AdS branes and Holography

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Strings 2002 - Cambridge

based mainly on:

CB, J. de Boer, R. Dijkgraaf, H. Ooguri
hep-th/0111210

CB, hep-th/0205115

* Rich extension of holography:

a AdS_n brane in AdS_m bulk reaches the boundary (∞ blueshift) where it appears as

↳ external source $(n<m-1)$

↳ domain wall, or defect line $(n=m-1)$

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* Relevant for warped Brane World Scenarios without perfect fine tuning of brane tension

Karch, Randall
Porrati
⋮

Can these be realized in string theory? see talk by Giddings
H. Verlinde, ...

Interesting because product-space compactifications so far failed to address stability + cosmo. const. problems

* Can take pp-wave limits

Skenderis, Taylor
Bergman, Gaberdiel, Green
Bain, Meessen, Zamaklar
Mateos, Ng
Seki
⋮

Here I will concentrate on first point, and comment briefly on second one.

General set up

AdS_m bulk supported by flux:

$$ds^2 = L^2 \left\{ \frac{du^2}{u^2} + u^2(-dt^2 + dx^2 + dy^2 + dz^2) \right\}$$

$$C = L^2 u^2 \, dt \wedge dx \wedge u^2 dy \wedge dz$$

$$(m = 3, 5)$$

Consider static p-brane, with embedding $u(x)$; y & z are spectator coordinates (either longitudinal or transverse). The brane energy is:

$$\mathcal{E} = T_{eff} \int \sqrt{u^4 + u'^2} \, u^{p-1}$$

depends on compact part

$$- \rho \int u^{m-1} + \cdots$$

only present for codimension 1 $(p = m-2)$

Extrema given by first-order eqns:

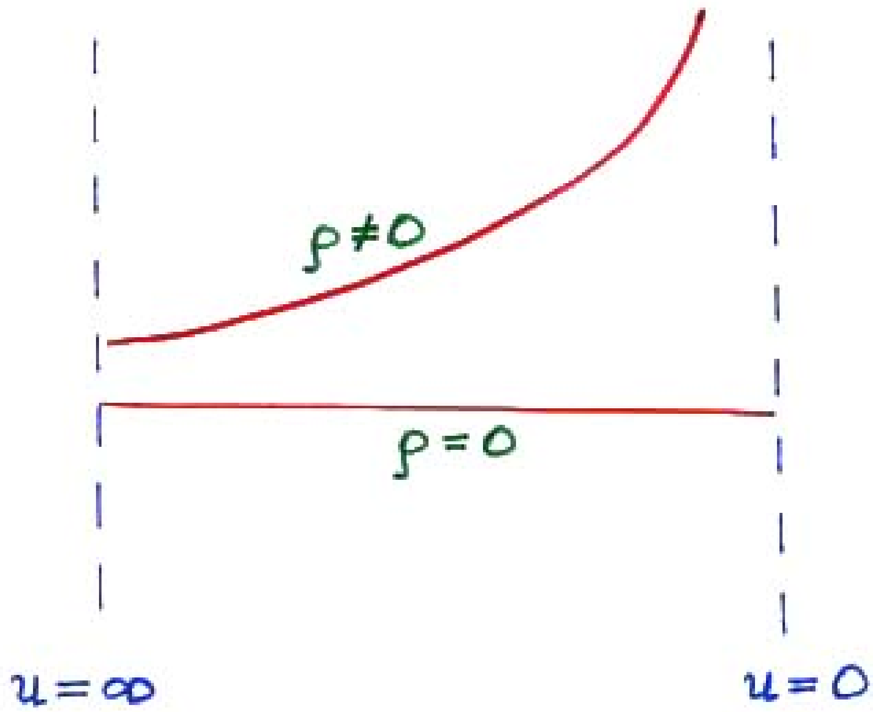
$$\frac{T_{eff}\, u^{p+3}}{\sqrt{u^4+u'^2}} - \rho\, u^{m-1} = -\Theta_{xx} \quad \text{constant}$$

if $p=m-2$

worldvolume energy-moment. tensor

Free endpoint b.cs (no applied force)

$$\Rightarrow \quad \boxed{x = \frac{C}{u}} \quad \text{with} \quad \boxed{C = \frac{\rho}{\sqrt{T^2-\rho^2}}}$$



$$L_{brane} = \sqrt{1+C^2}\, L \rightarrow \infty$$

for $T \rightarrow \rho$

CB + Petropoulos
Karch + Randall

Lift to (IIB) string theory

* $AdS_3 \times S_3 \times T_4$ (near-horizon of n_1 F1, n_5 NS5 branes)
↳ up to U-duality

Brane is a D3/D1/F1 bound state
↳ couple to background flux

with $AdS_2 \times S_2$ geometry.

Has been constructed as <u>exact</u> conformal boundary state (in Euclidean theory)

* Ponsot, Schomerus, Teschner

also: Rajaraman, Rozali
Parnachev, Sahakyan
Giveon, Kutasov, Schwimmer
Hikida, Sugawara
Petropoulos, Ribault
Lee, Ooguri, Park, Tannenhauser
⋮
Stanciu

* $AdS_5 \times S_5$ (near horizon of D3 branes)

probe: D5/D3 bound state
with $AdS_4 \times S_2$ geometry.

Many possible generalizations ...

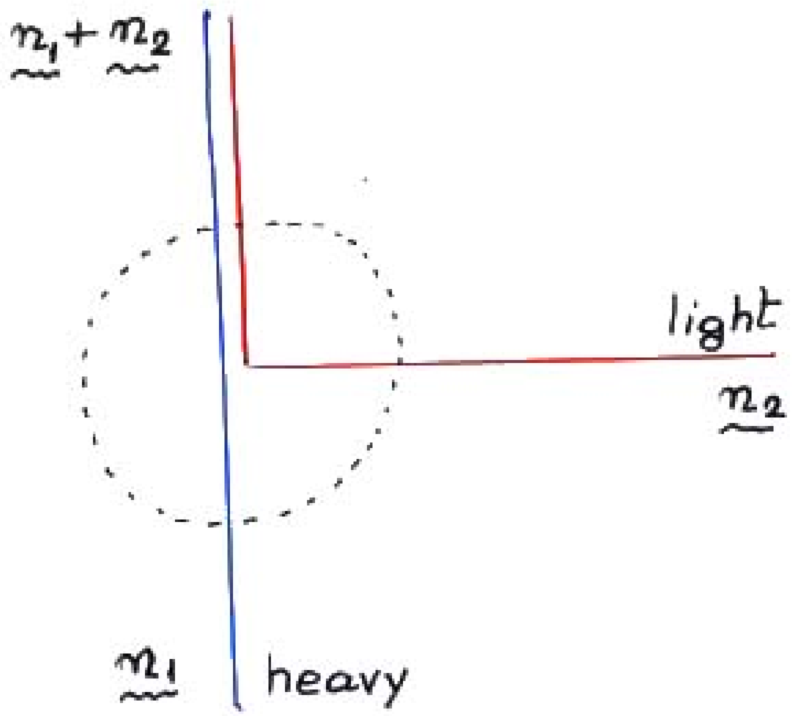
Holographic duals

↳ Consider first AdS_3 case.

- Type IIB theory compactified on T_4
  has variety of strings (D5, D3's, D1, NS5, F1)
  in fundamental rep. of U-duality
  group $SO(5,5|\mathbb{Z})$.

- These can form supersymmetric
  string junctions.

- What we describe is the nh blow-up
  of a junction between a heavy
  (background) string and a light (probe)
  one.



- On holographic screen see domain wall between CFTs with different central charge or different moduli.

  ↳ modified attractor mechanism

  Ferrara, Kallosh, Strominger

- Probe brane is AdS and supersymmetric

  ⟹ wall is superconformal

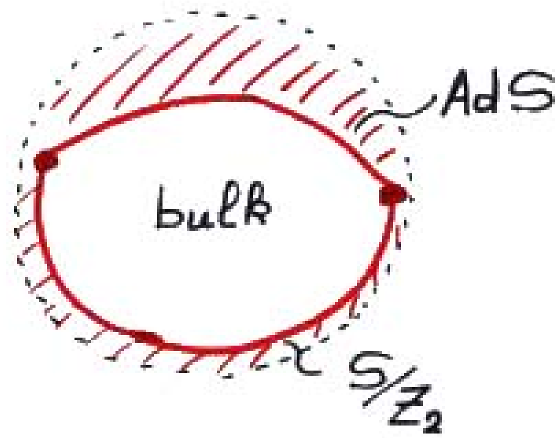
- Similar story in higher dimensions, but more complicated since wall can have interacting CFT on its worldvolume

  De Wolfe, Freedman, Ooguri
  Erdmenger, Guralnik, Kirsch
  Mateos, Ng, Townsend

Important clarification:

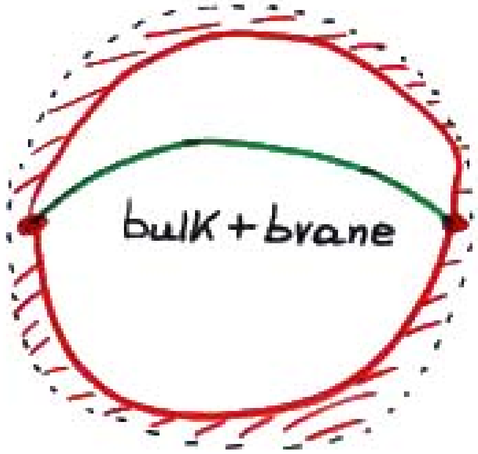
↳ many papers on subject take holographic screen to be $AdS \times S/Z_2$

Karch, Randall
Porrati
Bousso, Randall
Duff, Liu, Sati



gravity localized on AdS brane further mapped on $\partial(S/Z_2)$

↳ Here holographic screen will be S



Confirm with analysis à la Brown-Henneaux

or in 'modern version'

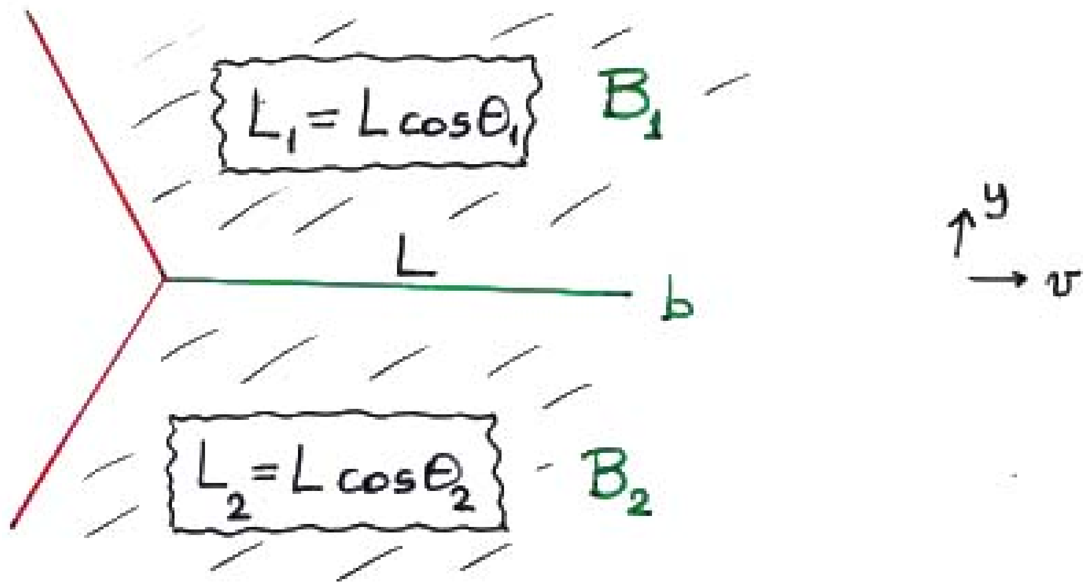
Henningson + Skenderis
Balasubramanian + Krauss
⋮

They showed that asymptotic symmetries of gravity in AdS_3 are two (left + right) Virasoro algebras with $c = 3L/2G_N$

Repeat analysis for gravity in two patches of AdS_3 spacetime glued along common AdS_2 boundary:

$$ds^2 = \frac{1}{f^2}\left(du^2 + dy^2 - dt^2\right)$$

with
$$f(u,y) = \begin{cases} u + \dfrac{y \tan\theta_1}{L} & y > 0 \\ u + \dfrac{y \tan\theta_2}{L} & y < 0 \end{cases}$$



Solves eqns. deriving from

$$S_{bulk} + S_{boundary} + S_{brane}$$

discontinuous cosmo. constant

Gibbons-Hawking & counterterm

$$S_{brane} = T\int\sqrt{-\hat{g}} - \frac{1}{8\pi G_N}\int\sqrt{-\hat{g}}\,[K]$$

due to thin brane approx.

<u>iff</u> $\tan\theta_1 - \tan\theta_2 = 8\pi G_N L \cdot T$

Change coordinates to

$$ds^2 = L^2\left\{\frac{du^2}{u^2} + u^2(-dt^2+dx^2) + 2\sin\theta_r\, dx\, du\right\}$$

in B_r

↳ One Virasoro algebra of asymptotic symmetries at $u \to \infty$:

$$\begin{cases} \delta x^{\pm} = -\xi^{\pm} - \frac{1}{2u^2}\partial_{\mp}^2 \xi^{\mp} + \frac{\sin\theta_r}{2u}\left(\partial_+\xi^+ - \partial_-\xi^-\right) \\ \delta u = \frac{u}{2}\left(\partial_+\xi^+ + \partial_-\xi^-\right) - \frac{\sin\theta_r}{2}\left(\partial_+^2\xi^+ - \partial_-^2\xi^-\right) \end{cases}$$

with $\xi^{\pm} = g(x^{\pm})$

↳ same function for continuity at brane

↳ Brown-York tensor

$$T_{ab} = \frac{2}{\sqrt{-\gamma}}\frac{\delta S}{\delta\gamma^{ab}}$$

↳ metric at boundary

$$\delta T_{\pm\pm} = \frac{L\cos\theta_r}{16\pi G_N}\partial_{\pm}^3\xi^{\pm} \quad \text{in } \partial B_r$$

$$\delta T_{+-} = 0$$

i.e. consistent with expected jump of central charge.

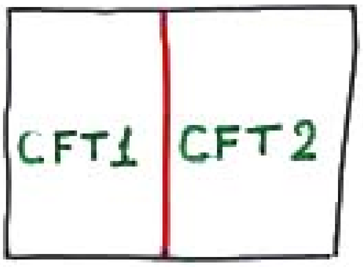
Permeable CFT walls

Can two CFTs interact non-trivially along conformal wall?

Is this generic RG fixed point?

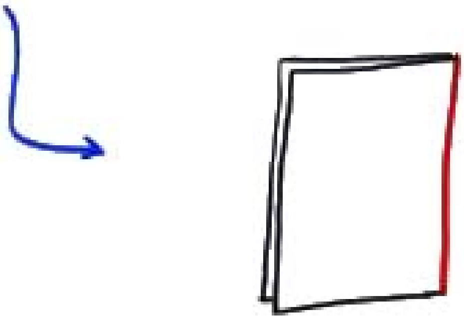
Folding trick:

Affleck, Oshikawa
⋮



$T^1_{xt} = T^2_{xt}$ (no net flow of energy to boundary)

Fold



$T^{tot}_{xt} = T^{tot}_{++} - T^{tot}_{--} = 0$

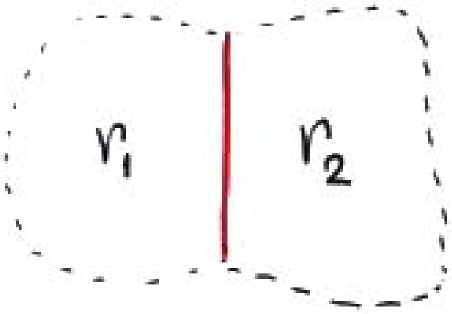
CFT1 ⊗ CFT2

∴ Conformal boundary state in tensor product theory.

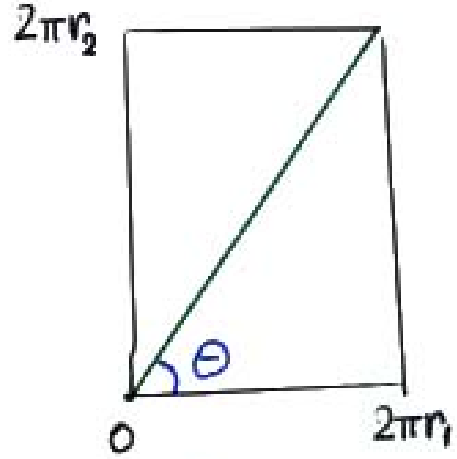
Permeable $\Leftrightarrow$ not $\sum \otimes \left(\begin{array}{l}\text{separate} \\ \text{Ishibashi states}\end{array}\right)$

Interesting classification problem for bCFT. Can construct explicit examples:

* Radius jump for free field



described after folding by diagonally-embedded D1-brane



$\tan\theta = r_2/r_1$

* Jump in Kähler moduli of CY CFTs

CY, CY$'$: same complex structure

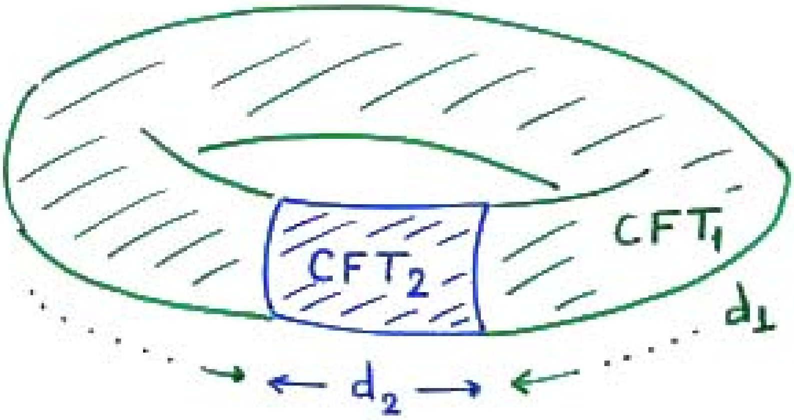
$\rightsquigarrow$ diagonal brane $z_i = z_i'$

* Some algebraic constructions

Quella, Schomerus
Recknagel

Casimir energy of conformal bubble

↳ CFT calculation:



$$Z = \langle\langle B |\, q_1^{L_0^{(1)} - \frac{c^{(1)}}{24}}\, q_2^{L_0^{(2)} - \frac{c^{(2)}}{24}}\, | B \rangle\rangle$$

with $q_i = \exp(-2\pi d_i / T)$

$$E_{\text{casimir}} = \lim_{T \to \infty} \lim_{d_1 \to \infty} -\frac{1}{T} \log \langle\langle \quad \rangle\rangle$$

* For free scalar with radius jump

$$\lim_{q_1 \to 0} \langle\langle \quad \rangle\rangle \propto \prod_{n=1}^{\infty} \left(1 - \cos^2 2\theta\, q_2^{2n}\right)^{-1}$$

quantum dilogarithm

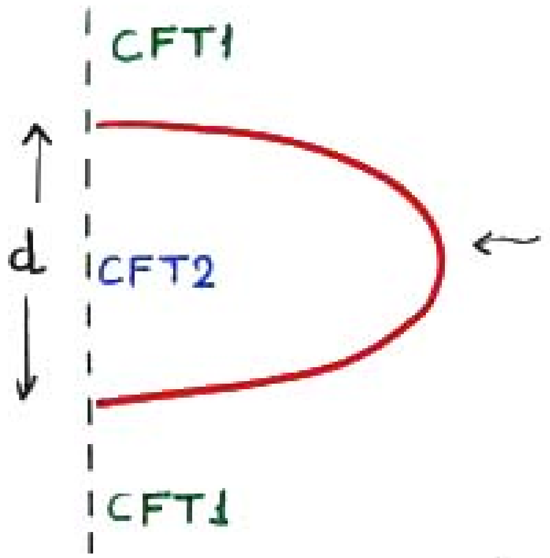
$$\Rightarrow \quad E_{Casimir} = -\frac{1}{8\pi d}\, Li_2(\cos^2 2\theta)$$

↳ classical dilogarithm

$-\frac{\pi}{48d}$ when $\frac{r_1}{r_2} \to 0, \infty$ (perfect reflection)

$-\frac{1}{8\pi d}\left(\frac{\delta r}{r}\right)^2$ for $r_1 \approx r_2$ (near perfect transmission)

↳ Supergravity calculation



← Solution with $\Theta_{xx} \neq 0$ (not free endpoint $\Rightarrow$ not AdS)

analogous to $q\bar{q}$ potential

Maldacena
Rey

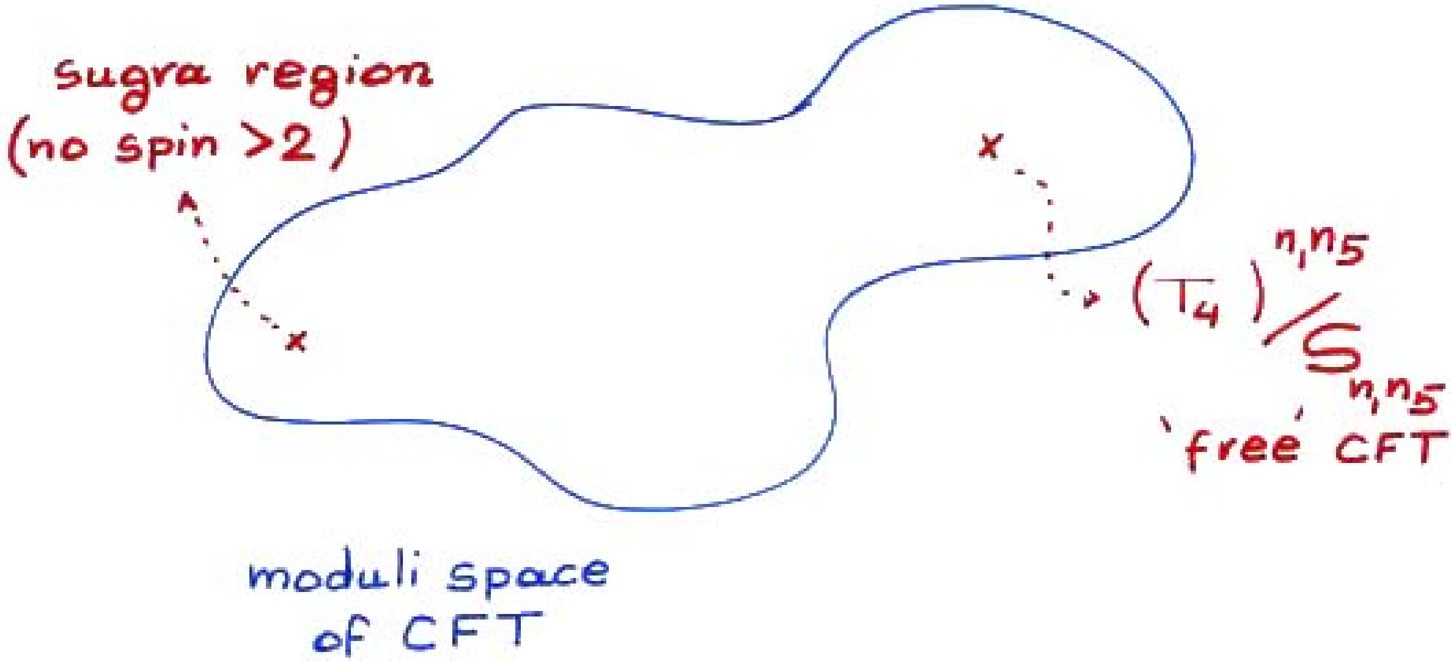
end result:

$$E_{\text{Casimir}} = -\frac{2LT}{d}\left\{2\,E(k) - K(k)\right\}^2$$

complete elliptic integrals

$$k^2 = \frac{T-\rho}{2T}$$

Cannot compare, since different regions of validity



↳ Strong coupling prediction. Need exact Conf. boundary state for asymp. AdS branes to approach symmetric-product orbifold.??

Speculation

* In 'RS-like' scenaria the Planck brane in AdS bulk serves to generate 4d gravity. But where does SM live?

* If it were realized holographically then N, $g_{YM}^2 N$ large.
  Uncomfortable idea $\Rightarrow$ better to realize SM with probe brane in susy 'RS-like' bulk.

* Tantalizing parametric suppression of cosmological constant

$$L_{brane} \sim \sqrt{\frac{T}{\delta T}} \cdot L_{bulk}$$

* Suppose that:

$$L_{bulk} \sim mm \sim 10^{+12}/GeV$$

$$T \sim (10^{18}\ GeV)^4$$

$$\delta T \sim (10^{3}\ GeV)^4$$

$\hookrightarrow m_{SUSY}^4$ from loops on the brane

$$\Longrightarrow \quad L_{brane} \sim 10^{30}\ mm$$

present Hubble radius of our universe

* Problem: how to generate

$$G_N^{(4)} \sim (10^{19}\ GeV)^{-2} \quad ??$$

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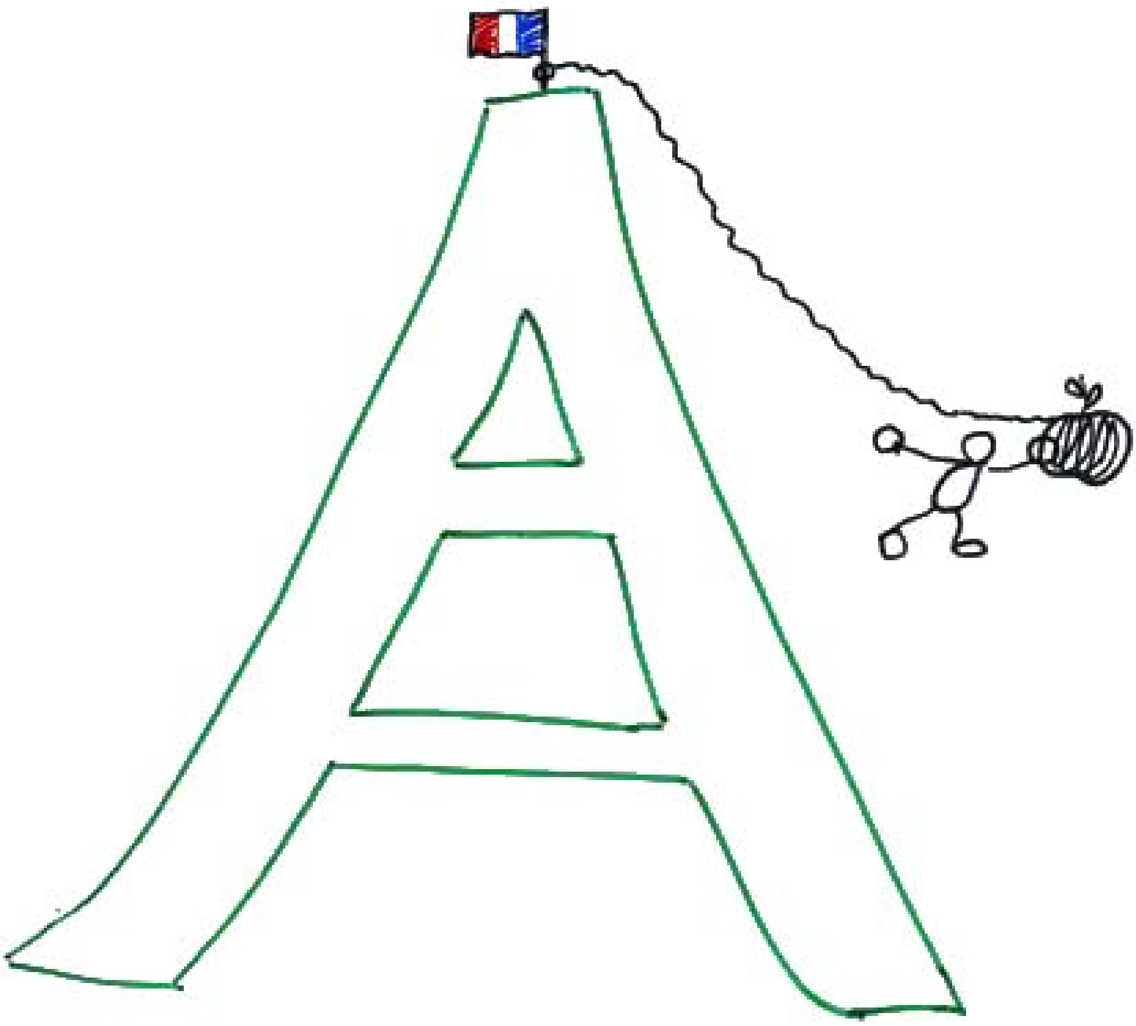

We hope to welcome
you all there, in 2 years