QUANTUM GRAVITY IN dS SPACE IS CHARACTERIZED BY AN INTEGER N , A VERY INTERESTING RESULT THAT HINTS THAT EINSTEIN'S THEORY

$$G_N \Lambda^{1/2}$$

$$\mathcal{L} = -\frac{1}{8\pi G_N} \int d^4x \sqrt{g} R - \Lambda \int d^4x \sqrt{g}$$

CANNOT BE QUANTIZED AND MUST BE DERIVED FROM A MORE FUNDAMENTAL THEORY THAT DETERMINES THE POSSIBLE VALUES OF $G_N \Lambda^{1/2}$.

(A SIMILAR ARGUMENT SEEMS POSSIBLE FOR $\Lambda < 0$ IN ODD DIMENSIONS OF SPACETIME)

WHAT VALUES OF N ARE IN

⑥

FACT POSSIBLE? IF CLASSICALLY

THERE EXISTED COMPACTIFICATIONS TO dS
SPACE (PERHAPS DEPENDING ON DISCRETE
FLUXES TO INTRODUCE AN INTEGER) WE WOULD
HAVE IN THE CLASSICAL LIMIT $N \rightarrow \infty$.

HOWEVER, AN IMPORTANT NO GO THEOREM

(MALDACENA & NUÑEZ) SAYS THAT IN

$D=10$ AND $D=11$ SUPERGRAVITY, THERE

IS NO CLASSICAL COMPACTIFICATION TO

dS SPACE OF ANY DIMENSION.

(SOME SUGGESTIONS BY SILVERSTEIN
ABOUT CIRCUMVENTING IT)

⑦

THE ABSENCE OF A CLASSICAL
LIMIT SUGGESTS THAT THE POSSIBLE
VALUES OF N ARE SPORADIC
AND THAT THERE MIGHT BE ONLY
FINITELY MANY CHOICES ...

IF SO I WOULD NOT EXPECT
IT TO BE POSSIBLE TO GET

$$N > 10^{10^{100}}$$

BUT AS SPACE WITH SUCH
LARGE N APPEARS TO BE THE
MOST OBVIOUS INTERPRETATION OF RECENT
ASTRONOMICAL DATA!

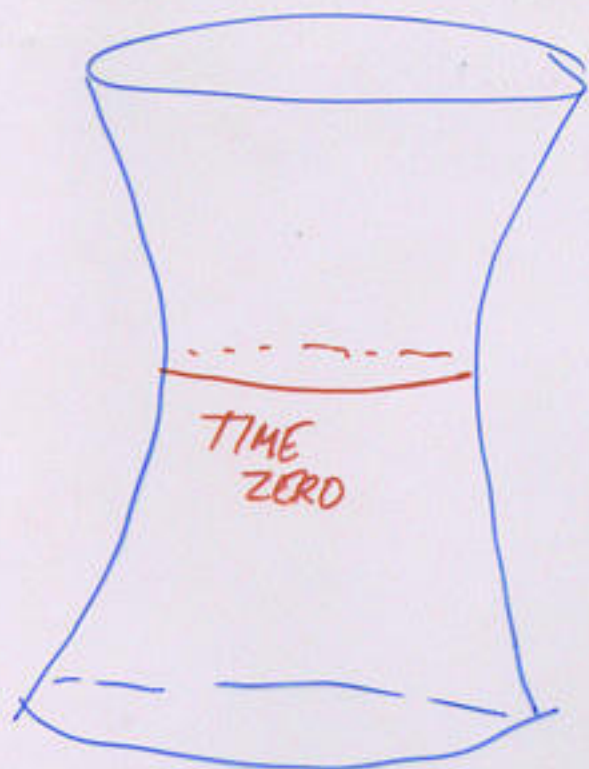
⑧

THE FACT THAT N , THE
DIMENSION OF THE HILBERT SPACE,
IS FINITE MEANS THAT THE dS
GROUP $SO(1, D)$ DOESN'T ACT -
IT HAS NO FINITE DIMENSIONAL
UNITARY REPRESENTATIONS!

THIS MAY SOUND LIKE A
PROBLEM, BUT IN FACT IT ISN'T.

THE dS GROUP DOESN'T ACT
BECAUSE THE SPATIAL SECTIONS OF
 dS SPACE ARE COMPACT,

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IN GENERAL RELATIVITY
THE SPACETIME SYMMETRY
GENERATORS (BEING
GAUGE CHARGES) CAN
BE EXPRESSED AS SURFACE

TERMS AT INFINITY HERE THERE IS NO
(spatial)
INFINITY AND HENCE THE dS

GENERATORS ARE ZERO,

THUS, WHAT I'VE INFORMALLY CALLED

"QUANTUM GRAVITY IN dS SPACE"

IS NOT dS -INVARIANT,

THAT IS THE IMPORTANCE OF THE FACT
THAT THE ONLY ASYMPTOTIA IS IN THE PAST AND FUTURE

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IN PERTURBATION THEORY, THE dS
GROUP IS A GROUP OF CONSTRAINTS,
i.e.

START WITH FREE FIELDS
IN dS SPACE.

SUCH A FREE FIELD
CAN BE QUANTIZED, AND
THOUGH THERE IS NO MINIMUM
ENERGY STATE, THERE IS A ~~MINIMUM~~
DISTINGUISHED dS -INVARIANT STATE,
WHICH ONE MIGHT CALL THE VACUUM, $| \Omega \rangle$

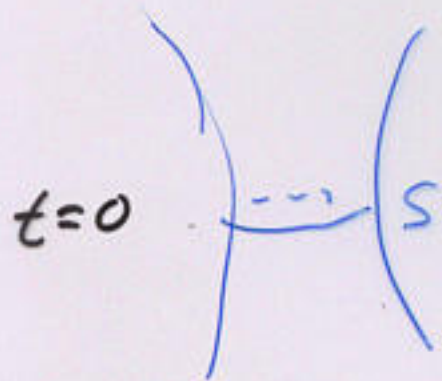


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$|\Omega\rangle$ IS THE UNIQUE
QUANTUM STATE WHICH
IS dS INVARIANT AND

GAUSSIAN, I.E. FOR A

SCALAR FIELD ϕ



$$\Omega(\phi(x)) = \exp - \int_{S \times S} dx dy \phi(x) \phi(y) D(x,y)$$

WHERE D IS UNIQUELY DETERMINED
BY dS INVARIANCE

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ONE CAN CHARACTERIZE (Ω)

BY THE TWO-POINT FUNCTION

$$G = \langle \Omega | \phi(x) \phi(y) | \Omega \rangle$$

WHICH CAN BE CONVENIENTLY

COMPUTED BY ANALYTICAL CONTINUATION

FROM EUCLIDEAN SIGNATURE



$$(\square + m^2)G = \delta$$

THE RESTRICTION

OF G TO S IS

(AS AN OPERATOR)

$$G = D^{-1}$$

OTHER QUANTUM STATES, IN

PERTURBATION THEORY, ARE

POLYNOMIALS TIMES A GAUSSIAN

$$\int dx_1 \dots dx_s \phi(x_1) \dots \phi(x_s) f(x_1, \dots, x_s)$$

$$\exp - \int_{S \times S} D(x, y) \phi(x) \phi(y)$$

IF WE ARE DOING QUANTUM

GRAVITY, WE MUST IMPOSE DS

INVARIANCE ON THE STATES.

THIS GIVES A (SEVERE) RESTRICTION

ON f BUT AS f DEPENDS ON AN

ARBITRARIALLY LARGE NUMBER OF

VARIABLES AND THE S GROUP

IS FINITE DIMENSIONAL, THE

HILBERT SPACE IN PERTURBATION

THEORY APPEARS TO BE INFINITE

DIMENSIONAL. IT IS NOT CLEAR HOW IT
MIGHT TURN OUT TO BE FINITE.

WHAT IS A NATURAL NONPERTURBATIVE

FRAMEWORK FOR CONSTRUCTING A

STATE VECTOR IN \mathcal{H} AND POSSIBLY

PROBING ITS ALLEGED FINITE-DIMENSIONALITY?

HERE I BELIEVE WE SHOULD
USE THE ds ANALOG OF

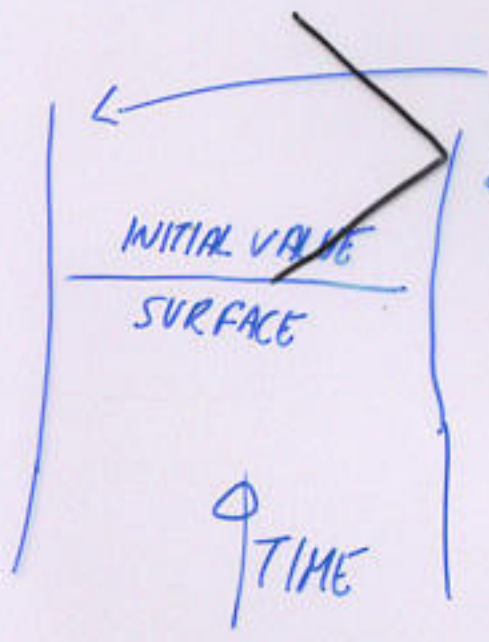
HOLOGRAPHY: SPECIFY INITIAL

CONDITIONS ^{FINAL} IN THE FAR PAST OR FUTURE

(VS. CONDITIONS AT SPATIAL INFINITY

IN THE AdS CASE).

AdS: N



SPATIAL INFINITY

NEAR INFINITY

$$ds^2 = dr^2 + e^{2r} dx^2$$

IN AdS

$$r \rightarrow \infty$$

$(dx)^2 = \text{const metric on } \text{bd'ry}$

TO PROBE THE BOUNDARY

CFT WE GENERALIZE THIS

TO

$$dr^2 + e^{2r} \underbrace{g_{ij} dx^i dx^j}$$

ANY CONFORMAL METRIC
ON THE BOUNDARY.

IN ds

ROUND SPHERE

$$ds^2 = dt^2 + \frac{1}{2} e^{t/2} \overbrace{d\Omega^2}$$

FOR $t \rightarrow \pm\infty$

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TO PREPARE AN INITIAL

OR FINAL STATE $|i\rangle$ OR $|f\rangle$

WE PICK A CONFORMAL

METRIC $g^{(i)}$ OR $g^{(f)}$ ON

THE SPHERE AND ASK THAT

THE SPACETIME METRIC BE

ASYMPTOTIC FOR $t \rightarrow -\infty$ (OR $t \rightarrow +\infty$)

TO ~~the~~ ~~metric~~ ~~is~~



$$dt^2 + e^{\pm 2t} g_{ab}^{(i \text{ OR } f)} dx^a dx^b$$

$$\frac{1}{2} d\Omega^2$$

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THE PATH INTEGRAL FOR



METRICS WITH
THIS

ASYMPTOTIA IN
THE PAST
AND FUTURE

GIVES A N

OBSERVABLE THAT

WE MAY CALL $\langle f | i \rangle$.

THESE ARE THE ONLY
OBSERVABLES I CAN SEE IN
ASYMPTOTICALLY dS SPACETIME

(JUST AS THE S-MATRIX IS THE ONLY EVIDENT OBSERVABLE IN AN ASYMPTOTICALLY LORENTZIAN SPACETIME, AND THE FAMILIAR BOUNDARY CFT OBSERVABLES ARE THE NATURAL OBSERVABLES IN ASYMPTOTICALLY AdS SPACE)

UPON LETTING $|i\rangle$ AND $|f\rangle$ VARY, THE RANK OF THE MATRIX

$$M(f, i) = \langle f | i \rangle \text{ IS THE}$$

DIMENSION OF THE QUANTUM HILBERT SPACE.

$$\sum c_i |i\rangle / \underline{\text{all vectors}}$$

20.5

THIS MAY NOT SEEM
LIKE MUCH, BUT IT
COULD HAVE BEEN WORSE.



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WE CAN ALSO IMAGINE
COSMOLOGY IN A SIMILAR
SPIRIT



ALL WE CAN MEASURE IS

$$\langle f | \Omega \rangle$$

WHERE $|\Omega\rangle$ IS A
DISTINGUISHED STATE OF
THE WORLD

$$|\Omega\rangle \in \mathcal{H}_{ds}$$

IF IT IS THE CASE THAT

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IN STRING THEORY, ^{TO GET} $\Lambda > 0$ BUT

~~THE~~ EXTREMELY SMALL IS IMPOSSIBLE,

I FOR ONE WON'T BE TOO UPSET

BECAUSE I FIND SUCH A UNIVERSE

UNPLEASANT TO CONTEMPLATE.

ASSUMING RECENT EXPERIMENTAL

FINDINGS HOLD UP, I'D HOPE FOR

A DIFFERENT INTERPRETATION OF THEM

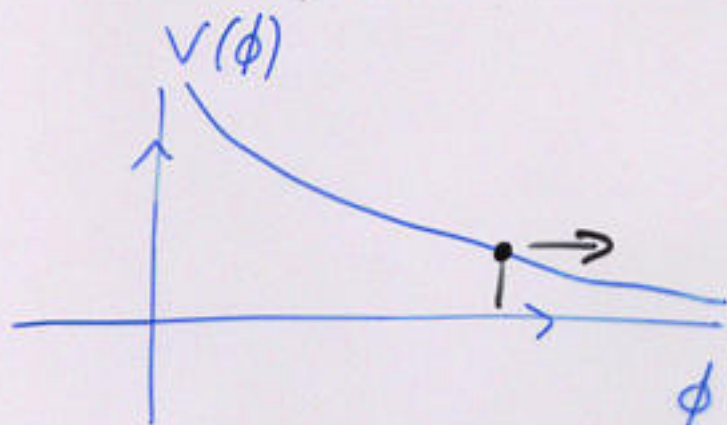
THERE MIGHT BE SOME MORE

RADICAL POSSIBILITIES, BUT TWO

CONSERVATIVE OPTIONS COME TO

MIND AT ONCE:

① RUNAWAY



EXPERIMENTAL CONSTRAINTS

.. EQUIVALENCE PRINCIPLE

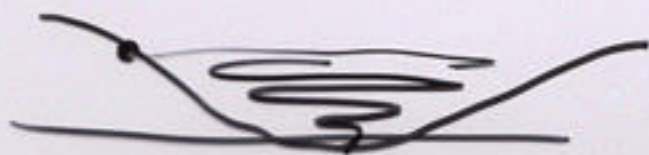
• NATURAL "CONSTANTS"

(but note Antonides)

② AXION

$$V(a) = m_{pl}^4 e^{-I} (\cos a + 1)$$

NEW ELEMENTARY PARTICLE



FINALLY, ... I NOTE THAT

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BANKS HAS PROPOSED THAT

$N =$ NUMBER OF QUANTUM STATES

CAN BE CHOSEN AS AN INPUT AND

THAT THE EFFECTIVE COUPLINGS

THEN ADJUST, VIA "GIANT GRAVITONS",

TO PRODUCE A COSMOLOGICAL CONSTANT

OF THE RIGHT MAGNITUDE FOR THAT N .

I DON'T HAVE ANY INTUITION ABOUT THIS

PROPOSAL AND WON'T COMMENT ON IT,

EXCEPT TO SAY THAT IF SO, I HOPE

THE GIANT GRAVITONS HAVE GONE ONE

STEP FARTHER TO PRODUCE A UNIQUE

SIGNATURE,