

0.
(SOME REMARKS ON THE)

GEOMETRY of

SUPERSYMMETRIC

-pp- WAVES

JOSÉ FIGUEROA-O'FARRILL
(U of Edinburgh)

STRINGS 2002, Cambridge

Based on:

hep-th/0105308	(GP)
0110242	(MB, CMH, GP)
0201081	(MB, CMH, GP)
0202111	(MB, GP)

and work in PROGRESS with GP

math.DG/0109162

GP = George PAPADOPOULOS

MB = Matthias BLAU

CMH = Chris HULL

MAXIMAL SUSY & PP-WAVES

eg: D=11 SUGRA (g, F)

$$\delta_{\epsilon} \psi_{\mu} \Big|_{\psi=0} =: \mathcal{D}_{\mu} \epsilon$$

$$= \nabla_{\mu} \epsilon + \Omega_{\mu} \epsilon$$

↑
SPIN CONNECTION

↑
F-dependent algebraic term

MAXIMAL SUSY $\Rightarrow \mathcal{D}$ is flat

\Rightarrow 1) F is PARALLEL
and DECOMPOSABLE

2) Riemann = $T(g, F)$
(algebraic)

1) + 2) $\Rightarrow (M, g)$ LOCALLY SYMMETRIC

Tracing 2) $\Rightarrow R \propto |F|^2$
↑
SCALAR CURVATURE

2.

TWO CASES:

$R \neq 0 \Rightarrow$ FREUND-RUBIN (using deRham-Wu
dec. theorem)

\therefore $AdS_4 \times S^7$ $AdS_2 \times S^4$

$R = 0 \Rightarrow$ (i) $F = 0 \Rightarrow$ FLAT

(ii) F NULL \Rightarrow CW pp-WAVE
(Kowalski-Glikson
'84)

Theorem (Cahen-Wallach '69)

Indecomposable Lorentzian symmetric spaces
of dim ≥ 3 are:

AdS dS GW pp-WAVE

CW pp-WAVES

$M^D = G/K$ depending on $A \in S^2 R^{D-2}$

$\mathfrak{g} = \langle e_i, e_i^*, e_+, e_- \rangle$ $i=1, \dots, D-2$

$\left. \begin{aligned} [e_i, e_j^*] &= A_{ij} e_+ \\ [e_-, e_i] &= e_i^* \\ [e_-, e_i^*] &= \sum_j A_{ij} e_j \end{aligned} \right\}$ SOLVABLE !

3.

$$\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$$

$$\mathfrak{k} = \langle e_i^* \rangle$$

$$\mathfrak{p} = \langle e_i, e_+, e_- \rangle$$

C-3-1033-4

$$\left\{ \begin{array}{l} [\mathfrak{k}, \mathfrak{k}] \subset \mathfrak{k} \quad (\text{abelian}) \\ [\mathfrak{k}, \mathfrak{p}] \subset \mathfrak{p} \\ [\mathfrak{p}, \mathfrak{p}] \subset \mathfrak{k} \end{array} \right.$$

$$\eta \in S^2 \mathfrak{p}^*$$

$$\left. \begin{array}{l} \eta(e_i, e_j) = \delta_{ij} \\ \eta(e_+, e_-) = 1 \end{array} \right\} \text{K-INVARIANT}$$

$\therefore (G/K, \eta)$ is a **LORENTZIAN SYMMETRIC SPACE**

FURTHER,

INDECOMPOSABLE \iff A_{ij} NON DEGENERATE

In natural coordinates (x^i, x^\pm)

CW
PP-WAVE
METRIC

$$g = 2dx^+ dx^- + \sum_{i,j} A_{ij} x^i x^j (dx^-)^2 + \sum_i (dx^i)^2$$

4. supplemented by:

$$D=11 \left\{ \begin{array}{l} F = dx^- \wedge C \\ C = \frac{1}{3!} \sum_{i,j,k} C_{ijk} dx^i \wedge dx^j \wedge dx^k \end{array} \right.$$

\swarrow CONSTANT
 $\in \Lambda^3 \mathbb{R}^9$

or

$$II B \left\{ \begin{array}{l} F = dx^- \wedge B \\ B \in \Lambda^4_+ \mathbb{R}^8 \end{array} \right.$$

or ...

Symmetries

Isometries of g : $G \times SO_A$
 \uparrow
isotropy of $SO(D-2)$ preserving A
 \nearrow subgroup

Symmetries of supergravity solution:

$$G \times (SO_A \cap SO_F)$$

\uparrow
 C $SO(D-2)$ preserving F

5.

CW pp-WAVES are natural Ansätze for
 SUGRA solutions preserving $\nu \geq \frac{1}{2}$ susy.

e.g. KOWALSKI-GLIKMAN SOLUTION of $D=11$

$$A_{ij} = \left(\begin{array}{c|c} -\mathbb{1}_3 & \\ \hline & -\frac{1}{4} \mathbb{1}_6 \end{array} \right)$$

$$\boxed{\nu = 1}$$

$$C = 3 dx^1 \wedge dx^2 \wedge dx^3$$

Symmetries: $G \times (SO_3 \times SO_6)$

$$\dim = 38$$

e.g. IIB PP-WAVE

$$A_{ij} = -\mathbb{1}_8$$

$$\boxed{\nu = 1}$$

$$B = \frac{1}{2} (dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 + dx^5 \wedge dx^6 \wedge dx^7 \wedge dx^8)$$

Symmetries: $G \times (SO_4 \times SO_4)$

$$\dim = 30$$

6.

Other $\frac{1}{2} < \nu < 1$ pp-waves

have been found by

Mitchelson (hep-th/0203140)

Cvetič, Lü, Pope (hep-th/0203229)

Gaiotto, Hull (hep-th/0202255)

⋮

Other $\nu=1$ pp-WAVES in lower dimension

have been found by

Meesseu (hep-th/0111031)

Similarities between $\nu=1$ solutions

D=11

AdS₉ × S²

AdS₇ × S⁴

CW pp-wave

} All are LSS & dimension of symmetry groups agrees:

$$\dim(SO(3,2) \times SO(8)) = \dim(SO(2,6) \times SO(5)) = 38$$

Closer inspection \Rightarrow CW pp-WAVE isometry algebra is a contraction of $SO(3,2) \times SO(8)$ & $SO(2,6) \times SO(5)$.

“LIMIT”

7.

IIB

$$\left. \begin{array}{l} \text{AdS}_5 \times \text{S}^5 \\ \text{PP-WAVE} \end{array} \right\} \text{ Also conformally flat.}$$
ALSO:

CW PP-WAVES embed isometrically
in $\mathbb{R}^{2,D}$:

$$\left\{ \begin{array}{l} U_1^2 + U_2^2 = 1 \\ U_1 V_1 + U_2 V_2 = \sum_{i,j=1}^{D-2} A_{ij} X^i X^j \end{array} \right.$$

$(\underbrace{U_1, V_1}_{\text{null}}, \underbrace{U_2, V_2}, X^i)$ coords. of $\mathbb{R}^{2,D}$

(cf. $\text{AdS} \times \text{S}$)

8.

PENROSE - GÜVEN LIMITS $\gamma \subset M$ null geodesic

Choose coordinates: (U, V, Y^i)

U : affine parameter
 V : twist-free
 Y^i : label geodesic congruence

$$g = dV \left(dU + \alpha dV + \sum_i \beta_i dY^i \right) + \sum_{ij} C_{ij} dY^i dY^j$$
Boost in (U, V) -plane + rescale (Y^i)

$$U = u$$

$$V = \Omega^2 v$$

$$Y^i = \Omega y^i$$

Define

$$\bar{g} := \lim_{\Omega \rightarrow 0} \Omega^{-2} g(\Omega)$$

Changing coordinates...

$$\bar{g} = 2dx^+ dx^- + \sum_{ij} \boxed{A_{ij}(x^-)} x^i x^j (dx^-)^2 + \sum (dx^i)^2$$

10.

$(\bar{g}, \bar{F}, \dots)$ inherits the following properties from (g, F, \dots) :

- CURVATURE EQUATIONS

eg: CONFORMAL FLATNESS (Weyl = 0)

LOCAL SYMMETRY ($\nabla \text{Riemann} = 0$)

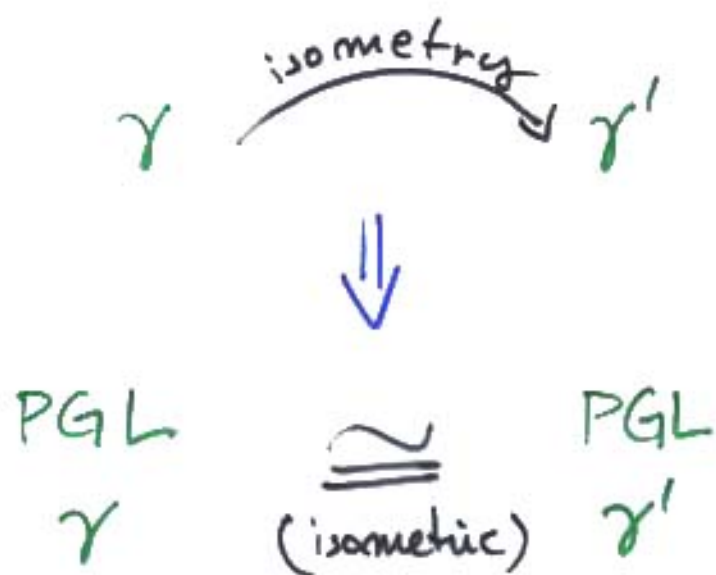
EINSTEIN \rightarrow RICCI-FLAT

- ISOMETRIES

- CONFORMAL ISOMETRIES

- SUPERSYMMETRIES

A COVARIANCE PROPERTY



11.

ALLOWS CLASSIFICATION OF PGL'S
OF SOLUTIONS WITH \wedge SYMMETRY :
ENOUGH

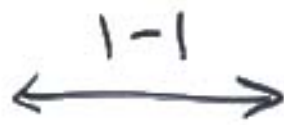
γ determined from

$$\gamma(0) = p \in M$$

$$[\dot{\gamma}(0)] \in \text{Celestial sphere in } T_p M$$

\uparrow a null direction at p

Different
PGL's



Orbits of isometry
group of M in
"celestial sphere
bundle"

eg: $AdS_p \times S^q$

• HOMOGENEOUS $\Rightarrow p$ arbitrary

• ISOTROPY at p has 2 orbits on
celestial sphere in $T_p M$

- BIG (generic) \Rightarrow CW pp-WAVE

- SMALL ($\dot{\gamma}(0) \cap T_p S = 0$) \Rightarrow flat

12.

eg: Brane solutions, ... (see paper!)

Finally ...

\exists PGL's for multi-time
spacetimes

'null planes' replace γ 's

⌞