Tachyon models and string field theory around the tachyon vacuum

– Strings 2001 –

B. Zwiebach

D-branes are made of tachyons !

(Except possibly for top dimensional D-branes)

String Field Theory is a nonperturbative formulation of string theory

OSFT, at least. CSFT is also likely to be nonperturbative.

We now know the open string tachyon potentials !

(Classical results in a field basis where only the tachyon needs to condense. Simple models confirmed by B-SFT.)

Do we have the SFT around the vacuum of the tachyon ?

This talk will have two main topics:

- Simple field theory models for tachyon dynamics in bosonic strings and in superstring theory. Coupling to gauge fields.

- Motivation and discussion of a proposal for an SFT describing the physics around the vacuum of the tachyon.

The first topic is interesting in that such models, derived by special limits of solvable models, turned out to give the **exact** (classical) tachyon potentials, as confirmed by B-SFT calculations.

The second topic is interesting in that such SFT could be expected to be more powerful and to possess more symmetries than the usual formulation of cubic string field theory.

Work on tachyon models is in collaboration with **Joseph Minahan**.

Work on SFT around the tachyon vacuum is in collaboration with **Leonardo Rastelli** and **Ashoke Sen**.

A good question concerning scalar field theory:

Can you find a scalar potential $V(\phi)$ such that

- At the maximum it has a tachyon of $M^2 = -1$
- At a minimum the tachyon acquires infinite mass.
- The field theory has a codimension one lump solution. On the lump world volume there is a tachyon, a massless scalar and a scalar for each mass level $M^2 = 1, 2, \cdots, \infty$.

The surprise is that the answer to this question (and an analogous one for superstrings) gives a simple potential that happens to be the exact tachyon potential for the bosonic string (and for the superstring).

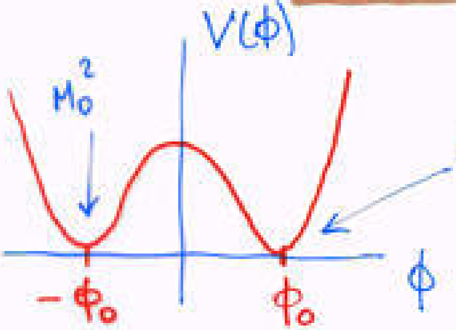
Given a scalar field theory

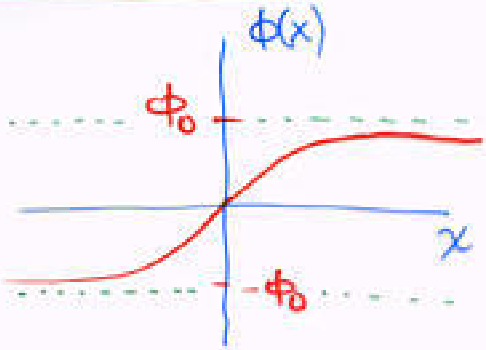
$$S = -\int d^d y dx \left(\frac{1}{2}(\partial\phi)^2 + V(\phi)\right)$$

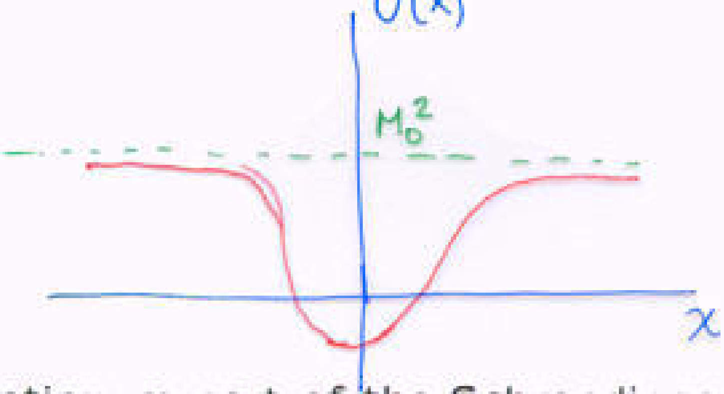
$\overline{\phi}(x)$ is a lump/kink solution along x

The mass spectrum of fields living on the lump is controlled by a Schroedinger potential

$$U(x) = V''(\overline{\phi}(x)) = M^2_{eff}(x)$$







The continuum part of the Schroedinger spectrum tells us about the vacuum!

An interesting class of Schroedinger potentials:

$$H_\ell = -\frac{d^2}{du^2} + U_\ell(x)\,, \quad U_\ell(x) = \ell^2 - \ell(\ell+1)\text{sech}^2 u$$

x x

$u \to x$

Energy spectrum:

$$E_n(\ell) = \ell^2 - (\ell - n)^2\,, \quad n = 0, 1, \cdots \ell - 1.$$

$$E_n(\ell) = 0, 2\ell - 1, \cdots, \ell^2 - 1\,.$$

Continuum spectrum sets in at $n = \ell$:

$$E_{cont}(\ell) \geq \ell^2$$

Solved recursively, with H_0 the free particle hamiltonian using

$$a_\ell = \ell \tanh u + \frac{d}{du}, \quad a_\ell^\dagger = \ell \tanh u - \frac{d}{du}.$$

x x x x

$$\rightarrow H_\ell = a_\ell^\dagger a_\ell \quad H_{\ell-1} = a_\ell a_\ell^\dagger - (2\ell - 1)$$

For $V(\phi) = \frac{1}{3}\phi^3 - \frac{1}{2}\phi^2 + \frac{1}{6}$ the Schroedinger potential is $U_3(x/2)$ up to an additive constant.

$$x = Ax' \quad \text{then} \quad U'(x') = A^2 U(Ax'), \quad E'_n = A^2 E_n$$

Choose $A = 1/\sqrt{2\ell}$

$$\lim_{\ell\to\infty} \frac{1}{2\ell} U_\ell\left(\frac{x'}{\sqrt{2\ell}}\right) = \frac{1}{4}(-2 + x'^2), \quad \lim_{\ell\to\infty} \frac{1}{2\ell} E_n(\ell) = n = 0, 1, \cdots \infty$$

Get the SHO !

Bosonic model

$$\overline{\phi}(x) = \mathcal{L}(x^2) \quad \text{Lump profile}$$

$$\phi = \mathcal{L}(T), \quad \overline{T} = x^2, \quad \text{new field variable}$$

$$S = -\int dtdx (\mathcal{L}'(T))^2 \Big(\frac{1}{2}(\partial T)^2 + 2T\Big) \quad \text{Action}$$

$$\mathcal{L}_\ell(x^2) \sim \text{sech}^{\ell-1}(x)\,, \quad \mathcal{L}'_\infty(T) \sim \exp(-T/4)$$

$$S_\infty = -\int dtdx\; e^{-T/2}\Big(\frac{1}{2}(\partial T)^2 + 2T\Big)$$

In the original field variables:

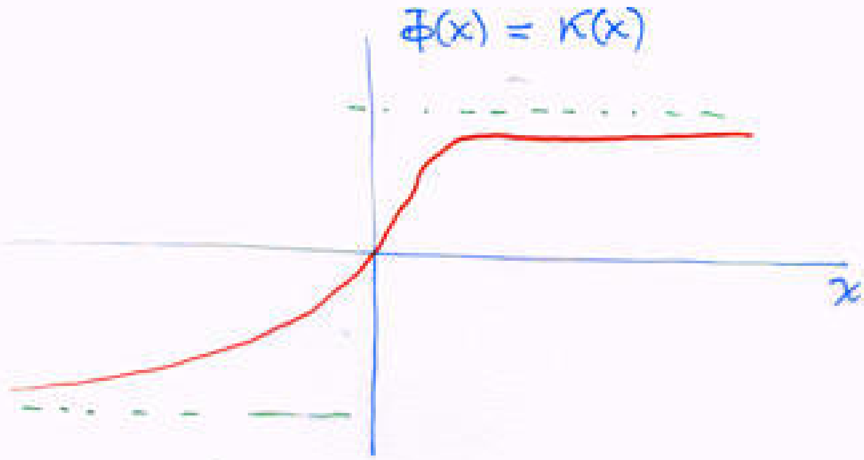
$$S_\infty(\phi) = -\int dtdx\; \Big(\frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\phi^2 \ln(\phi^2)\Big)$$

This model (Minahan + BZ, 0008231) was found to describe the exact potential in B-SFT (Gerasimov, Shatashvili 0009103, Kutasov, Marino, Moore 0009148).

The original model has a tachyon of $M^2 = -1$ and the lump has a spectrum $M^2 = -1, 0, 1, 2, \cdots \infty$. At the tachyon vacuum the tachyon acquires infinite mass (it is strongly coupled –Kleban, Lawrence, Shenker, 0012081).

Superstring Model

$$\overline{\phi}(x) = \mathcal{K}(x) \quad \text{kink profile}$$



$$\frac{1}{2}(\overline{\phi}'(x))^2 = V(\overline{\phi}(x))$$

Let $\phi = \mathcal{K}(T)$, $\overline{T} = x$ new field variable

$$V(\overline{\phi}(x)) = \frac{1}{2}(\mathcal{K}'(x))^2 = \frac{1}{2}(\mathcal{K}'(\mathcal{K}^{-1}(\overline{\phi})))^2 = \frac{1}{2}(\mathcal{K}'(\overline{T}))^2$$

$$S = -\int dt dx (\mathcal{K}'(T))^2 \Big((\partial T)^2 + 1\Big) \quad \text{Action}$$

$$\mathcal{K}_\ell(x) = \sqrt{T}\,\mathrm{sech}^\ell(x)\,, \quad \mathcal{K}'_\infty(T) = \sqrt{T}\exp(-T^2/4)$$

$$S_\infty = -\int dt dx\; e^{-T^2/2}\Big((\partial T)^2 + 1\Big)$$

This model (Minahan + BZ, 0009246) was argued to describe the exact potential in B-SFT (Kutasov, Marino, Moore, 0010108).

Tachyon fluctuations

$$S_{quad} = -\int dt d^p y dx \Big\{ (\partial_\mu \hat{T})^2 + \hat{T}\Big(-\frac{\partial^2}{\partial x^2} + \frac{\mathcal{K}'''(x)}{\mathcal{K}'(x)}\Big)\hat{T}\Big\}$$

$$\frac{\mathcal{K}_\ell'''(x)}{\mathcal{K}_\ell'(x)} = U_\ell(x)$$

$$\frac{\mathcal{K}_\infty'''(x)}{\mathcal{K}_\infty'(x)} = \frac{1}{4}x^2 - \frac{1}{2}$$

In this latter case $M_T^2 = -1/2$ and $M_n^2 = 0, 1, 2, \cdots$.
This is good!

Coupling to gauge fields

Motivated by: $V(T)\sqrt{-\det(\eta_{\mu\nu}+F_{\mu\nu})}$ (Sen, 9909062)

B-SFT analysis of Cornalba (0010021) Okuyama (0010028), Gerasimov Shatashvili (0011009), Tseytlin (0011033).

We propose (Minahan, BZ, 0011226)

$$S = -\int dt d^{p+1}x (\mathcal{K}'(T))^2\Big((\partial T)^2 + 1 + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Big) \quad \text{Action}$$

Note that at the tachyon vacuum $T=\infty$ the gauge kinetic term prefactor $\mathcal{K}'_\ell(\infty)\to 0$ (for any ℓ).

What lives on the lump ? $x^\mu = (x^{\bar{\mu}}, x)$

$$A_x = 0\,, \quad \text{Gauge condition}$$

$$\partial_x(\partial_{\bar{\mu}}A^{\bar{\mu}}) = 0\,, \quad \text{Subsidiary condition}$$

Go to the lump $\overline{T}=x$ and then redefine

$$B_{\bar{\mu}} = \mathcal{K}'(x)A_{\bar{\mu}}, \quad \widetilde{F} = \mathrm{d}B$$

$$S_{quad} = -\int dt d^p y dx \Big\{\frac{1}{4}(\widetilde{F})^2 + \frac{1}{2}B_{\bar{\mu}}\Big(-\frac{\partial^2}{\partial x^2} + \frac{\mathcal{K}'''(x)}{\mathcal{K}'(x)}\Big)B^{\bar{\mu}}\Big\}$$

Schroedinger potential is the same as the tachyon's !

We get a massless gauge field, and massive gauge fields, with a **continuum** if ℓ is finite.

For finite ℓ despite the vanishing of the prefactor of F^2 we see evidence of conventional massive gauge field states on the tachyon vacuum. Those states would become infinitely heavy as $\ell \to \infty$.

This, in turn is evidence that in the $\ell = \infty$ model there are no conventional gauge field excitations on the tachyon vacuum.

Additional couplings/models

- Can couple fermions that will localize correctly at the lump

$$S = -\int d^{10}x (\mathcal{K}'(T))^2 \left[\frac{i}{2}\bar{\psi}\Gamma^\mu \overleftrightarrow{\partial}_\mu \psi - \frac{\mathcal{K}''(T)}{\mathcal{K}'(T)} \bar{\psi}\psi \right],$$

- The generalized Born-Infeld forms (Garousi 0003122, Bergshoeff, Roo, deWit, Eyras, Panda 0003221)

$$S = -\int d^{p+2}x \ \exp(-T^2/2)\sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu} + 2\partial_\mu T \partial_\nu T)}$$

Get $\overline{T} = ux$, with $u \to \infty$. The spacing of the gauge field fluctuations is half the expected one.

- There are two possible couplings of gauge fields in the bosonic model

$$S = -\int d^{p+2}x (\mathcal{L}'(T))^2 \left(\frac{1}{2}(\partial T)^2 + 2T + \begin{Bmatrix} 2T \\ 1 \end{Bmatrix} \frac{1}{4} F_{\mu\nu}F^{\mu\nu} \right),$$

While top model is more BI type, and is solvable for finite ℓ the gauge field spacing is twice the expected one. The lower model is not solvable for finite ℓ but the gauge field spacing is correct.

There are very recent works on super-connections (combine the gauge field and tachyon).

Consider the p-adic string model (Freund, Olson, Witten 1987, ...) revisited by Ghoshal and Sen (0003278) for a study of tachyon condensation

$$S = \frac{1}{g^2}\frac{p^2}{p-1}\int d^dx\Big(-\frac{1}{2}\phi e^{-\frac{1}{2}\ln p \frac{\partial^2}{\partial x^2}}\phi + \frac{1}{p+1}\phi^{p+1}\Big)$$

G. M. Hardy and E. M. Wright *An Introduction to the theory of numbers* : " A number p is said to be *prime* if (i) $p > 1$, (ii) p has no positive divisors except 1 and p.

Let $p = 1 + \epsilon$, and $\epsilon \to 0$ (Gerasimov, Shatashvili)

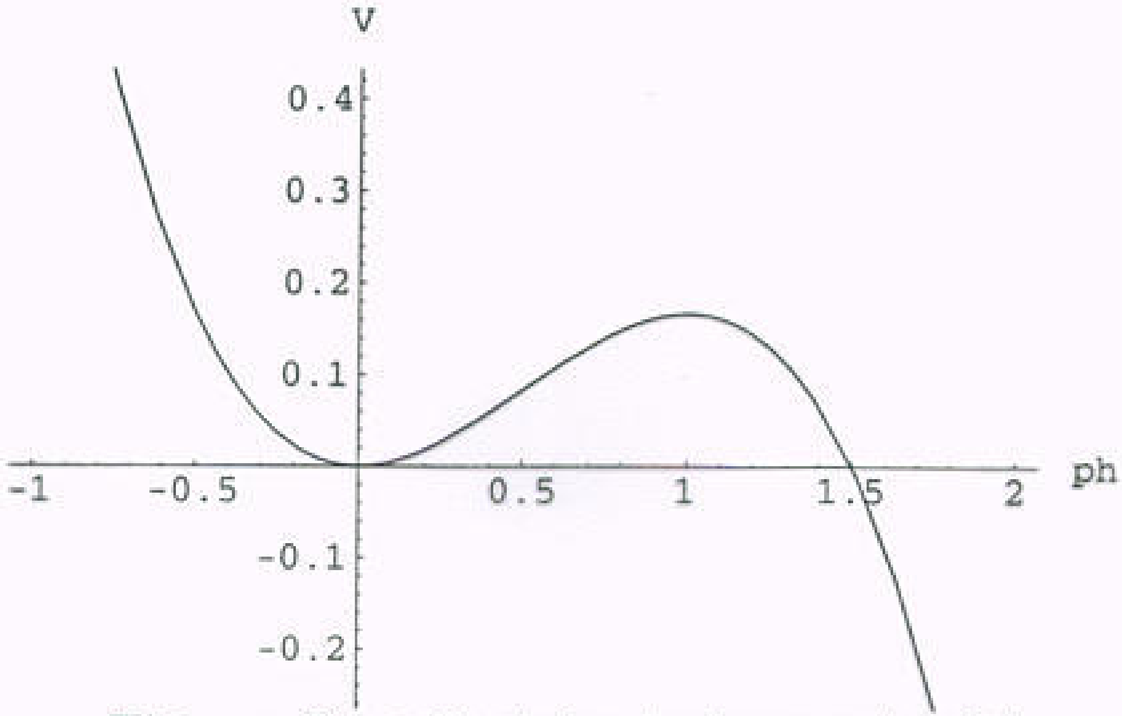
$$S = \frac{1}{g^2}\int d^dx\Big(\frac{1}{4}\phi\frac{\partial^2}{\partial x^2}\phi + \frac{1}{4}\phi^2(-1+\ln\phi^2)\Big)$$

This is the $\ell = \infty$ model

Consider now $p = 2$ giving

$$S = \frac{4}{g^2}\int d^dx\Big(-\frac{1}{2}\phi e^{-\frac{1}{2}\ln 2 \frac{\partial^2}{\partial x^2}}\phi + \frac{1}{3}\phi^3\Big)$$

$$\to \quad V(\phi) = \frac{1}{2}\phi^2 - \frac{1}{3}\phi^3$$



The $p = 2$ p-adic string tachyon potential

Redefine the tachyon $\phi \to e^{\frac{1}{4}\ln 2 \frac{\partial^2}{\partial x^2}} \phi$ and get

$$S = \frac{4}{g^2} \int d^d x \Big(-\frac{1}{2} \phi \phi + \frac{1}{3} (e^{\frac{1}{4}\ln 2 \frac{\partial^2}{\partial x^2}} \phi)^3 \Big)$$

Note the kinetic term is now regular and the interactions have a momentum dependence quite similar to that of cubic open string field theory.

String Field Theory around the vacuum of the Tachyon

Rastelli, Sen and Zwiebach (0012251)

There are two aims for this study

- Confirm the complete dissappearance of perturbative open string excitations on the tachyon vacuum – This is the second aspect of Sen's tachyon conjectures.
- Find a more flexible formulation of open string field theory.

It is natural to begin this analysis in cubic open string field theory

$$S(\Phi) = -\frac{1}{g_o^2}\left[\frac{1}{2}\langle\Phi\,,\,Q_B\,\Phi\rangle + \frac{1}{3}\langle\Phi\,,\,\Phi * \Phi\rangle\right], \qquad (1)$$

As opposed to B-SFT:

- we have here all open string fields, and thus we can ask what is their fate (inclusion of massive fields in B-SFT is not yet under control).
- there is no evidence of singularities of the field variables at the tachyon vacuum.

This action is consistent on account of:

$$Q_B^2 = 0,$$
$$Q_B(A * B) = (Q_B A) * B + (-1)^A A * (Q_B B),$$
$$\langle Q_B A, B \rangle = -(-)^A \langle A, Q_B B \rangle,$$

$$\langle A, B \rangle = (-)^{AB} \langle B, A \rangle$$
$$\langle A, B * C \rangle = \langle A * B, C \rangle$$
$$A * (B * C) = (A * B) * C.$$

Gauge Tr: $\delta\Phi = Q_B \Lambda + \Phi * \Lambda - \Lambda * \Phi,$

Let Φ_0 describe the tachyon vacuum:

$$Q_B \Phi_0 + \Phi_0 * \Phi_0 = 0.$$

Expand around the tachyon vacuum: $\Phi = \Phi_0 + \widetilde{\Phi}$, and up to a constant (representing the brane mass) get

$$S_0(\widetilde{\Phi}) \equiv -\frac{1}{g_o^2} \left[\frac{1}{2} \langle \widetilde{\Phi}, Q \widetilde{\Phi} \rangle + \frac{1}{3} \langle \widetilde{\Phi}, \widetilde{\Phi} * \widetilde{\Phi} \rangle \right].$$

where

$$Q\widetilde{\Phi} = Q_B \widetilde{\Phi} + \Phi_0 * \widetilde{\Phi} + \widetilde{\Phi} * \Phi_0.$$

$\rightarrow Q$ satisfies the same properties as Q_B.

A closed form expression for Φ_0 is unknown and a field redefinition may be necessary to simplify the kinetic term. For example:

$$\widetilde{\Phi} = e^{K}\,\Psi\,,$$

with

$$\begin{aligned} &K(A * B) = (KA) * B + A * (KB)\,, \\ &\langle KA, B \rangle = -\langle A, KB \rangle\,. \end{aligned}$$

would preserve the cubic term and have

$$Q \to \mathcal{Q} = e^{-K} Q e^{K}$$

This suggests that if a simple form exists for the SFT action around the tachyon vacuum it might be *easier to guess it than to derive it*.

Thus, propose:

$$S(\Psi) \equiv -\frac{1}{g_0^2}\left[\frac{1}{2}\langle \Psi , \mathcal{Q}\Psi\rangle + \frac{1}{3}\langle \Psi , \Psi * \Psi\rangle\right],$$

where $\mathcal{Q}$ will be required to satisfy:

- Gauge invariance conditions

$$\mathcal{Q}^2 = 0,$$
$$\mathcal{Q}(A * B) = (\mathcal{Q}A) * B + (-1)^A A * (\mathcal{Q}B),$$
$$\langle \mathcal{Q}A, B \rangle = -(-)^A \langle A, \mathcal{Q}B\rangle .$$

- $\mathcal{Q}$ must have vanishing cohomology.

- $\mathcal{Q}$ must be universal.

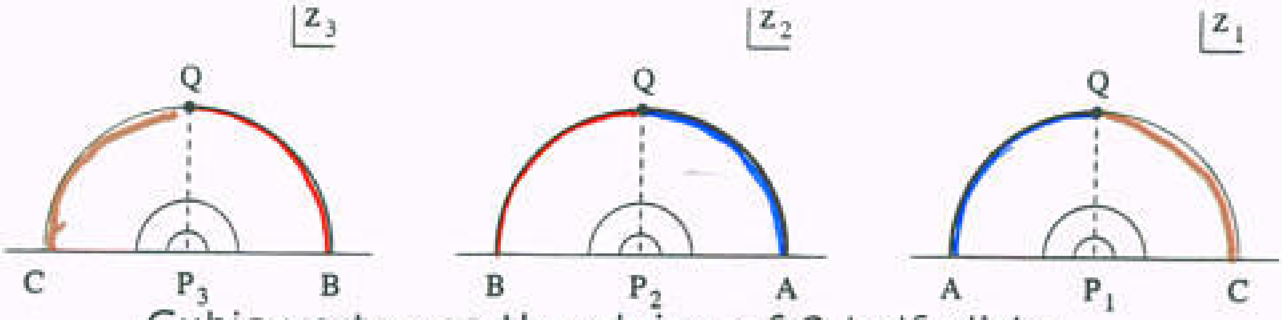
The simplest possibility would be to set $\mathcal{Q} = 0$ giving purely cubic SFT (Horowitz, *et.al* 1986). Very problematic!

Next simplest choice is $\mathcal{Q} = c_0$, the zero mode of the ghost field $c(z)$.

It has ghost number zero, it is BPZ odd, it squares to zero, and it has no cohomology since $\{c_0, b_0\} = 1$:

$$c_0|\Phi\rangle = 0 \to |\Phi\rangle = c_0 b_0|\Phi\rangle = \mathcal{Q}(b_0|\Phi\rangle)$$

Also, c_0 is universal.



Cubic vertex as the gluing of 3 half–disks.

$$\begin{aligned} z_1 z_2 &= -1\,, && \text{for } |z_1| = 1,\ \Re z_1 \leq 0 \\ z_2 z_3 &= -1\,, && \text{for } |z_2| = 1,\ \Re z_2 \leq 0 \\ z_3 z_1 &= -1\,, && \text{for } |z_3| = 1,\ \Re z_3 \leq 0 \end{aligned}$$

The common interaction point P, $z_i = i$ (for $i = 1, 2, 3$) is the mid–point of each open string $|z_i| = 1$, $\Im z_i \geq 0$.

There is a globally defined (Jenkins-Strebel) quadratic differential:

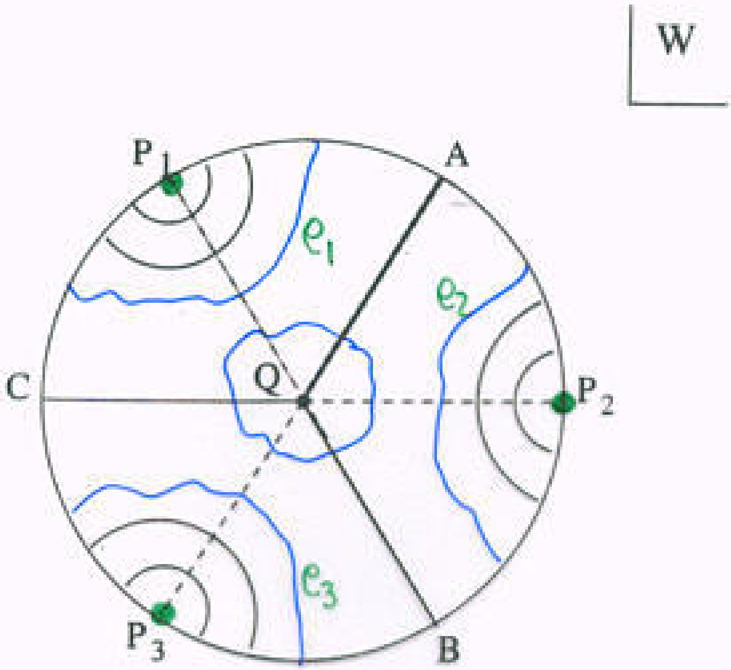
$$\varphi = \phi(z_i) dz_i^2 = -\frac{1}{z_i^2}\, dz_i^2\,, \quad i = 1, 2, 3.$$

φ has second order poles at the punctures $z_i = 0$.

The lines along which φ is real and positive (horizontal trajectories) cover the surface and represent the open strings.

φ has a first order zero at the interaction point P $(z_i = i)$.

The metric $ds^2 = |\phi(z_i)||dz|^2$ turns the three half disks into three semi-infinite strips of width π.



Conservation law (c primary of dimension -1 and φ dimension $+2$).

$$\langle V_3| \sum_{i=1}^{3} \oint_{C_i} dz_i \, c^{(i)}(z_i)\phi^{(i)}(z_i) = 0\,.$$

With $c(z_i) = c_0 z_i + \cdots$ and with $\phi^{(i)} \sim -1/z_i^2$

$$\langle V_3| \left(c_0^{(1)} + c_0^{(1)} + c_0^{(1)}\right) = 0\,.$$

$$\rightarrow c_0(A * B) = (c_0 A) * B + (-)^A A * (c_0 B)$$

Thus $\mathcal{Q} = c_0$ satisfies all requisite properties !

We conclude that:

$$S(\Psi) \equiv -\frac{1}{g_0^2}\left[\frac{1}{2}\langle \Psi, c_0 \Psi \rangle + \frac{1}{3}\langle \Psi, \Psi * \Psi \rangle\right],$$

Is a good SFT, with gauge invariance, and correct spectrum !

In fact, the operators

$$\mathcal{C}_n \equiv c_n + (-)^n c_{-n}, \quad n = 0, 1, 2, \cdots$$

are also possible choices for $\mathcal{Q}$.

General choice:

$$\mathcal{Q} = \sum_{n=0}^{\infty} a_n \mathcal{C}_n, \quad a_n \,\text{constant}$$

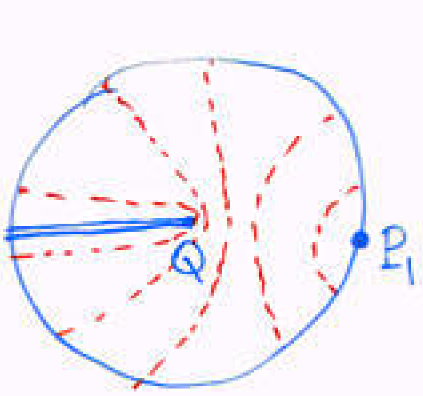
Field redefinitions relate most. With
$K_n = L_n - (-)^n L_{-n}$

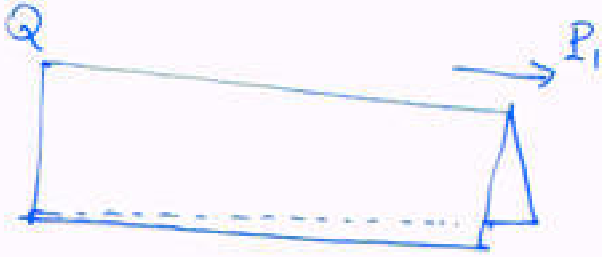
$$[K_n, \mathcal{C}_m] = -(2n+m)\mathcal{C}_{m+n} + (-1)^{n+1}(2n-m)\mathcal{C}_{m-n}$$

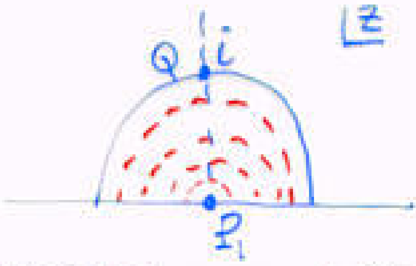
A subclass of the above kinetic operators were considered by Horowitz *et.al.* as solutions that emerge from purely cubic SFT. They noted the absence of excitations.

Identity string field $\mathcal{I}$ satisfies $A * \mathcal{I} = \mathcal{I} * A = A$

$$|\mathcal{I}\rangle = \exp(L_{-2} - \frac{1}{2}L_{-4} + \frac{1}{2}L_{-6} - \frac{7}{12}L_{-8} + \cdots)|0\rangle \,.$$







The quadratic differential $\varphi = -dz^2/z^2$ actually has a pole at $z = i$ and therefore

$$\langle \mathcal{I}| \oint c(z)/z^2 dz \neq 0, \quad c_0|\mathcal{I}\rangle \neq 0$$

Surprising since derivations should annihilate the identity. On the other hand

$$\Big(c_0 + \frac{1}{2}(c_2 + c_{-2})\Big)|\mathcal{I}\rangle = 0$$

So expect c_0 and $c_0 + \frac{1}{2}(c_2 + c_{-2})$ not to be related by (regular) field redefinitions.

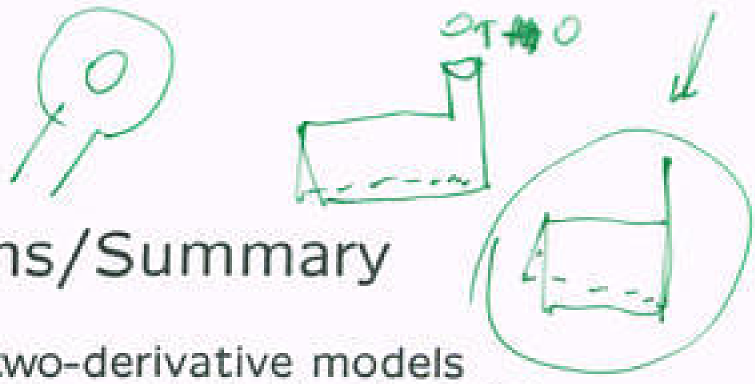
Some observations/questions

- Must find a way to normalize the kinetic term. **Is** the kinetic operator uniquely determined ?

- With purely ghost kinetic operator, one can find a simple enough propagator and the matter dependence of n-tachyon Green's functions is calculable exactly. This calculation is essentially impossible in standard cubic OSFT.

- Can study the existence of lump solutions, and their tensions. This is a nontrivial analysis based on the above n-tachyon Green's functions and some analyticity assumptions. It gives strong evidence for the type of action, **as will be explained by Ashoke Sen**

$$Q\phi_0 + \phi_0 * \phi_0 = 0$$



Conclusions/Summary

- Tachyon models are two-derivative models obtained as special limits of field theories with potentials having solitons with solvable spectrum. They anticipated the exact tachyon potentials of B-SFT, and may provide a framework to study how the gauge field acquires a mass (infinite ?) on the tachyon vacuum.

- Upon lump localization, the degeneracies of massive fields arising from the tachyon and gauge fields in the models are consistent with the spectrum of D-branes.

- Tachyon models give a natural explanation for the apparent finite range of marginal parameters in SFT (CFT marg. → SFT marg. is two to one).

- We have proposed a very simple class of SFT actions that may describe the physics of the tachyon vacuum.

- The actions, reminiscent of p-adic string models, implement manifestly the lack of conventional open string excitations and gauge invariance.

- The simplicity of the n-tachyon Green's functions in this theory allows a nontrivial test via the computation of ratios of brane tensions.