Strings 2002

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Elliptic Models at Large-N, (Little) Worldsheet Instantons and Integrable Systems

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SUSY gauge theory $\longleftrightarrow$ Integrable systems

$\mathcal{N}=2$: Donagi + Witten
Marshakov et al
Martinec + Warner

$\mathcal{N}=1^*$ SUSY Yang-Mills $G=U(N)$ $\longleftrightarrow$ Elliptic Calogero-Moser (ECM) system

$\mathcal{N}=1^*$ vacua $\longleftrightarrow$ equilibrium configurations of ECM

chiral condensates $u_{2k}=\langle \mathrm{Tr}\,\Phi^{2k}\rangle$ $\longleftrightarrow$ Lax matrix
Calogero + Perelemov

$N\to\infty \quad \lambda=g^2N\gg 1$

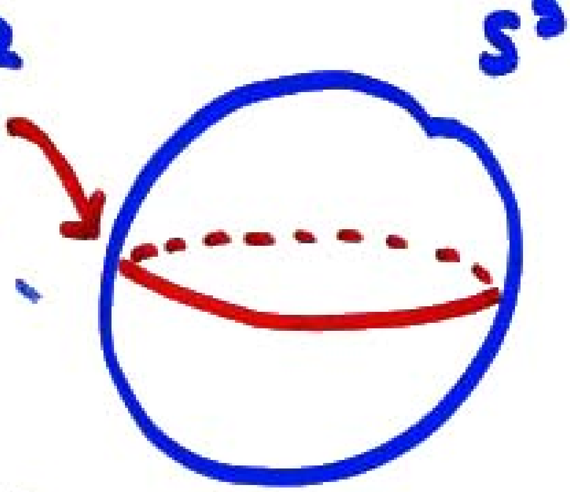
- comparison with AdS dual
- Exact results for worldsheet instanton sum

$\mathcal{N}=4 \xrightarrow[\text{breaking}]{\text{soft}} \mathcal{N}=1^*$ $G=U(N)$

conformal ($\mathcal{N}=4$) — confinement ($\mathcal{N}=1^*$)

AdS dual Polchinski + Strassler

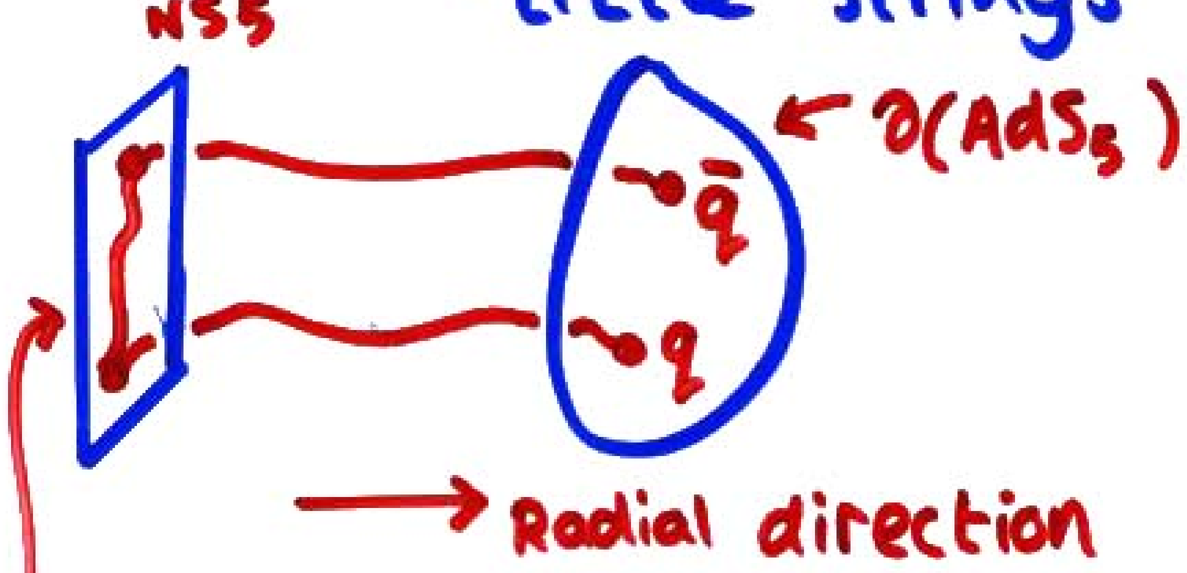
NS5 wrapped on S^2
+ N units of D3 charge



$\lambda = g^2 N >> 1$

UV physics: strongly-coupled SCFT
$AdS_5 \times S^5$ + boundary sources

IR physics: confinement due to "little" strings



IIB string
binds with NS5

F1/NS5 boundstate $\Rightarrow$ world sheet instantons on S^2

$$\text{action} = \frac{\text{area of } S^2}{\alpha'} = \lambda/2$$ 't Hooft coupling

holomorphic quantities have large-λ expansion:

modular in $\lambda/4\pi i$!

$$\langle \mathrm{Tr}\, \Phi^2 \rangle \cong E_2(\lambda/4\pi i)$$

adjoint scalar

$$= 1 - 24 \sum_{k=1}^{\infty} c_k\, e^{-k\lambda/2}$$

$$c_k = \sum_{d|k} d$$

– $u_{2k} = \langle \mathrm{Tr}\, \Phi^{2k} \rangle \quad k = 1, \dots N/2$

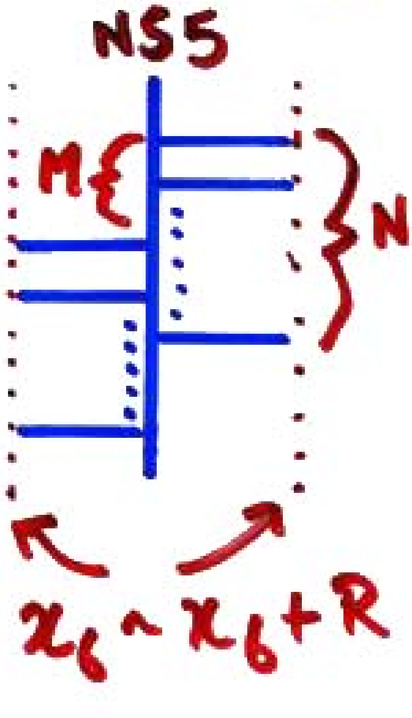
from equilibrium configurations of elliptic Calogero-Moser model ND + A. Sinkovics

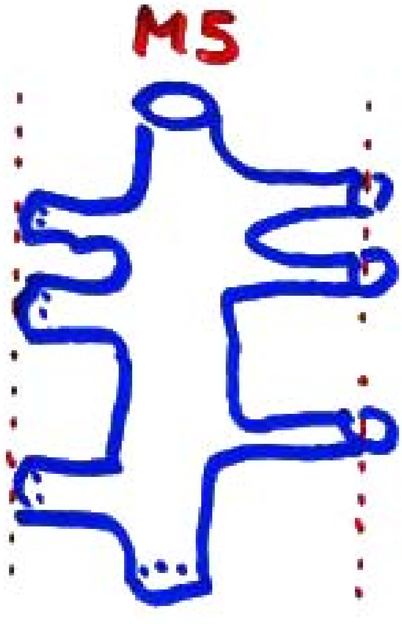
$\mathcal{N}=4 \quad G=U(N) \quad \tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$

$$\mathcal{N}=4 \xrightarrow{\;M \neq 0\;} \mathcal{N}=2^*$$

- Coulomb branch governed by Riemann surface Σ ← genus N

M-theory construction: Witten





Σ is branched N-fold cover of "bare" torus in spacetime

$$E(\tau) = \frac{\mathbb{C}}{\mathbb{Z} \oplus \tau \mathbb{Z}}$$

Σ is also spectral curve of

N-body Calogero-Moser system

$$H_2 = \sum_{i=1}^{N} \frac{p_i^2}{2} + \sum_{i>j} \wp_\tau (x_i - x_j)$$

Weierstrass function on $E(\tau)$

"positions" $x_i \in E(\tau)$

"momenta" $p_i \in T^* E(\tau)$

Lax formulation:

N×N matrices

$$\exists \quad L_{ij}(P,X) \;,\; M_{ij}(P,X)$$

Hamiltons equations $\Rightarrow \dot{L} = [M, L]$

Lax equation

$\Rightarrow$ N conserved quantities

$$H_k = \mathrm{Tr}_N [L^k] \quad k = 1, \dots, N$$

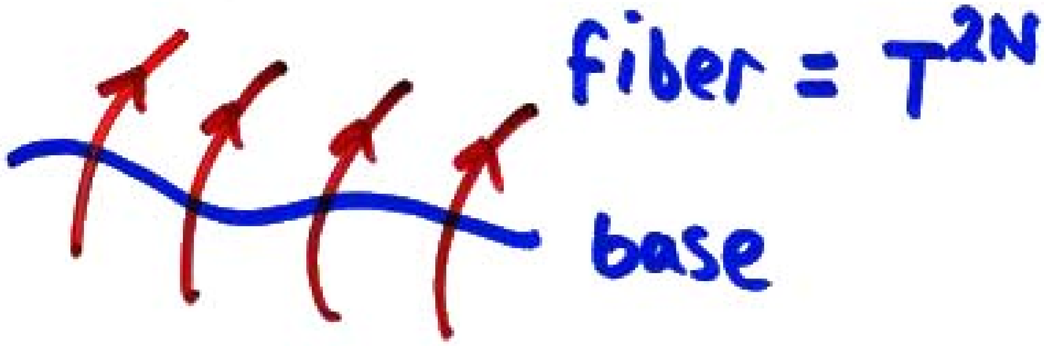
Correspondence: Donagi + Witten
Martinec + Warner
Marshakov et al

$$u_k = \langle \mathrm{Tr}_N \Phi^k \rangle \longleftrightarrow H_k = \mathrm{Tr}_N L^k$$

adjoint scalar

Explicit Integration:

Phase space ≡ Toric fibration



base parametrized by "action" varibles " H_k ← complex moduli of Σ

fiber ≡ "angle" variables

$$= \mathcal{J}(\Sigma)$$ ← Jacobian variety

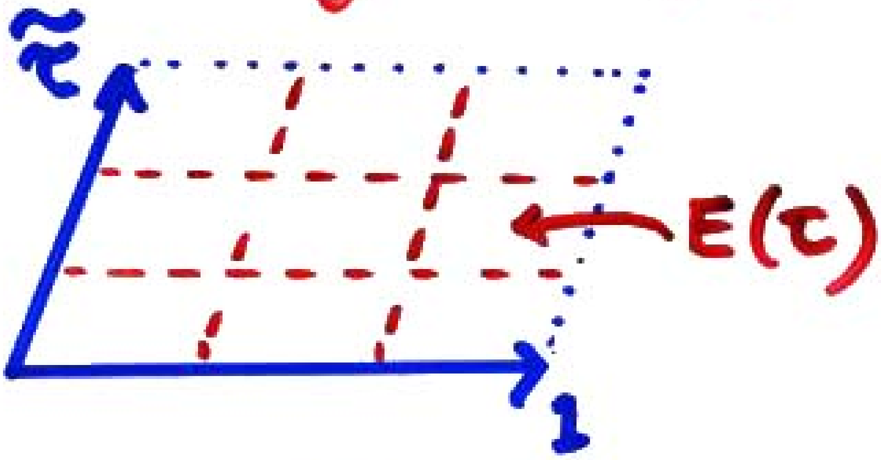
Solution: free motion on fiber

<u>Soft breaking</u>: $\mathcal{N}=2^* \xrightarrow[\mu\neq 0]{} \mathcal{N}=1^*$

massive vacua $\longleftrightarrow$ maximal degenerations of Σ, $\tilde{\Sigma}$

$\tilde{\Sigma}$ = unbranched N-fold cover of $E(\tau)$ Donagi + Witten

$$\tilde{\Sigma} = E(\tilde{\tau})$$



$$\tilde{\tau} = \frac{p\tau + k}{q} \qquad pq = N, \quad k = 0, \dots, q-1$$

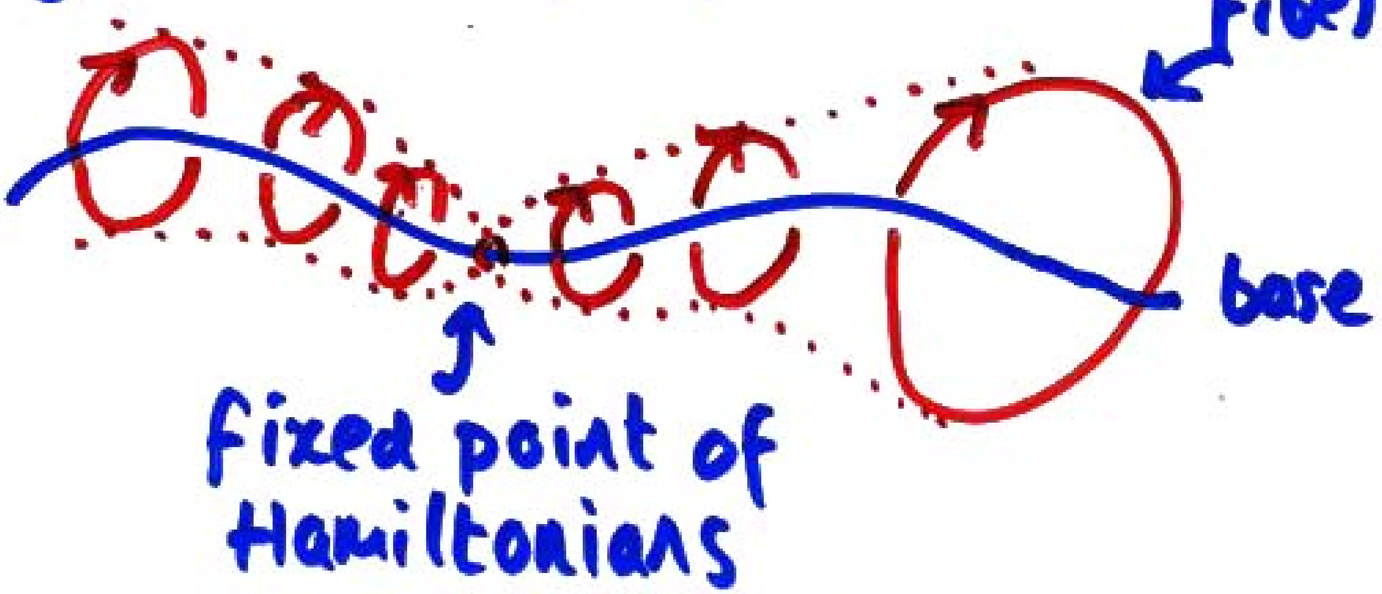
- $\sum_{d|N} d$ massive vacua permuted by $SL(2,\mathbb{Z})$
- confining vacuum:

$$\tilde{\tau} = \tau/N \sim \frac{4\pi i}{\lambda}$$

't Hooft coupling

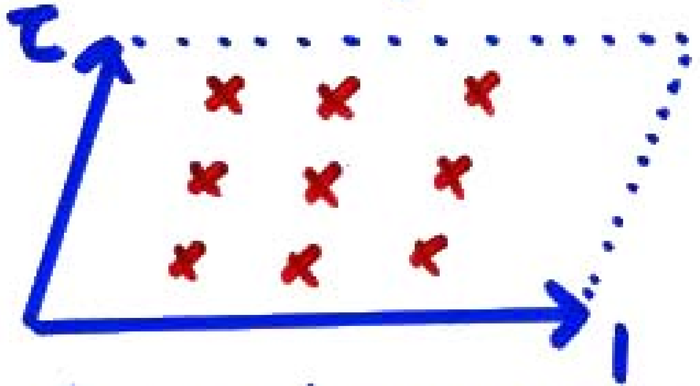
Calogero-Moser phase space:

degeneration of Σ,



$\mathcal{N}=1^*$ vacuum $\equiv$ Equilibrium configuration

massive vacua :



- matches Donagi-Witten

Condensates:

$$\langle \Phi_i \rangle \sim \widetilde{L}$$

Lax matrix at equilibrium

Trig limit: Calogero + Perelemov

Result

Large-N eigenvalue distribution of adjoint scalar Φ

$\lambda_k \quad k=1,\dots,N$

$N\to\infty, \ \frac{k}{N} \to x \in [0,1]$

$\lambda_k \to \mu(x)$

expansion for large 't Hooft coupling:

uniform distribution ~ fuzzy S^2

$$\mu(x) = \left(\tfrac{1}{2} - x\right) + \sum_{\ell=1}^{\infty} \left[\frac{e^{-\frac{\lambda}{2}(\ell - x)}}{1 + e^{-\frac{\lambda}{2}(\ell - x)}} + \frac{e^{-\frac{\lambda}{2}(\ell - 1 + x)}}{1 + e^{-\frac{\lambda}{2}(\ell - 1 + x)}} \right]$$

(litte) worldsheet instantons