Dimensional Deconstruction and Physics in Theory Space

Nima Arkani-Hamed

Strings 2002

Higher Dimensions have offered two important ingredients for extending the Standard Model:

(A) Higher dimensional local symmetries

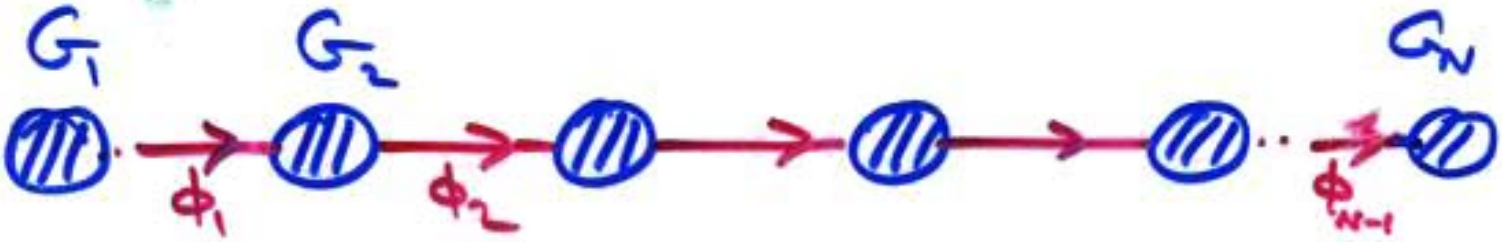
e.g. 5D gauge symmetry

5D gen. covariance

(B) Locality as a tool to suppress unwanted interactions exponentially

e.g. $Q_1 \leftarrow d \rightarrow Q_2 \rightarrow O, O_2 e^{-Md}$

These features can be abstracted away from extra dim, into purely 4D models, which can further generalize in interesting but entirely non-extra dimensional ways.



"Mooses" (Dimopoulos–Susskind, Georgi)

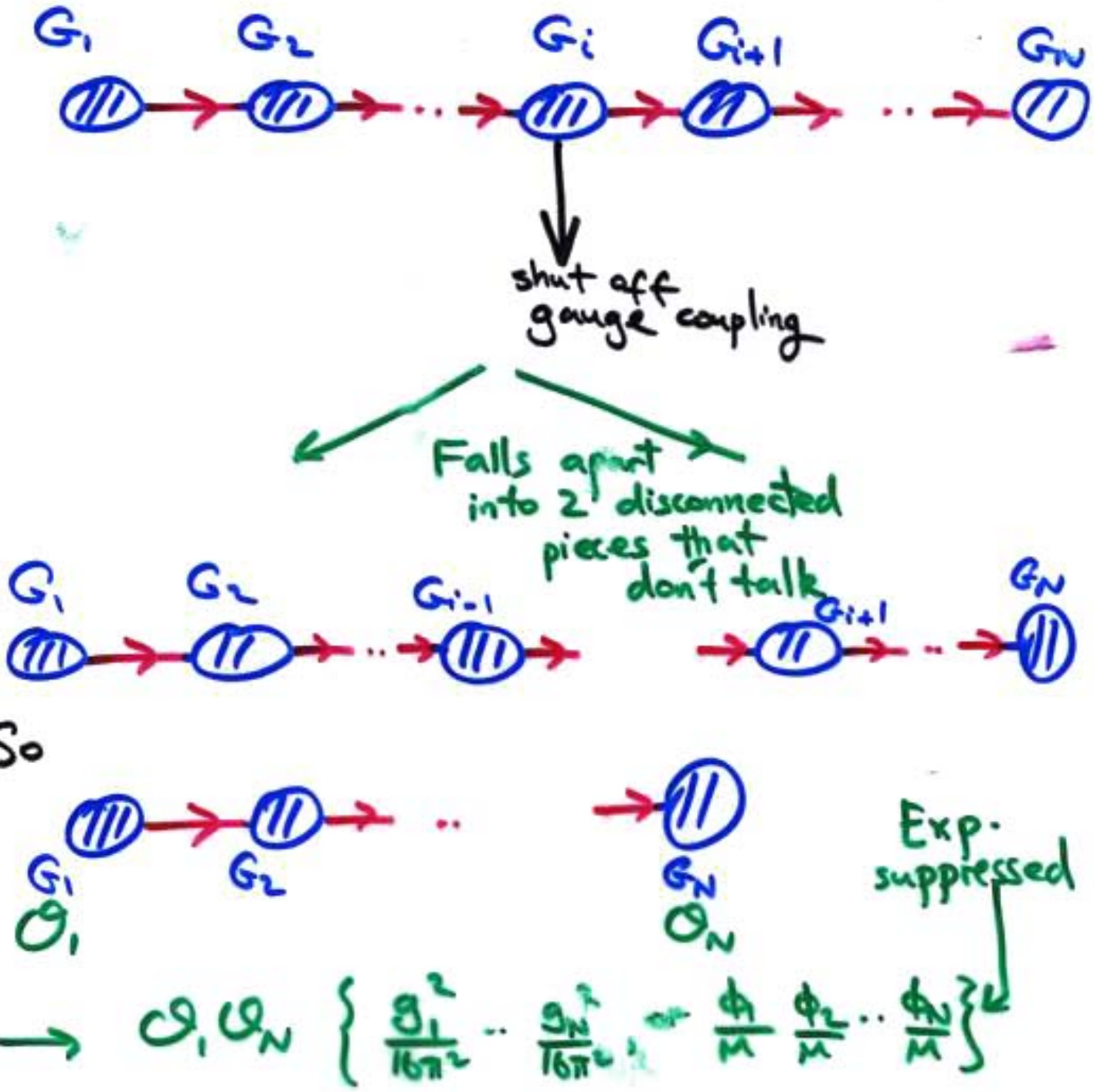
or

"Quivers" (Douglas + Moore)

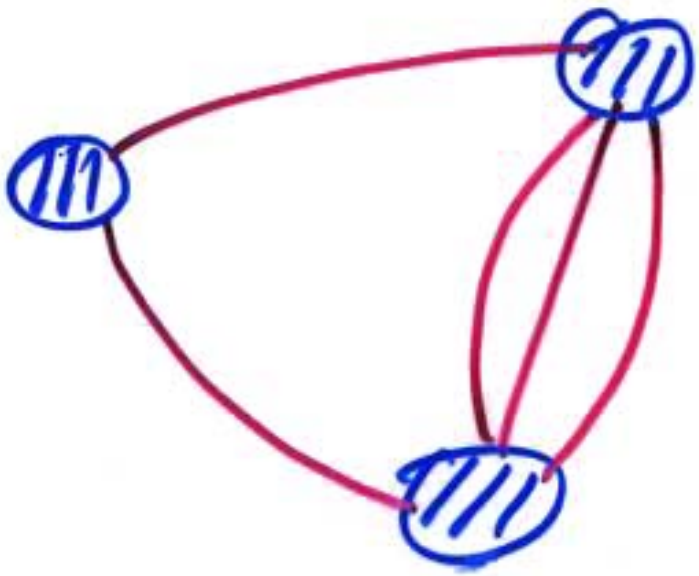
Exemplify this in a simple way

(A) Big gauge symmetry $G_1 \times \cdots \times G_N$

(B) Locality

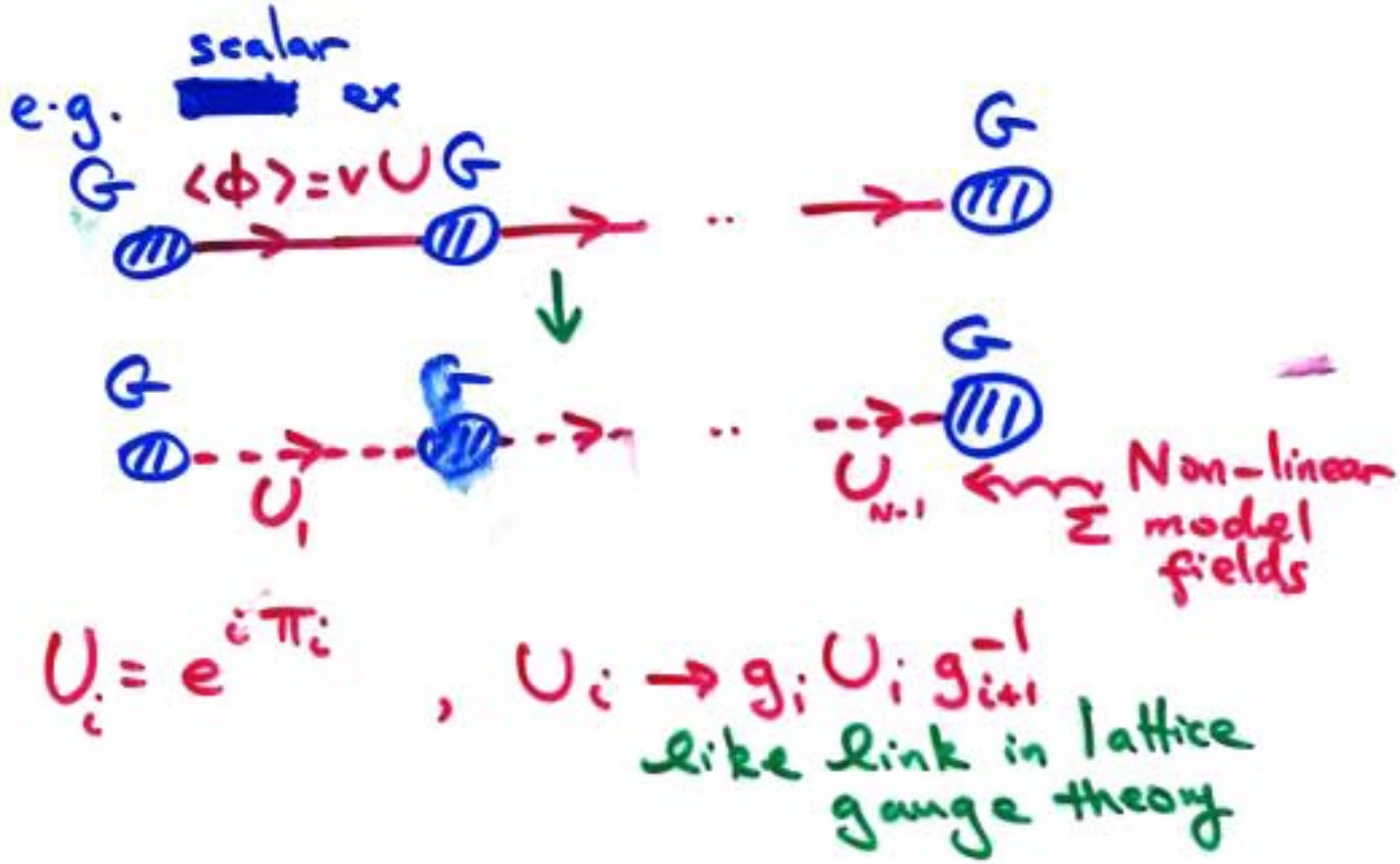


Motivates thinking about theories with a general site – link structure



"Theory Space"

In some cases, when link fields are in a Higgs phase, theory spaces transmute into discretized extra dimensions



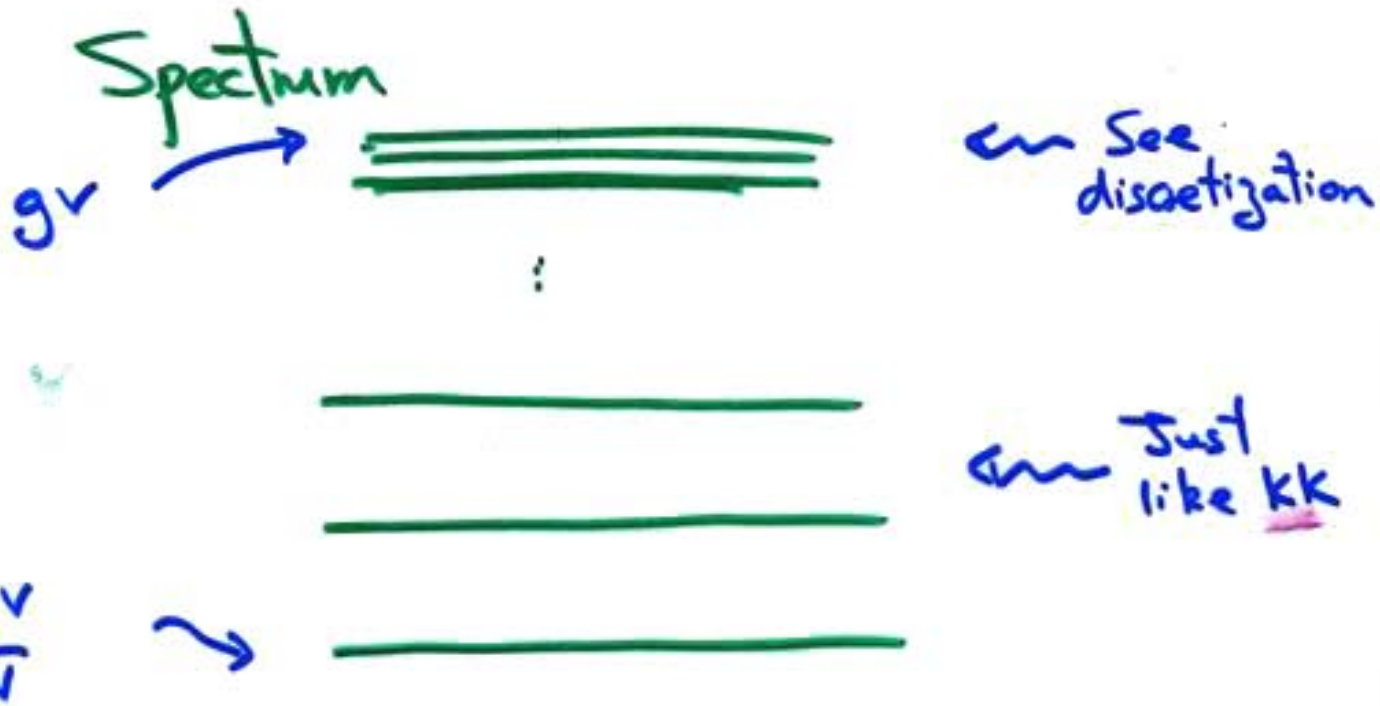
$U_i = e^{i\pi_i}$, $U_i \to g_i U_i g_{i+1}^{-1}$

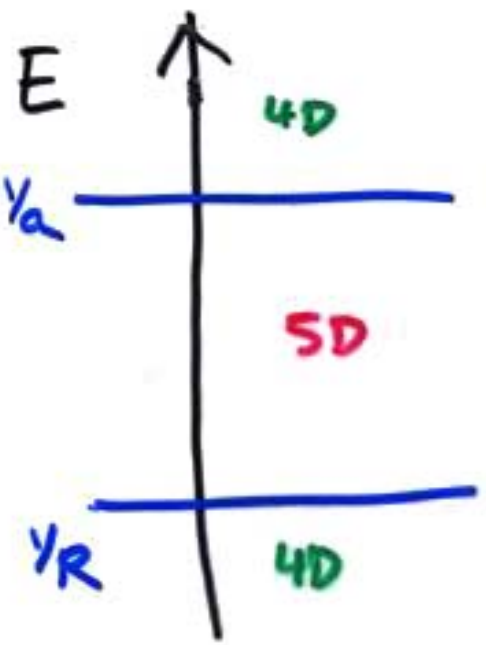
like link in lattice gauge theory

⇒ Discretized 5'th dim

. U's Higgs $G^N \to G$,

N-1 massive gauge bosons





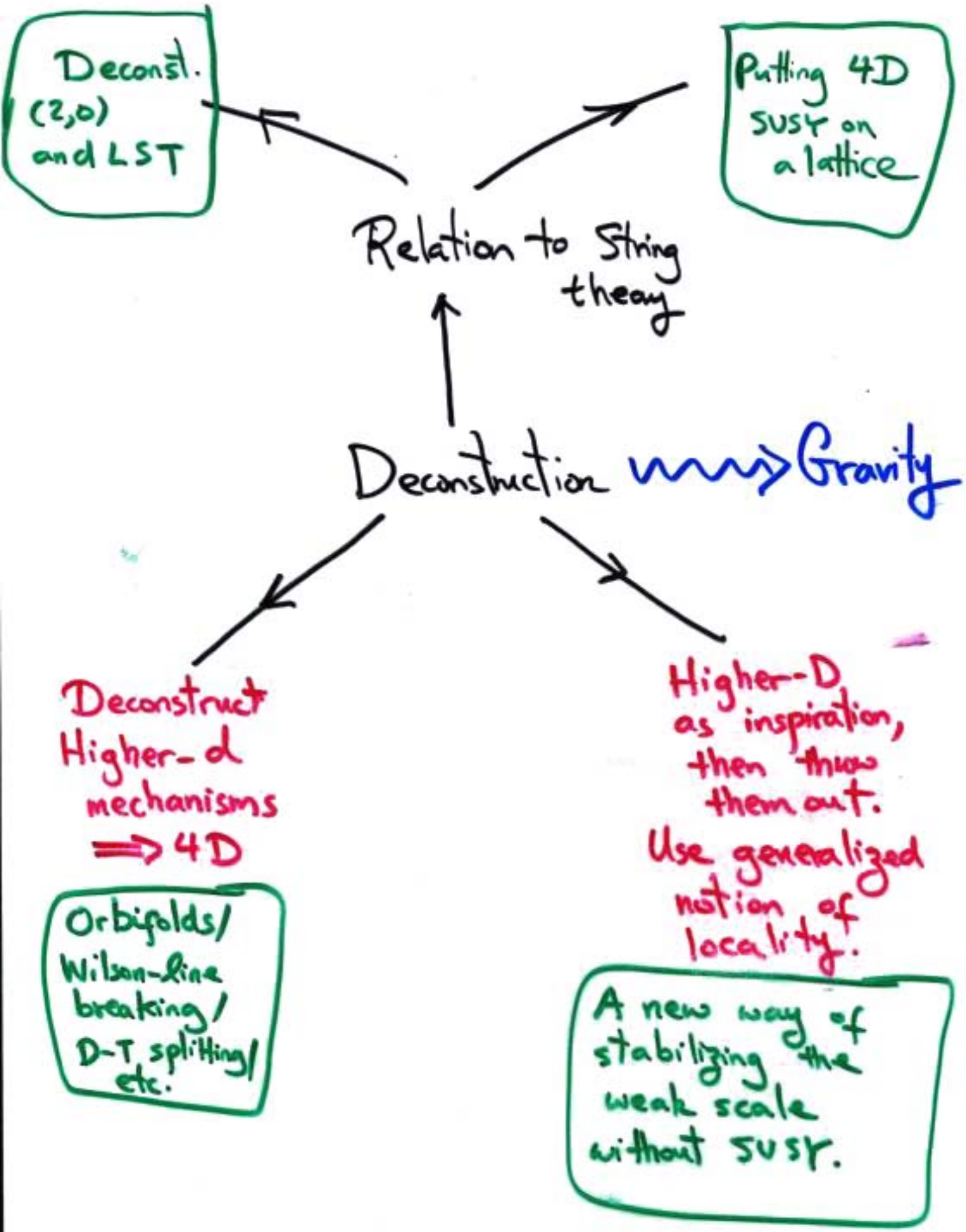
$\frac{1}{a} = gv$

$\frac{1}{g_5^2} = \frac{g}{v}$

$R = Na$

Dimensional Deconstruction

NAH, Cohen, Georgi
Hill, Pokorski, Wang


Deconstl. (2,0) and LST
Putting 4D susy on a lattice
Relation to String theory
Deconstruction
Gravity
Deconstruct Higher-d mechanisms ⟹ 4D
Orbifolds/ Wilson-line breaking/ D-T splitting/ etc.
Higher-D as inspiration, then throw them out.
Use generalized notion of locality.
A new way of stabilizing the weak scale without susy.

(I) Higher D $\Rightarrow$ 4D mechanisms.

Many examples ... let's pick SUSY-GUT's.

By early 80's, it was clear that SUSY GUT's had ....

- Great predictions: Explanation of SM quant #'s, gauge coupling unification
- Some problems: Doublet-triplet splitting, bad relationship between different Fermion masses ...

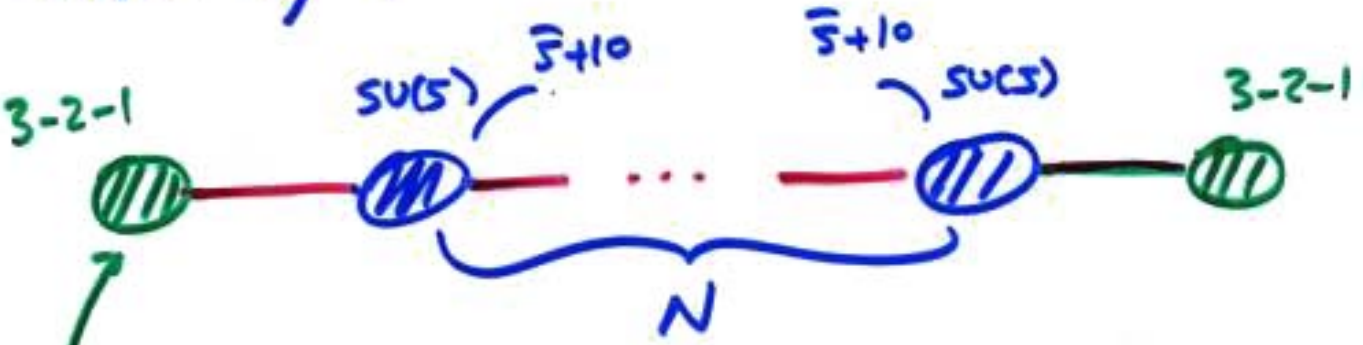
... Then string theory + it's extra dimensions came along and offered novel ways out of the problems while keeping good features, using

1. Discrete Wilson-line breaking in CY compactifications
2. Projecting out unwanted states by orbifolding

⋮

- Many of these mechanisms can be reproduced in 4D models with 2 sites e.g. for 1) Witten Dine, Nir, Shadmi (Barbieri, Drali, Strumia)

Also 2) :



H_u, H_d ~ $S^1/\mathbb{Z}_2$ Orbifold

Not a real GUT! But, all good features preserved

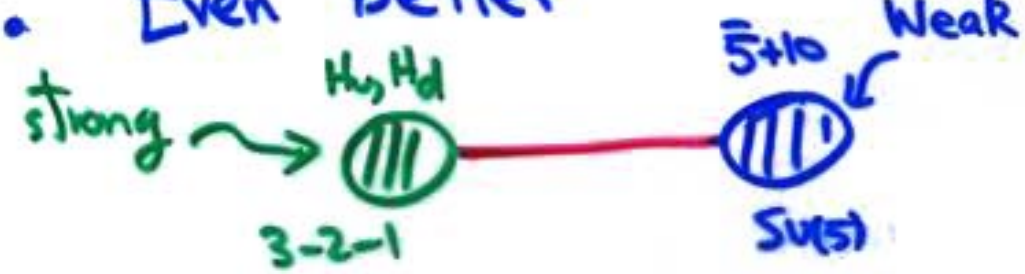
Unification of couplings:

$$\frac{1}{g_{i,D}^2} = 2/g_i^2 + \frac{N}{g_5^2} + \text{SM quant \#'s}$$

$\frac{N}{g_5^2}$ dominates

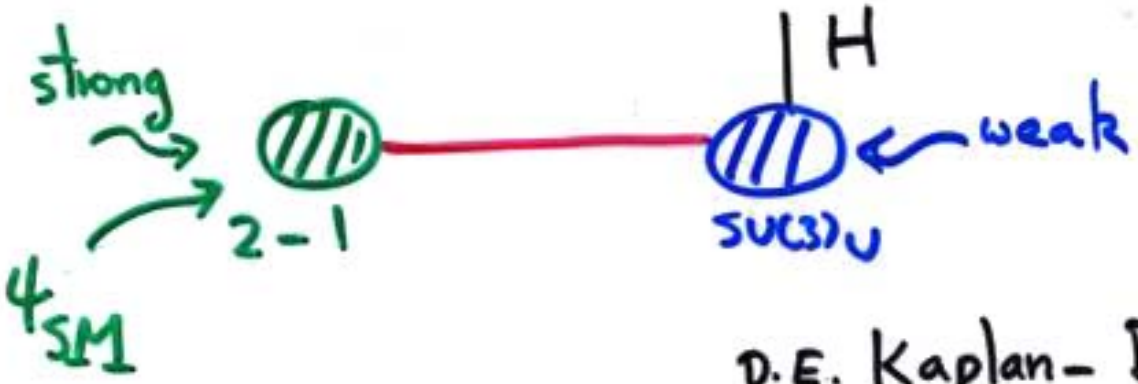
But no DT splitting problem!

- Even better



Weiner

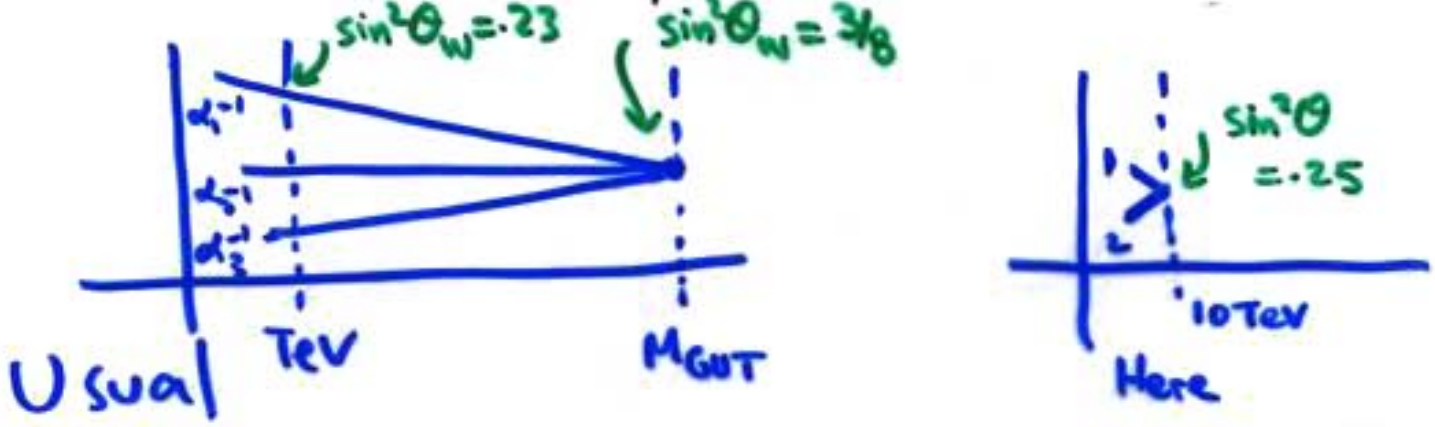
- Same idea, different group theory + motivation : Electroweak Unification $SU(2) \times U(1) \subset SU(3)_U$



D.E. Kaplan - Dimopoulos

- Predicts $\sin^2\theta_W|_{tree} = \frac{1}{4} = .25$!
  - But no difficulty with Fermion quant #'s.

- TeV scale unification:



(II) Relation to string theory

Deconstruction provides a UV completion of higher-d gauge theories, which are discretized in IR. Can we take $a \to 0$ limit but keep interacting theory?

e.g. 

N=1 model

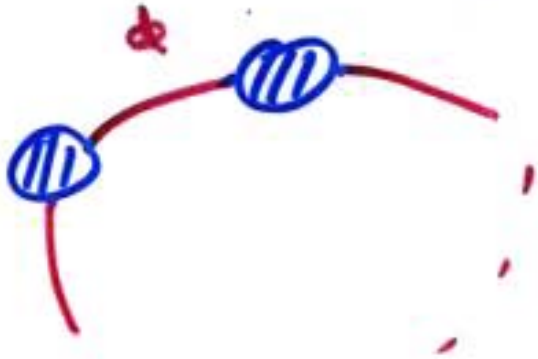
$$a = \frac{1}{g\langle\phi\rangle}$$

$$\frac{1}{g_5^2} = \frac{g}{\langle\phi\rangle}$$

$a \to 0$ keeping g_5^2 fixed

$\Rightarrow$ $g(\langle\phi\rangle) \Rightarrow \infty$.

But a SCFT might work!



$N=2$ Superconformal

we can at least try + take $g \to \infty$, $\phi \to \infty$, $N \to \infty$ so that

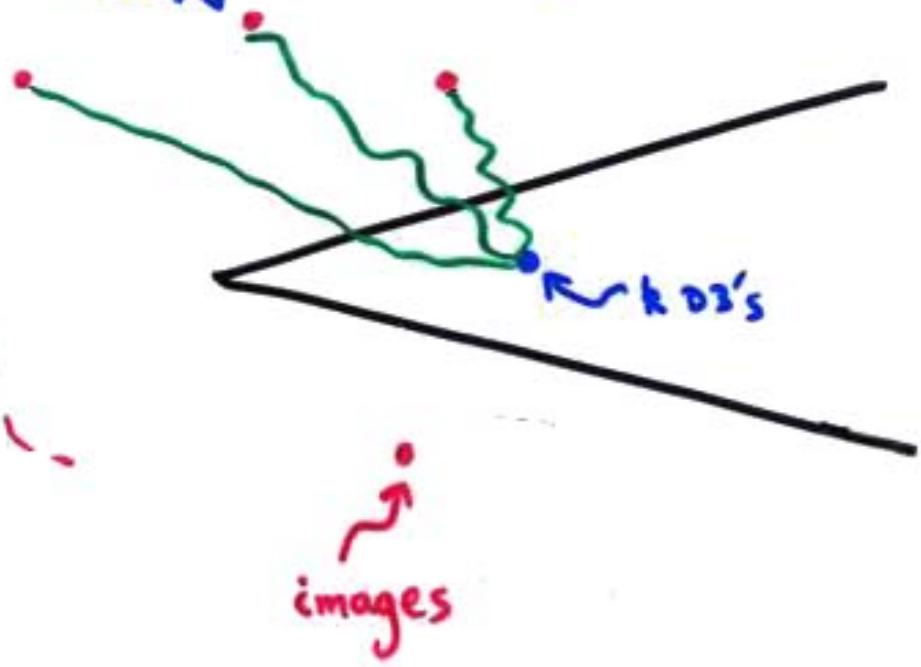
$$\frac{1}{g_5^2} = \frac{\phi}{g}$$

$$\frac{1}{R_5} = \frac{g}{N}\phi$$

are fixed.

. If this deconstructs anything, by SUSY's it must be (2,0) theory in 6D compactified on a torus $\left(R_5, R_4 = g_5^2\right)$

String embedding: k D3 branes on $\mathbb{C}^3/\mathbb{Z}_N$ orbifold



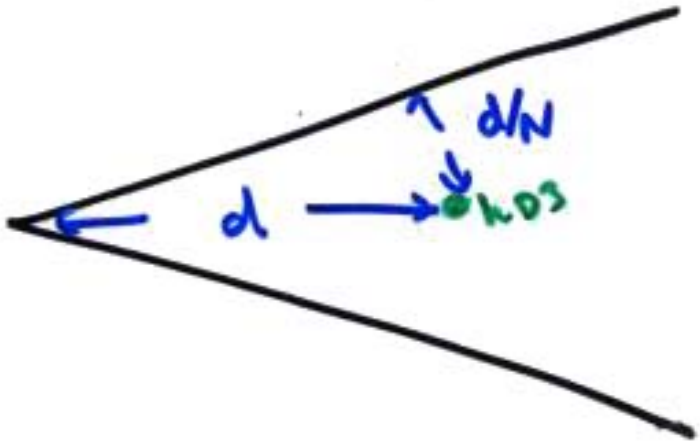
$$m_n^2 \propto T \left| 1 - e^{2\pi i \frac{n}{N}} \right|^2$$

$$= T \sin^2 \frac{n\pi}{N} \approx T \frac{\pi^2 n^2}{N^2} \quad \text{for } n \ll N$$

KK spectrum

Stretch Fund strings: massive W's = KK spectrum of deconstructed dim.

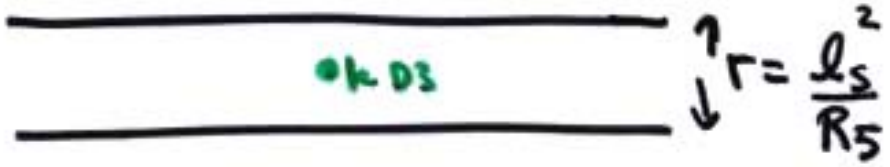
Stretch D-strings: massive monopoles = KK spectrum of hidden "magnetic" dimension



Limit: $N \to \infty$, $\ell_s \to 0$ $(d \ll \ell_s)$

$$\frac{d}{N\ell_s^2} \equiv \frac{1}{R_5} \quad \text{fixed}$$

$$\frac{d}{N\ell_s^2 g_s} = \frac{1}{R_6}$$



Orbifold regularization of a small circle.

As $\ell_s \to 0$, $\to$ T-dualize to wrapped D4 (2,0) 11Theory

but g_s gets huge $\to$ lift to M theory $\to$ Decoupled M_5 branes

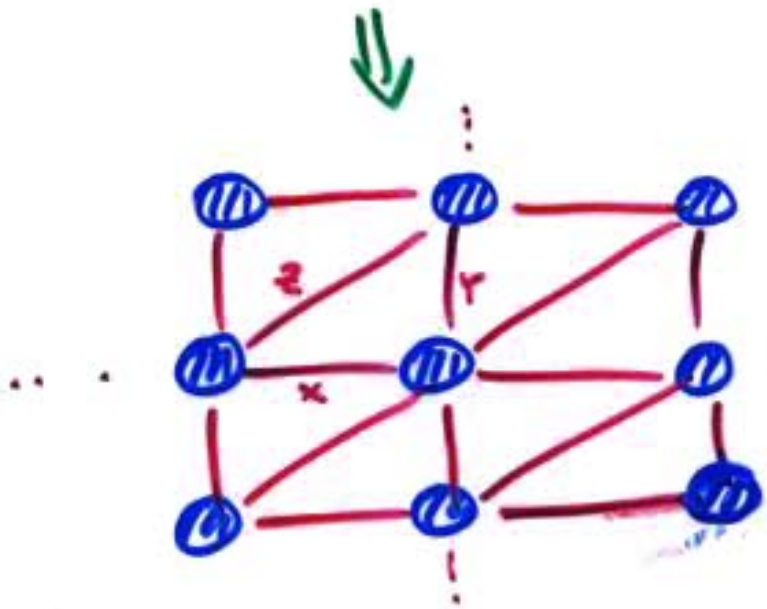
So, we appear to have deconstructed (2, 0) theory NAH, Cohen, Kaplan, Karch, Motl

[various subtleties to do with U(1) factors Distler]

[Note: it is good that one dimension generated "magnetically", since (2,0) is chiral and direct discretizations in terms of a 2D Moose would surely run into doubling problems...]

Same thing can be done to get (1,1) LST:

Start with $\mathbb{C}^3/\mathbb{Z}_N^2$ orbifold



N=1 SCFT

$W = \mathrm{tr}\, xyz + \cdots$

↑ generate "plaquette" interactions

Makes 6D theory ... same argument shows its (1,1) LST.

W bosons	=	KK modes
Monopoles	=	Winding modes
S-duality	=	T-duality

Do the same thing starting with Do branes on $\mathbb{C}^4/\mathbb{Z}_N^3$ orbifold. $\Rightarrow$ $\mathcal{N}=4$ SYM in 4D

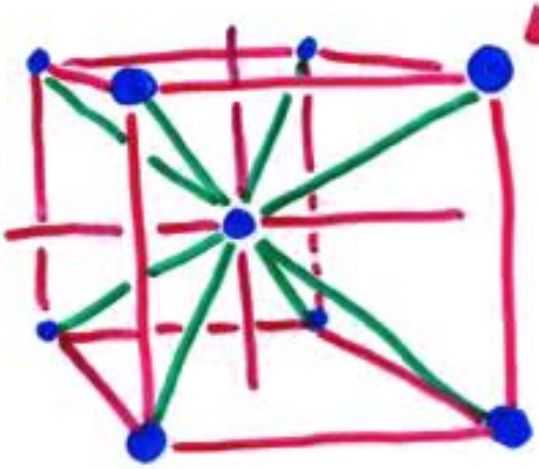
QM definition for $\mathcal{N}=4$ SYM

=

Hamiltonian SUSY Lattice!

* Long-standing problem in lattice gauge theory: how do you put SUSY on a lattice? Problem: generate relevant ops (e.g. scalar masses) that destroy SUSY [contrast w/ Lorentz inv].

... But this is nothing like a normal lattice! A piece of the quiver is



B.C.C. symmetry

QM has (2,0) SUSY ——— chiral superfields
——— Fermi superfields

... Plus, $\mathcal{N}=4$ SYM correlation functions are to be computed in an excited state of the QM

- Do branes away from fixed point

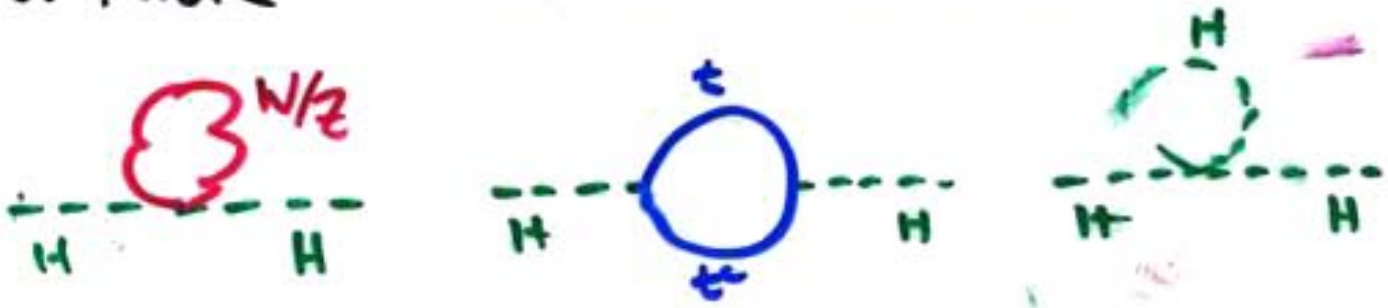
Claim: [D.B. Kaplan, E. Katz, M. Unsal]

The residual SUSY + crystal symmetries of the lattice, plus some appropriate limits, guarantee that there are no relevant ops that break $\mathcal{N}=4$.

- To make it practical, need spacetime lattice ... D-instantons on $\mathbb{C}/\mathbb{Z}_N$ orbifold ?? ...

(III) Generalized "locality" and Electroweak Symmetry Breaking.

- In Standard Model, EW scale ustable to radiative corrections



Something should at this off @ TeV scale.

- Precision data ⟶ Light Higgs, perturbative TeV scale physics ⟶ Low energy SUSY.

FOLK THEOREM

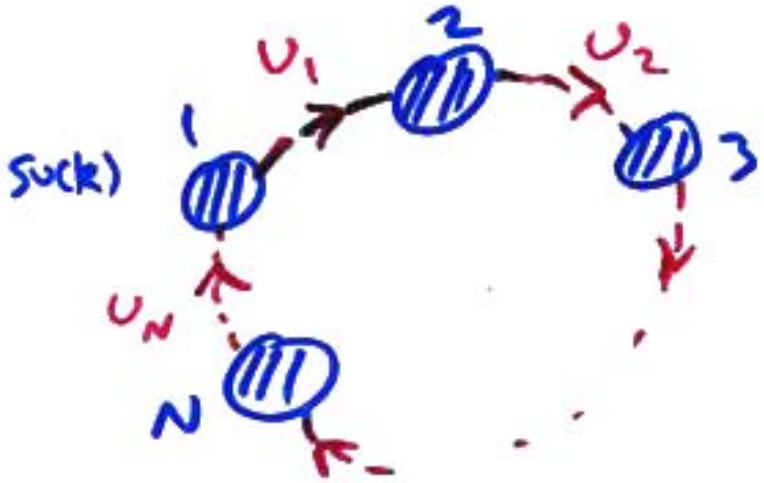
An alternative possibility has been an old dream for many model-builders: Can the Higgs be light because it is a pseudo-Goldstone Boson?

Georgi-Pais '74-'75
Kaplan-Georgi '84-'86
many others
⋮

Never realized. Doesn't look much like H is pseudo

are all ~ 1.

- We will see, inspired by deconstruction and a generalized notion of locality, that we can finally realize this possibility, in fully realistic models, with far fewer new parameters and d.o.f. than any extension of the SM to date.
- Perturbative, calculable physics @ TeV scale. Quadratic divergences cancelled by "partners" of same statistics.



U_i : NLΣM fields $\equiv e^{i\pi_i/f}$

Higgses $SU(k)^N \to SU(k)$

$(N-1)$ U's eaten.

1 Classically massless $\equiv \phi$

$(\Leftrightarrow U_1 \cdots U_N)$

Naively, 1-loop quad div

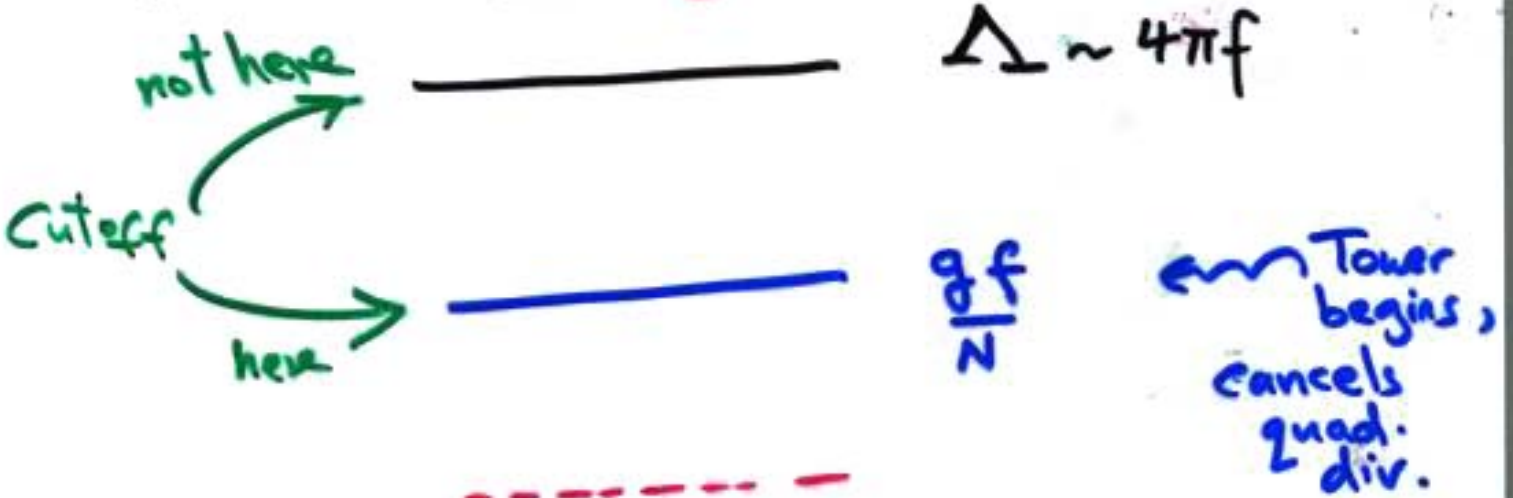
But can not be! In limit where any of $g_i \to 0$, circle is "cut", there is a larger global symmetry, ϕ is EXACT Goldstone Boson.

$\Rightarrow$ No quad. divergence till N-loops! (UV physics analytic in couplings)

$$m_\varphi^2 \propto \frac{g_1^2}{16\pi^2} \cdots \frac{g_N^2}{16\pi^2} \Lambda^2$$

Finite 1-loop mass (IR physics not analytic in couplings)

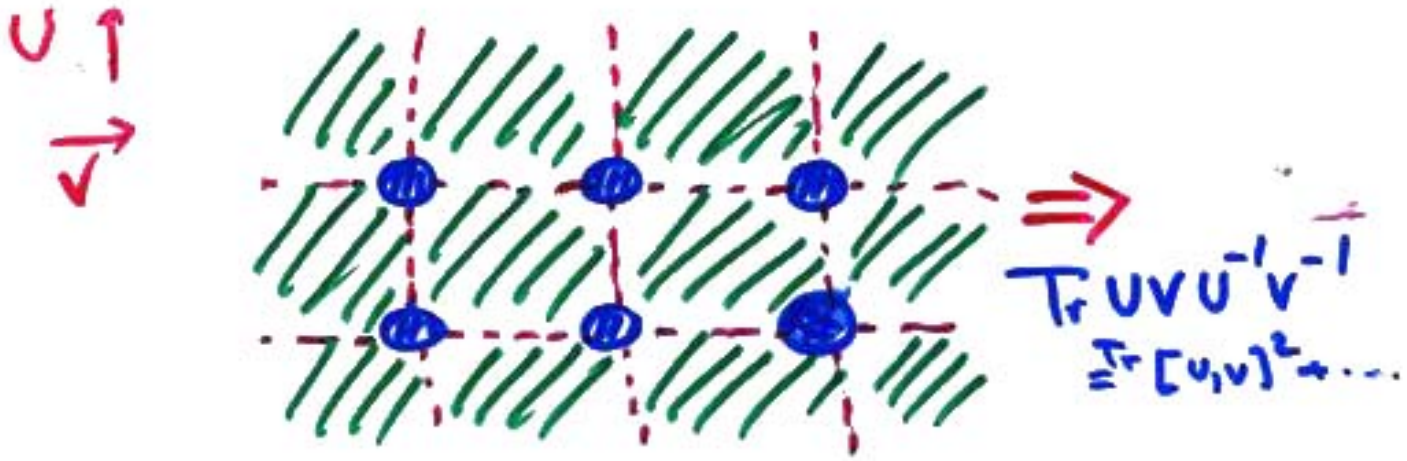
$$m_\varphi^2 \propto \frac{g^4}{16\pi^2 N^3} f^2$$



$$m_\varphi^2 \sim \frac{g_{LE}^2}{16\pi^2}\left(\frac{gf}{N}\right)^2$$: A "little Higgs"

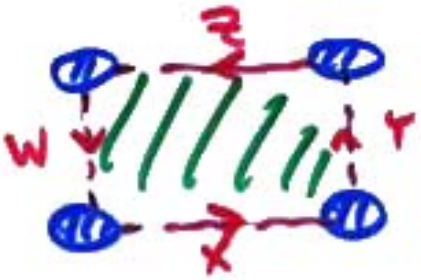
. Realistic theories must have Higgses with quartic couplings

Two dimensional Theory Spaces



e.g. a torus, with "plaquettes" added

A "face"



$\Rightarrow \mathcal{L} \supset \lambda\, \mathrm{Tr}\, XYZW$

Given a general Theory Space T with sites, links and Faces :

(1) What is the little Higgs content ?

(2) What is the potential for the little Higgses?

Given a general Theory Space T with sites, links and Faces :

(1) What is the little Higgs content ?

Little Higgses $\longleftrightarrow$ $\pi_1(T)$ elements

(2) What is the potential for the little Higgses?

$V_{\text{little Higgs}}$ $\longleftrightarrow$ Relations of the Homotopy group.

Gregoire, Wacker

e.g. Toms : $\pi_1 : \{x, y \mid xyx^{-1}y^{-1} = 1\}$

Little Higgses X, Y, $V_{LH} = -\lambda f^4 X f^4 X^{-1} Y^{-1}$

When is the theory space

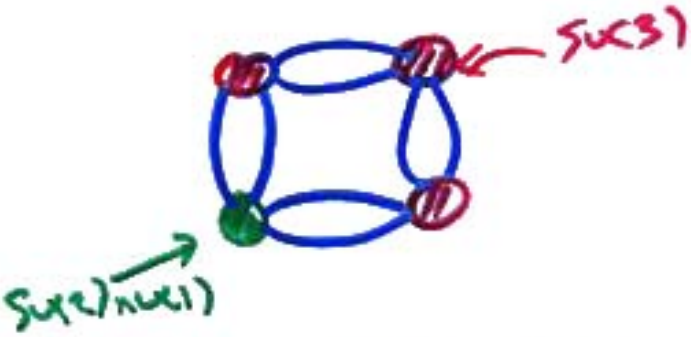
$\mathcal{T}$ free of quad. divergences,

at least @ 1-loop?

Simple Rules

- Every link connects two different points
- Every Plaquette contains a link only once.

- Fully realistic models were constructed in '01 NAH, Cohen, Georgi



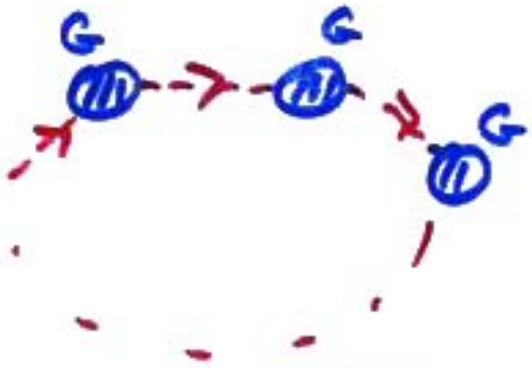
But still <u>too</u> extra-dimensional...

i.e. relying too much on theory space picture for getting locality... to finally sever all extra-dimensional ties and build the most economical models, we need a more sharp generalization of locality

Consider a general NLΣM based on a coset G_{gl}/H.

Suppose there are also a set of couplings $(\lambda_1, \ldots, \lambda_N)$ that explicitly break $G_{gl} \to G$.

ex:



$$G_{gl} = G^{2N}, \quad H = G^N$$

gauge couplings break $G_{gl} \to G^N = H$

$\Rightarrow$ NO EXACT GOLDSTONES.

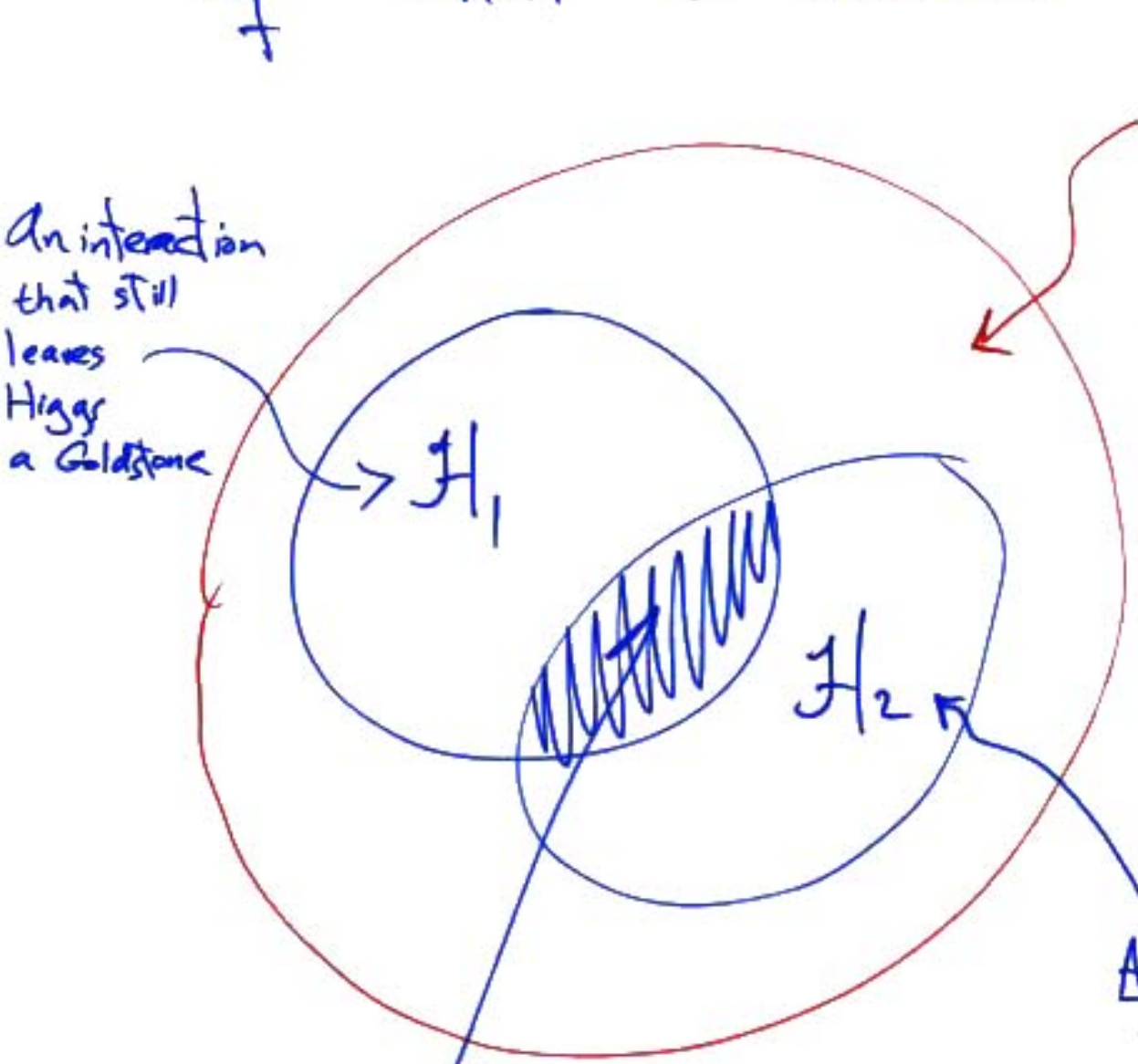
Locality is loosely

"symmetry breaking spurions are sparse"

Concretely: in limit where any $\lambda_i \to 0$, $G \to G_{bigger}$ such that G_{bigger}/H makes Higgs an exact goldstone boson.

- Group Theoretic question; "theory space" useful only in special case where $G/H \sim g^N/g^M$.

Schematic Structure of what is needed:

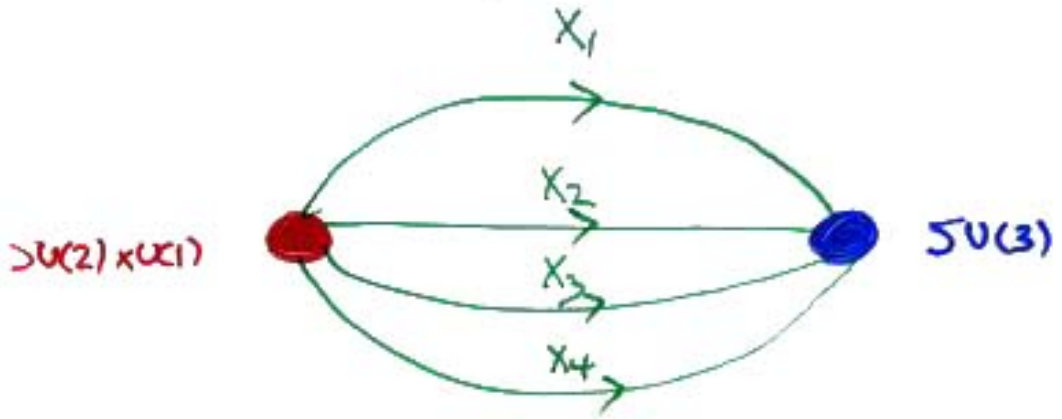


But together: Higgs not a Goldstone!

Theory space: Special case where $G = G_0^{N_{sites}}$

Minimal Theory Space

NAH
Cohen
Kate
Nelson
Gregoire
Wacker



$$\mathcal{L} = \mathcal{L}_{kinetic} + \mathcal{L}_{scalar} + \mathcal{L}_{Yuk}$$

$$\mathcal{L}_{scalar} = -\lambda\, Tr\left[X_1 X_2^\dagger X_3 X_4^\dagger + X_1 X_4^\dagger X_3 X_2^\dagger\right] + \epsilon\, Tr\, X_1 X_2^\dagger + h.c.$$

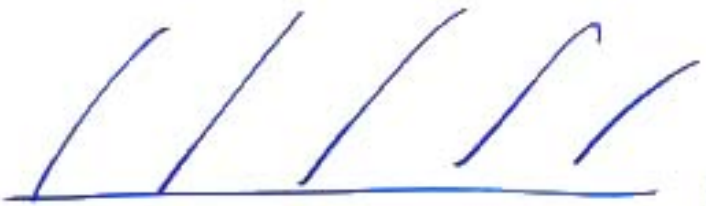
$$\mathcal{L}_{Yuk} = h\,(0\ 0\ v^c)\, X_1 X_3^\dagger \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} + M v v^c$$

leave Higgs a Goldstone

leave Higgs a Goldstone

Works!

Spectrum:



1 Higgs
1 real triplet+scalar
1 heavy fermion
1 heavy W'/Z'/V'

2 Higgs doublets
1 complex triplet
+ singlet

• 4 parameters beyond SM.

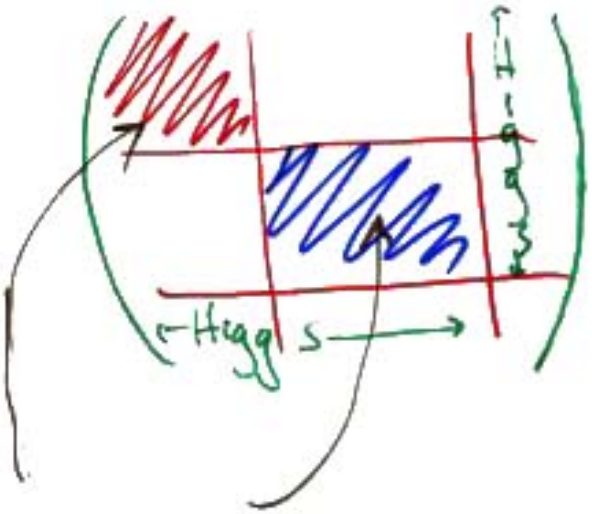
The littlest Higgs

NAH
Cohen
Kaplan
Nelson

One "site"

Goldstones of $SU(5) \to SO(5)$ breaking: $G/H = SU(5)/SO(5)$

$$\Sigma = e^{i\Pi}, \quad \pi^T = \pi .$$

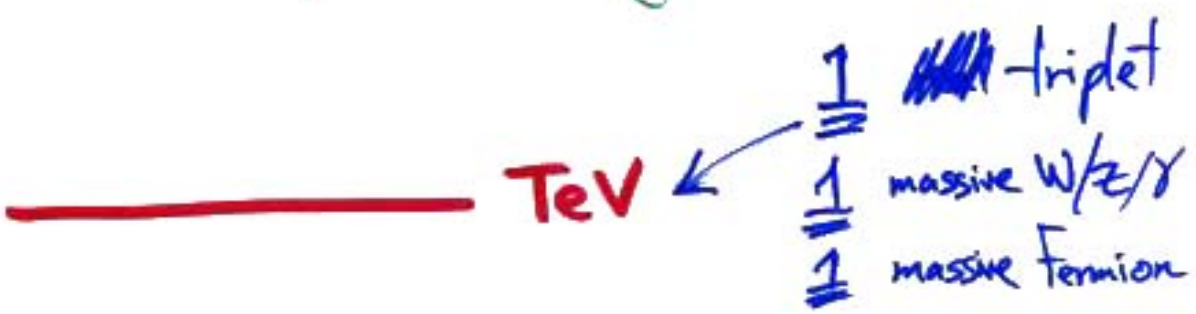


[Group structure of original attempt by Georgi/Kaplan in '84, to get Higgs as pseudo]

Gauge $[SU(2) \times U(1)]^2$

$\mathcal{L} = \mathcal{L}_{kinetic} + \mathcal{L}_{Yuk}$!

Spectrum of theory:



1 Higgs Standard Model!

~100 GeV

- Tiniest extension of SM that addresses stabilizing the Higgs.

3 new param. beyond SM

A silly table:

Theory	# of real d.o.f	# parameters
Standard Model	118	19
SUSY S.M	+ 126	$\sim$ +100 (generic) $\sim$ +5 → 10
	+56	$\sim$ +4
SU(5)/SO(5)	+32	$\sim$ +3

4D Theories with a "site-link" structure are remarkably rich, and generalize higher-dimensional locality and gauge symmetry in fruitful ways... in this talk we saw, from one circle of ideas,

- UV completion of 5D gauge theories
- 2-site cousins of extra-dimensional cures for GUT problems
- (2,0) theory + LST from 4D $N=2$ and $N>1$ SCFT
- First progress on putting 4D SUSY on a lattice
- First alternative to weak scale SUSY with naturally light Higgs + pert. TeV scale physics, smallest ext. of SM

A number of other things I didn't talk about..