Spontaneous Breaking of Rotational Symmetry in IIB Matrix Model

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0. Introduction

The IIB Matrix Model

(dimensional reduction of 10D SYM reduced to 0D)

$$S=-\frac{1}{g^2}Tr(\frac{1}{4}[A^\mu,A^\nu]^2+\frac{1}{2}\overline{\Psi}\Gamma^\mu[A^\mu,\Psi])$$

A^μ ($\mu=1\sim10$),

Ψ (10D Majorana-Weyl) : N×N hermitian

is one of the candidates of the constructive definition of string theory.

Although we have no rigorous proof, there are a few arguments which support this conjecture.

(i) World sheet regularization

The first argument is that the IIB matrix model can be regarded as a matrix regularization of the Green-Schwartz action in the Schild Gauge.

Green-Schwartz action in the Schild Gauge

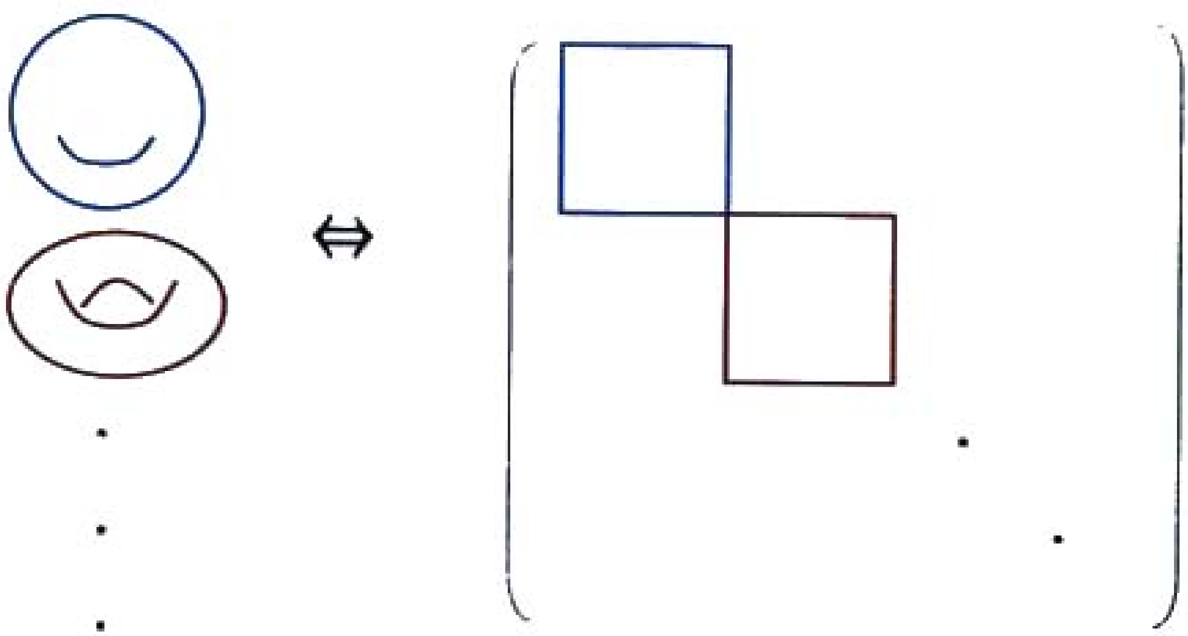
$$S=\int d^2\xi\,(\frac{1}{4}\{X^\mu,X^\nu\}^2+\frac{1}{2}\overline{\Psi}\gamma^\mu\{X^\mu,\Psi\})$$

Regularization by matrix

$$\downarrow \quad \{\ ,\ \}\rightarrow[\ ,\ ] \qquad \int \rightarrow Tr$$

$$S=-\frac{1}{g^2}Tr(\frac{1}{4}[A^\mu,A^\nu]^2+\frac{1}{2}\overline{\Psi}\gamma^\mu[A^\mu,\Psi])$$

Furthermore we can expect that multistring states are naturally embedded, if the size of the matrix is large enough.

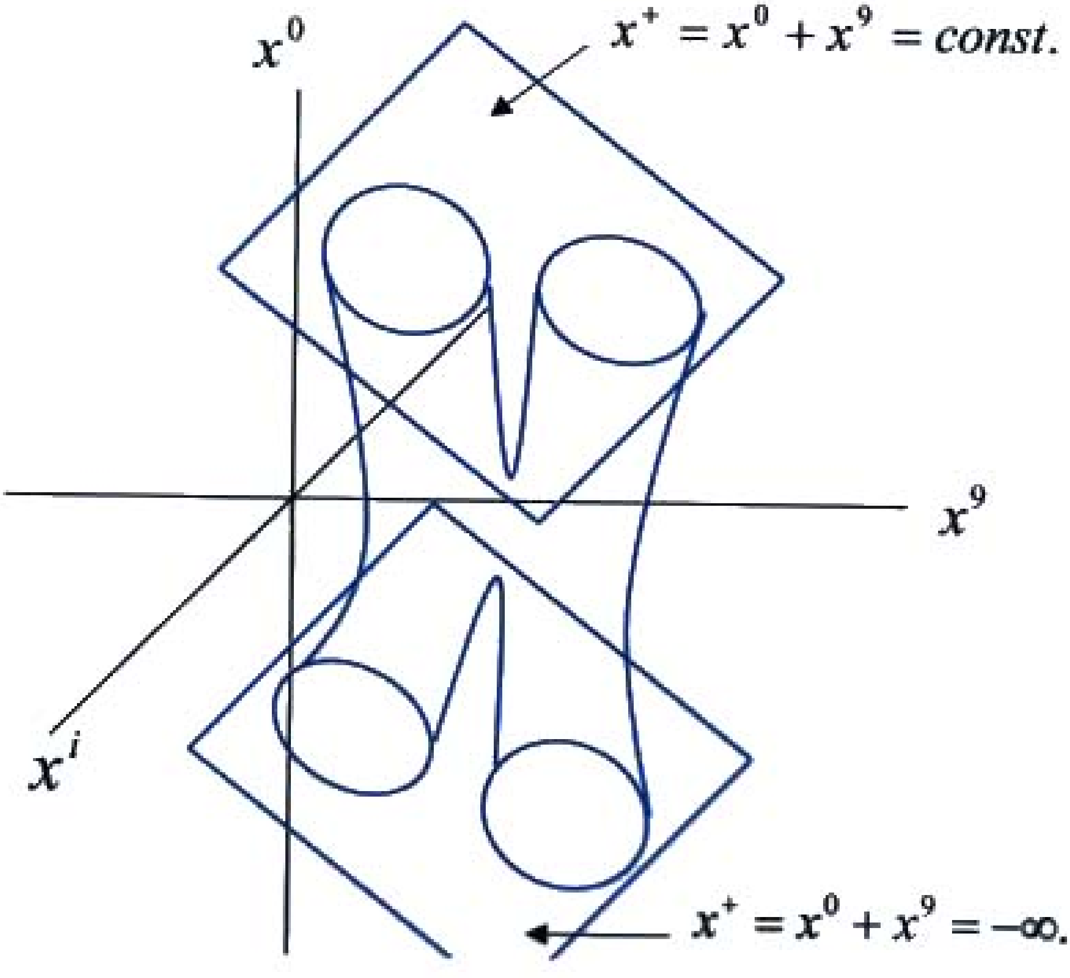


(ii) Loop equation and string field

The second argument is the relation between the loop equation and the string field theory. Although it is not complete, by combining the loop equations we can show that the x^+ dependence of the Wilson loop correlators is identical to that of the light-cone string field.

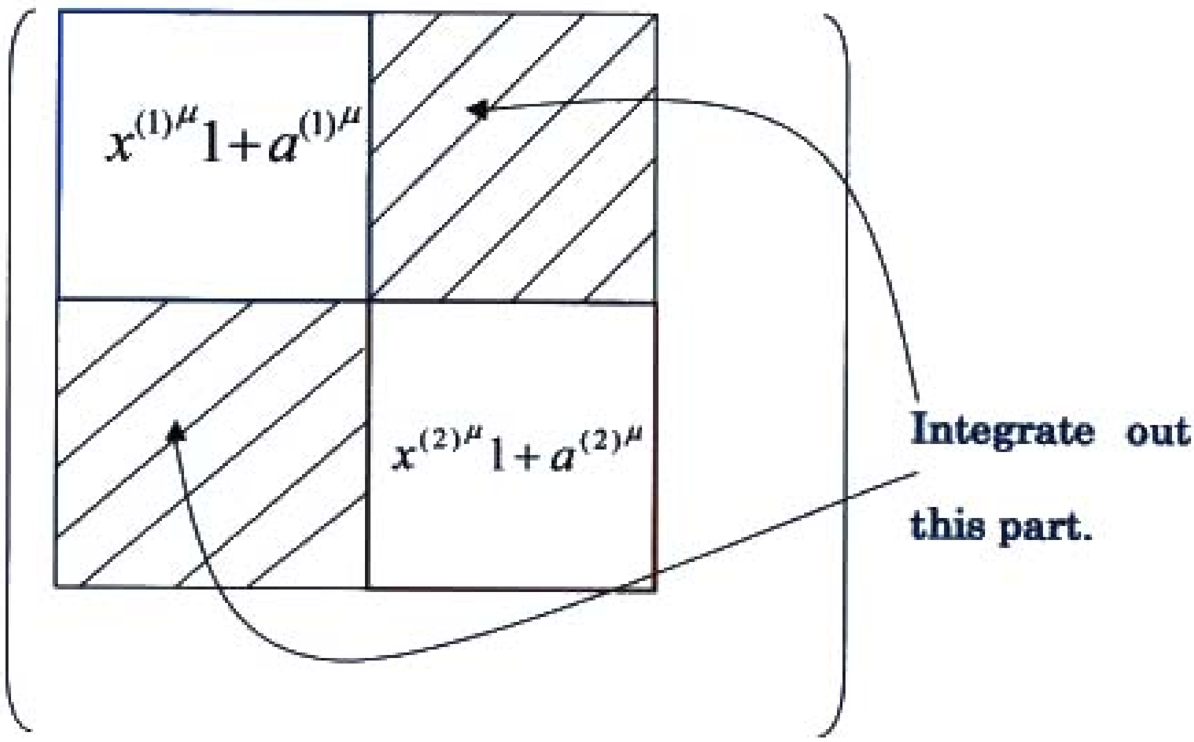
Wilson loop $w(k_\mu(\cdot))=Tr(P\exp(i\oint d\sigma\, k_\mu(\sigma) A^\mu + fermion))$

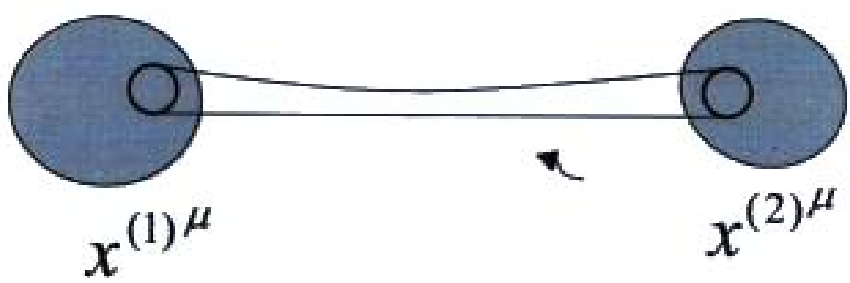
$\Leftrightarrow$ **creation annihilation operator of** $|k_\mu(\cdot)>$



(iii) effective Lagrangian and gravity

The third argument is that the one-loop integral reproduces the exchange of massless states of type IIB string theory, which is reminiscent of open string field theory.





$$S_{eff} = \frac{1}{(x^{(1)} - x^{(2)})^8}\{const \cdot tr(f^{(1)}{}_{\mu\lambda} f^{(1)}{}_{\nu\lambda})\, tr(f^{(2)}{}_{\mu\lambda} f^{(2)}{}_{\nu\lambda})$$
$$- const \cdot tr(f^{(1)}{}_{\mu\nu} f^{(1)}{}_{\mu\nu})\, tr(f^{(2)}{}_{\lambda\rho} f^{(2)}{}_{\lambda\rho}) + \cdots\}$$

(iv) finiteness of the path integral

Finally it should be mentioned that the finiteness of the path integral for various operators

$$\int dA\, d\Psi\, O\, e^{-S}$$

is rigorously proved. in spite of the flat directions

Austing and Wheater,
Krauth, Nicolai and Staudacher,
Suyama and Tsuchiya,
Ambjorn, Anagnostopoulos, Bietenholz, Hotta and Nishimura,
Bialas, Burda, Petersson and Tabaczek,
Green and Gutperle,
Moore, Nekrasov and Shatashvili.

On the other hand the IIB matrix model has the following difficulties.

(i) compactified space

One problem is that the action

$$S=-\frac{1}{g^2}Tr(\frac{1}{4}[A^\mu, A^\nu]^2+\frac{1}{2}\overline{\Psi}\Gamma^\mu[A^\mu,\Psi])$$

looks like an expansion around the flat space, and it is not clear how to describe the compactified space.

⇒ a. Still it is OK?

b. We need a modification?

c. It is correct, but there is a better formulation?

(ii) method for calculation

Another problem is that we do not have a good way to evaluate the correlation functions. Even the Monte Carlo method is hard to apply because the fermion determinant is not positive definite. Because of this difficulty we can not easily determine the N-dependence of the physical scale l_{string} :

$$l_{string} = N^{\alpha} g^{\frac{1}{2}}, \qquad \alpha\,?$$

1. The improved perturbation theory

However Nishimura and Sugino have obtained promising results by using the improved perturbation theory, which was originally applied to the BFSS M(-atrix) theory by Kabat and Lifschytz.

Since the IIB matrix model

$$S=-\frac{1}{g^2}Tr(\frac{1}{4}[A^\mu,A^\nu]^2+\frac{1}{2}\overline{\Psi}\gamma^\mu[A^\mu,\Psi])$$

does not have a quadratic term, it seems impossible to apply the perturbation theory.

However we can artificially add and subtract a quadratic term S_0 to the action S such that $S=(S-S_0)+S_0$ and apply the perturbation theory regarding $(S-S_0)$ as the perturbation.

An example of the improved perturbation theory

In order to show the idea we consider a large-N one-matrix model for a while

$$\frac{S}{N} = \frac{1}{4}Tr(\phi^4) + \frac{m^2}{2}Tr(\phi^2).$$

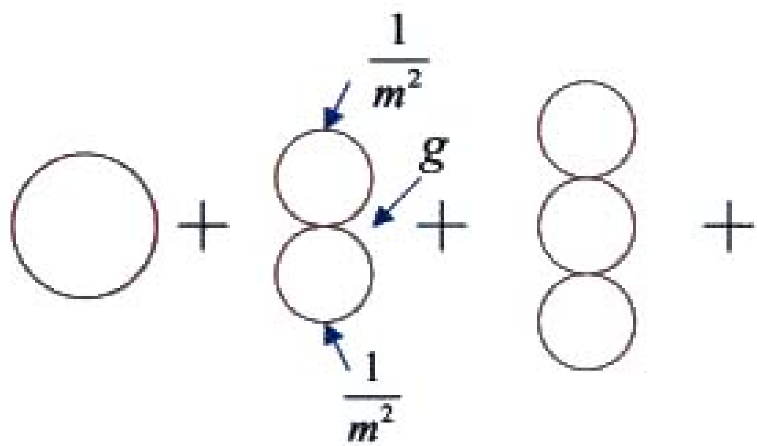
We introduce a formal expansion parameter g as

$$\frac{S}{N} = \frac{g}{4}Tr(\phi^4) + \frac{m^2}{2}Tr(\phi^2),$$

and set g=1 after applying the perturbative expansion.

The naïve perturbation theory gives an expansion with respect to $\frac{g}{m^4}$. For example, the free energy is given by

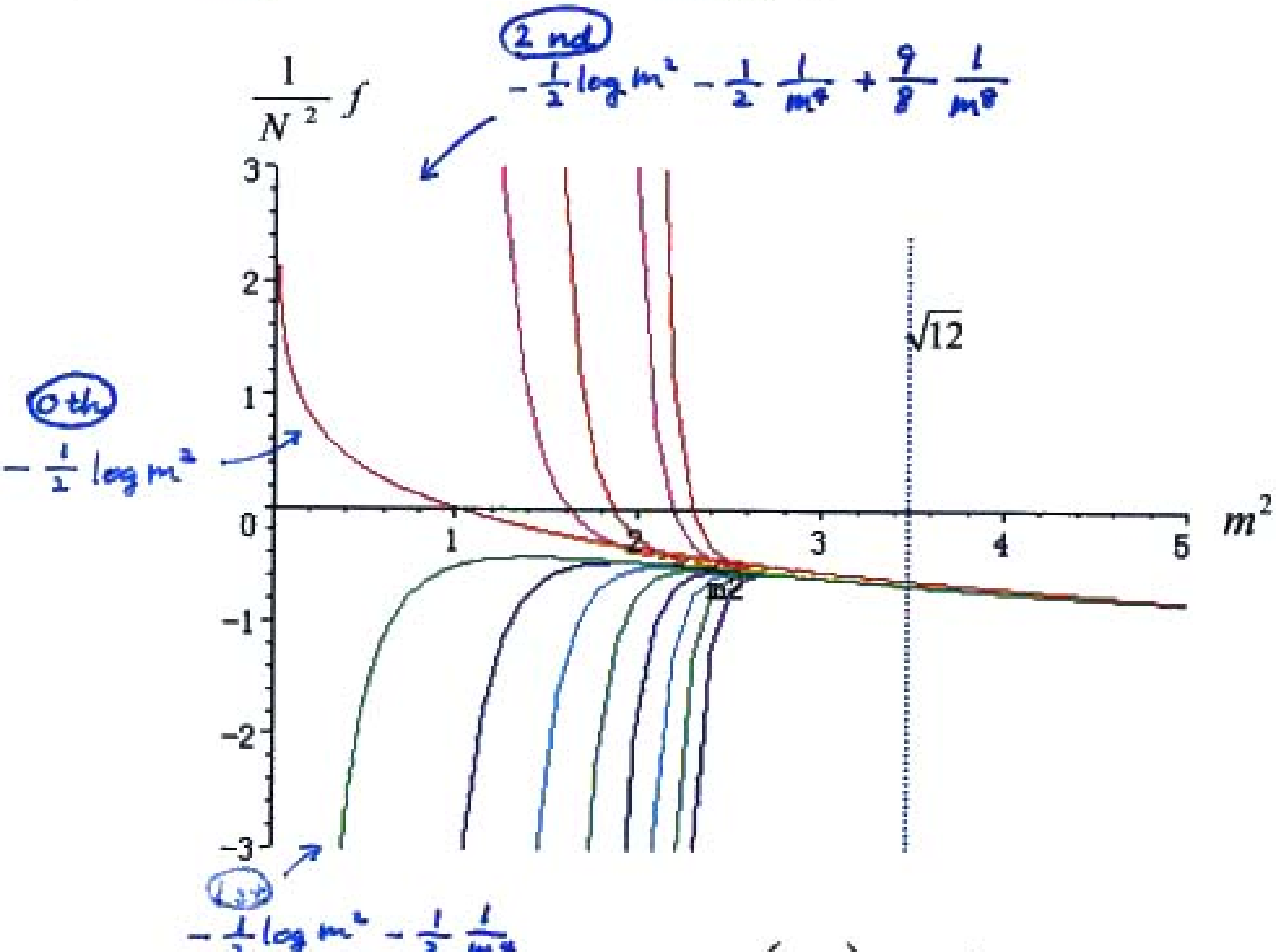
$$\frac{1}{N^2}f(g,m^2) =$$



,

$$\frac{1}{N^2} f(g,m^2) = -\frac{1}{2}\log(m^2) - \frac{1}{2}\frac{g}{m^4} + \frac{9}{8}\frac{g^2}{m^8} - \frac{9}{2}\frac{g^3}{m^{12}} + \frac{189}{8}\frac{g^4}{m^{16}}$$
$$-\frac{729}{5}\frac{g^5}{m^{20}} + \frac{8019}{8}\frac{g^6}{m^{24}} - \frac{104247}{14}\frac{g^7}{m^{28}} + \frac{938223}{16}\frac{g^8}{m^{32}} + \cdots .$$

The n-th order perturbation which is obtained by truncating this expression with respect to g up to order n and setting g=1 is depicted in the following graph.



Since the convergence radius of $\left(\frac{g}{m^4}\right)_{crit} = \frac{1}{12}$, the series for g=1 $f(g=1, m^2)$ converges only if $m^2 > \sqrt{12}$. Therefore the naïve perturbation theory does not work if $m^2 < \sqrt{12}$.

Improvement of the perturbation series

However as we will see, the improved perturbation theory works even for $m^2 = 0$.

In the naïve perturbation theory we decompose the action as

$$\frac{S}{N} = \underbrace{\frac{1}{4}Tr(\phi^4)}_{\text{perturbation}} + \underbrace{\frac{m^2}{2}Tr(\phi^2)}_{\text{unperturbed action}}$$

But in the improved perturbation theory we introduce an artificial mass parameter $m_0{}^2$ and decompose the action as

renormalization,
self-consistent approximation,
mean-field approximation···

$$\begin{aligned}\frac{S}{N} &= \frac{1}{4}Tr(\phi^4) + \frac{m^2}{2}Tr(\phi^2) \\ &= \underbrace{\frac{1}{4}Tr(\phi^4) + \frac{\left(m^2 - m_0{}^2\right)}{2}Tr(\phi^2)}_{\text{perturbation}} + \underbrace{\frac{m_0{}^2}{2}Tr(\phi^2)}_{\text{unperturbed action}}\end{aligned}$$

In other words, we introduce the formal expansion parameter g as

$$\frac{S}{N}=g\left(\frac{1}{4}Tr(\phi^4)+\frac{\left(m^2-m_0{}^2\right)}{2}Tr(\phi^2)\right)+\frac{m_0{}^2}{2}Tr(\phi^2).$$

and expand $e^{-\frac{S}{N}}$ with respect to g.

On the other hand the naïve perturbation series was given by $\frac{S}{N}=\frac{g}{4}Tr(\phi^4)+\frac{m^2}{2}Tr(\phi^2)$.

Therefore in order to obtain the improved perturbation series what we have to do is to replace m^2 in the naïve perturbation series $f(g,m^2)$ with $m_0{}^2+g\left(m^2-m_0{}^2\right)$ and reexpand it with respect to g:

$$f_{improved}(g,m^2,m_0{}^2)=f(g,m_0{}^2+g(m^2-m_0{}^2)).$$

Note that $f_{improved}$ depends not only on g and m^2 but also on $m_0{}^2$, which is an analogue of the renormalization point.

The concrete procedure is as follows.

We start with the naïve perturbation series.

$$\frac{1}{N^2}f(g,m^2)=-\frac{1}{2}\log(m^2)-\frac{1}{2}\frac{g}{m^4}+\frac{9}{8}\frac{g^2}{m^8}-\frac{9}{2}\frac{g^3}{m^{12}}+\frac{189}{8}\frac{g^4}{m^{16}}$$
$$-\frac{729}{5}\frac{g^5}{m^{20}}+\frac{8019}{8}\frac{g^6}{m^{24}}-\cdots$$

$$\Big\downarrow \quad m^2 \rightarrow m_0^{\,2}+g\left(m^2-m_0^{\,2}\right)$$

We then apply the substitution and reexpand the result with respect to g to obtain the improved series.

$$\frac{1}{N^2}f_{improved}(g,m^2,m_0^{\,2})$$

$$=-\frac{1}{2}\log(m_0^{\,2}+g(m^2-m_0^{\,2}))-\frac{1}{2}\frac{g}{(m_0^{\,2}+g(m^2-m_0^{\,2}))^2}$$
$$+\frac{9}{8}\frac{g^2}{(m_0^{\,2}+g(m^2-m_0^{\,2}))^4}-\frac{9}{2}\frac{g^3}{(m_0^{\,2}+g(m^2-m_0^{\,2}))^6}+\frac{189}{8}\frac{g^4}{(m_0^{\,2}+g(m^2-m_0^{\,2}))^8}$$
$$-\frac{729}{5}\frac{g^5}{(m_0^{\,2}+g(m^2-m_0^{\,2}))^{10}}+\frac{8019}{8}\frac{g^6}{(m_0^{\,2}+g(m^2-m_0^{\,2}))^{12}}-\cdots$$

$$=-\frac{1}{2}\log(m_0^{\,2})+\left(-\frac{1}{2}\frac{1}{m_0^{\,4}}-\frac{1}{2}\frac{m^2-m_0^{\,2}}{m_0^{\,2}}\right)g+\left(\frac{9}{8}\frac{1}{m_0^{\,8}}+\frac{m^2-m_0^{\,2}}{m_0^{\,6}}+\frac{1}{4}\frac{(m^2-m_0^{\,2})^2}{m_0^{\,4}}\right)g^2$$
$$+\left(-\frac{9}{2}\frac{1}{m_0^{\,12}}-\frac{9}{2}\frac{m^2-m_0^{\,2}}{m_0^{\,10}}-\frac{3}{2}\frac{(m^2-m_0^{\,2})^2}{m_0^{\,8}}-\frac{1}{6}\frac{(m^2-m_0^{\,2})^3}{m_0^{\,6}}\right)g^3+\cdots$$

Finally we truncate it up to order n and set g=1 to get the improved perturbation series at order n.

0th order: $f_{improved}^{(0)} = -\frac{1}{2}\log(m_0^{\ 2})$

1st order: $f_{improved}^{(1)} = -\frac{1}{2}\log(m_0^{\ 2}) + \left(-\frac{1}{2}\frac{1}{m_0^{\ 4}} - \frac{1}{2}\frac{m^2 - m_0^{\ 2}}{m_0^{\ 2}}\right)$

2nd order:

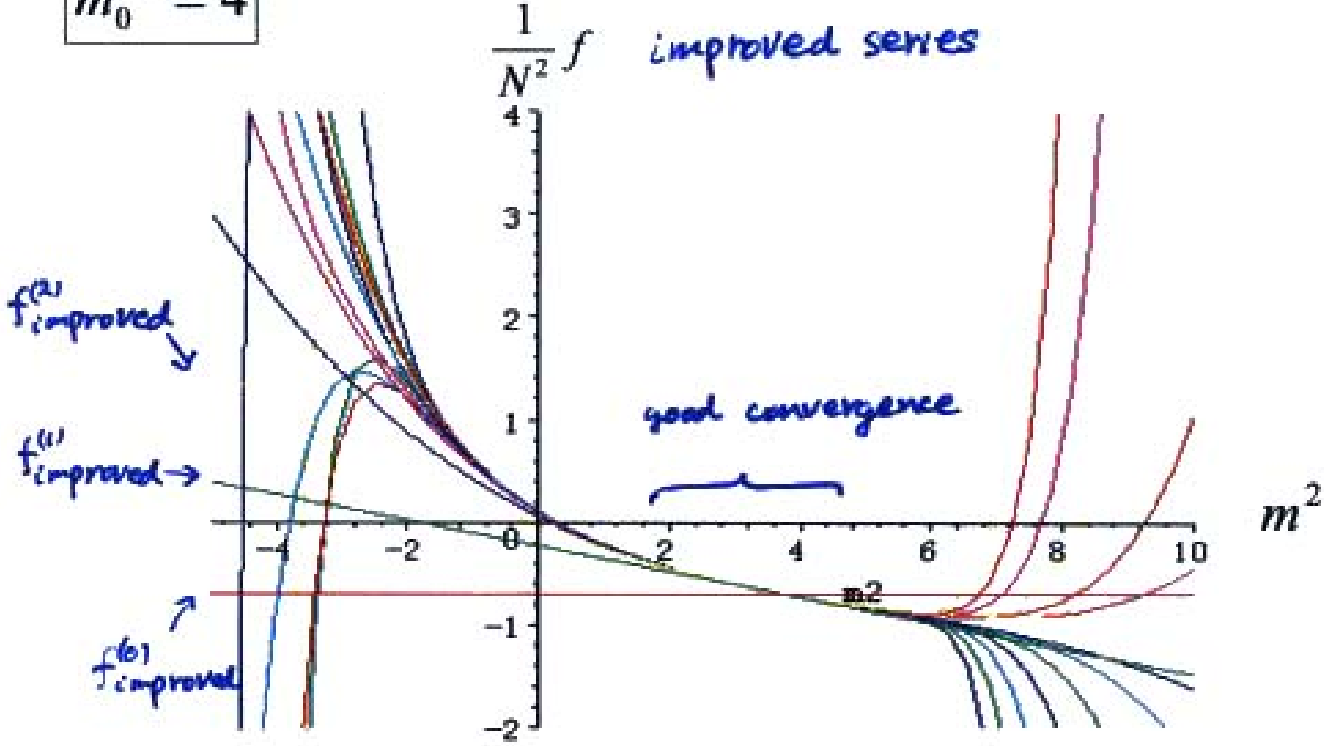
$$f_{improved}^{(2)} = -\frac{1}{2}\log(m_0^{\ 2}) + \left(-\frac{1}{2}\frac{1}{m_0^{\ 4}} - \frac{1}{2}\frac{m^2 - m_0^{\ 2}}{m_0^{\ 2}}\right) + \left(\frac{9}{8}\frac{1}{m_0^{\ 8}} + \frac{m^2 - m_0^{\ 2}}{m_0^{\ 6}} + \frac{1}{4}\frac{(m^2 - m_0^{\ 2})^2}{m_0^{\ 4}}\right)$$

. . .

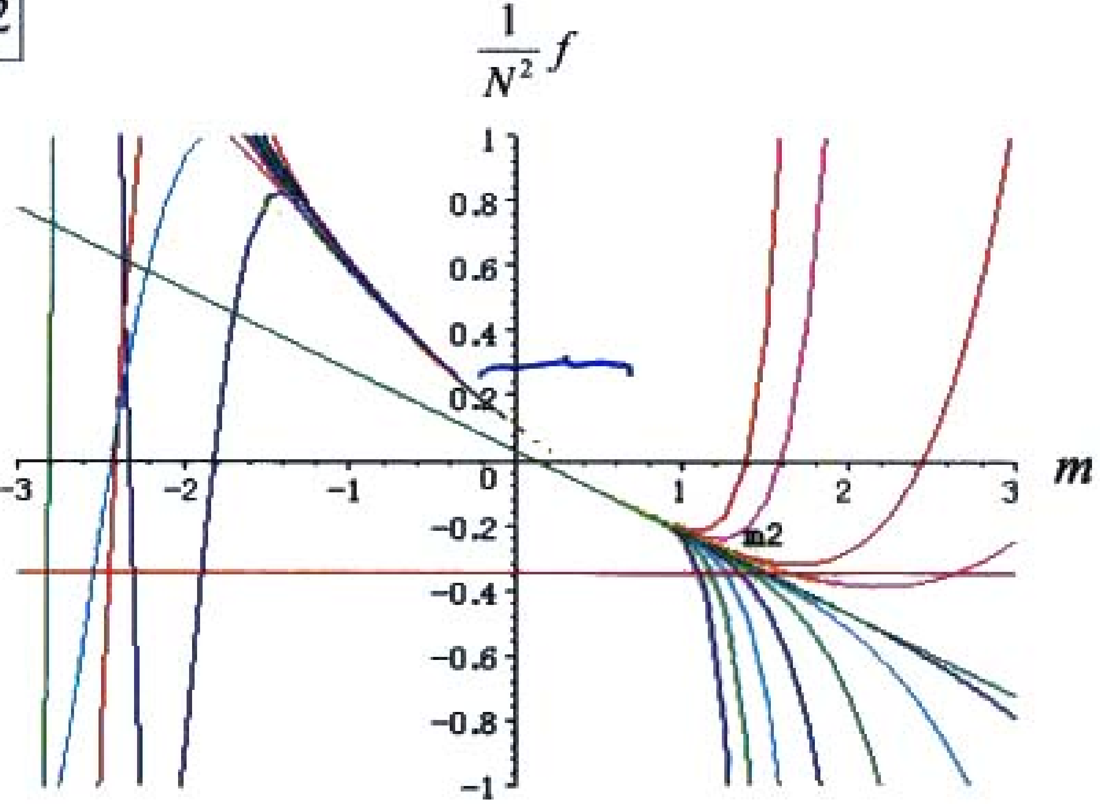
The behavior of this improved series for various values of $m_0^{\ 2}$ is depicted in the following graphs.

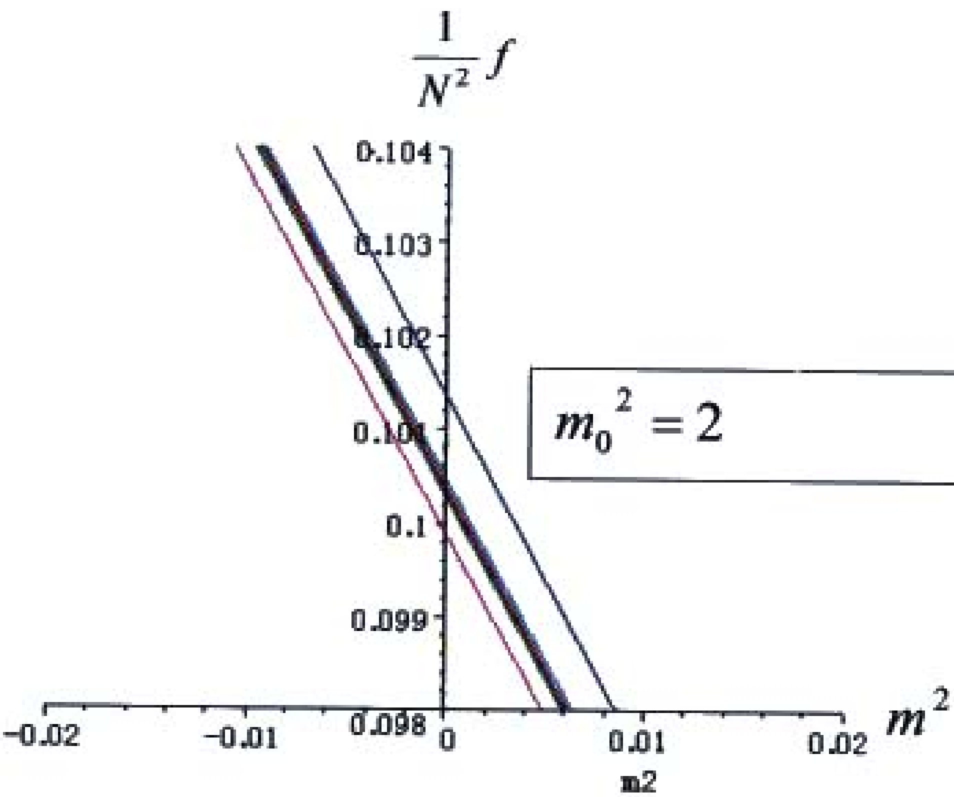
The improved perturbation series for various values of $m_0^{\ 2}$

$m_0^{\ 2} = 4$

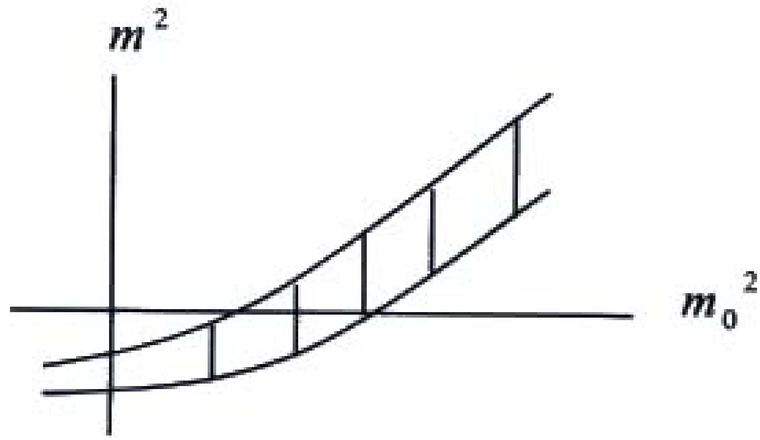


$m_0^{\ 2} = 2$





It seems that for each value of m_0^2 the improved series converges on a finite domain of m^2.



If we choose the value of $m_0^{\ 2} \approx 2$, the improved perturbation theory gives a convergent series even for the massless case $m^2 = 0$, which corresponds to the strong coupling limit $\frac{g}{m^4} \to \infty$. (We have set g=1.)

Free energy as a function of m_0^2

We have found that we can tune "the renormalization point" m_0^2 in such a way that the improved series converges well. In order to avoid this tuning procedure we can regard the free energy as a function of m_0^2.

Here to be concrete we consider the massless case $m^2 = 0$.

Namely we consider $f_{improved}(g=1, m^2=0, m_0^2)$ as a function of m_0^2.

By setting $m^2 = 0$ in the previous expressions, we obtain the improved series as a function of m_0^2 as follows:

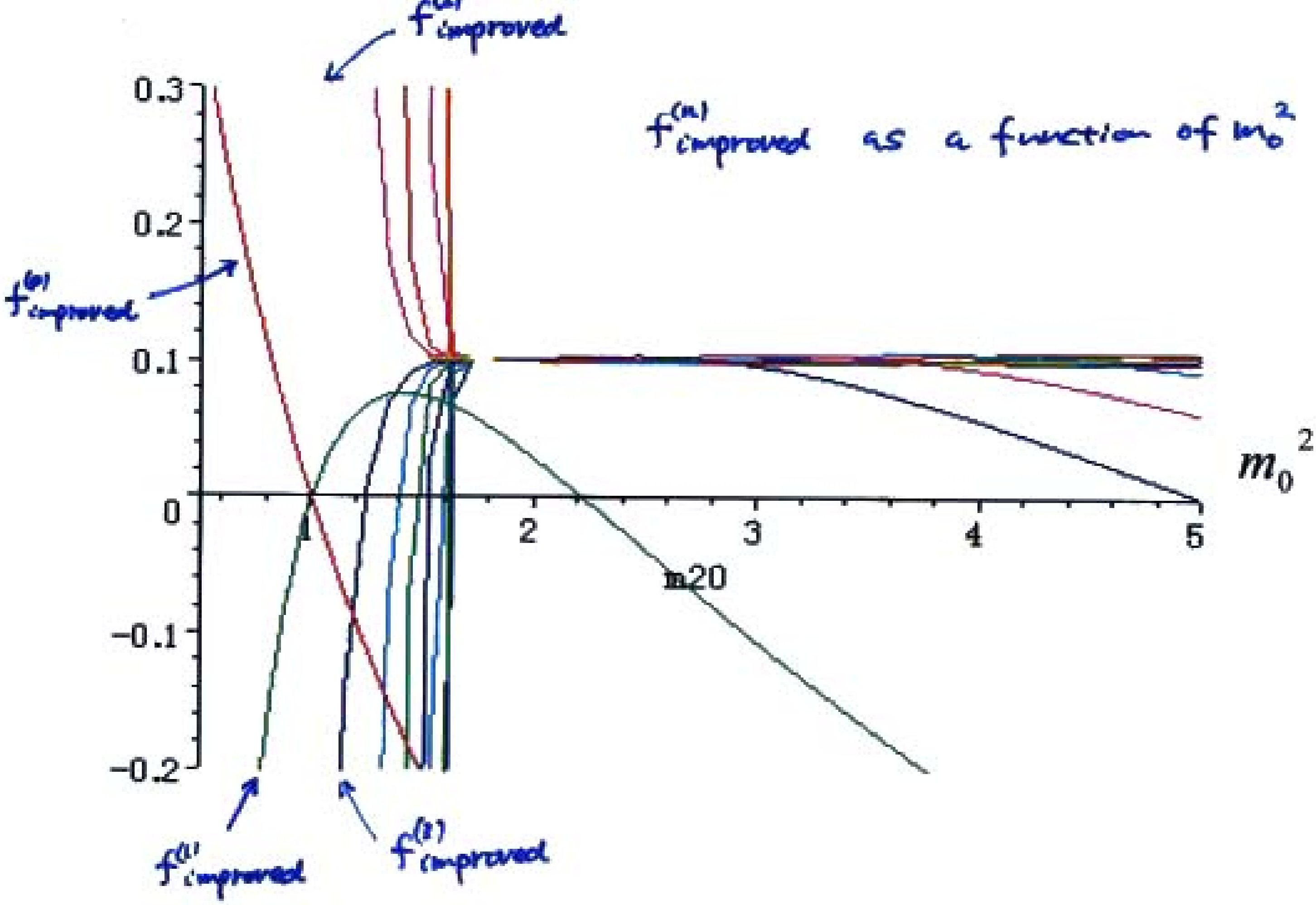
0th order: $f_{improved}^{(0)} = -\frac{1}{2}\log(m_0^2)$

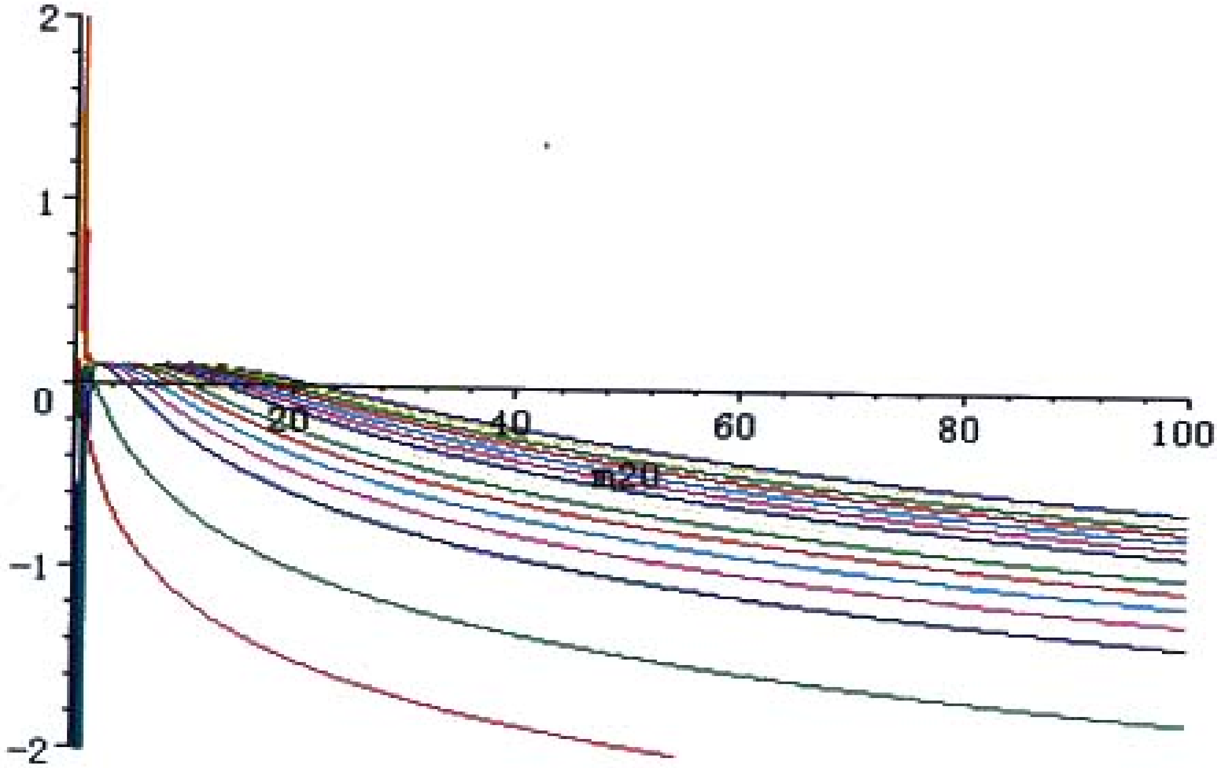
1st order: $f_{improved}^{(1)} = -\frac{1}{2}\log(m_0^2) + \left(-\frac{1}{2}\frac{1}{m_0^4} + \frac{1}{2}\right)$

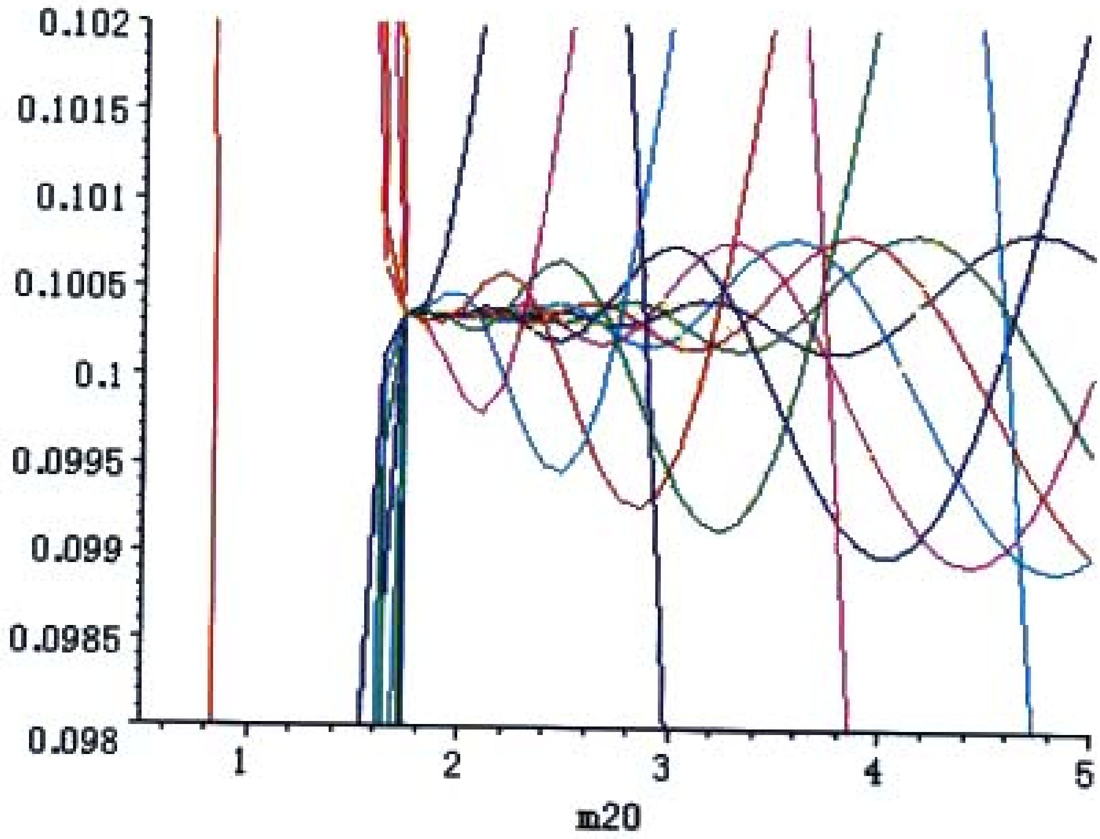
2nd order: $f_{improved}^{(2)} = -\frac{1}{2}\log(m_0^2) + \left(-\frac{1}{2}\frac{1}{m_0^4} + \frac{1}{2}\right) + \left(\frac{9}{8}\frac{1}{m_0^8} - \frac{1}{m_0^4} + \frac{1}{4}\right)$

...

As is clear from the following graphs, there is a plateau region where the function $f_{improved}(g=1, m^2=0, {m_0}^2)$ stays at a constant, and we can check it is close to the exact value. Furthermore the plateau becomes more stable as we go to higher orders, and the value converges to the exact one.




2
1
0
-1
-2
20
40
60
80
100
m20


0.102
0.1015
0.101
0.1005
0.1
0.0995
0.099
0.0985
0.098
1
2
3
4
5
m20

Why do we have a plateau?

There is no rigorous proof, but it is expected since $m_0^{\,2}$ is an artificially introduced parameter and the physical quantities should not depend on it.

This was stated by Stevenson some time ago (1981) as "the principle of minimum sensitivity", and his claim is that $m_0^{\,2}$ should be chosen as a stationary point of $f_{improved}(g, m^2, m_0^{\,2})$.

$$\frac{\partial}{\partial m_0^{\,2}} f_{improved}(g, m^2, m_0^{\,2}) = 0 .$$

However it would be more precise to express this principle in terms of "the plateau", because in general there are many extrema as we go to higher orders and it is the value of the plateau that converges to the exact value.

2. Application to the IIB matrix model

Nishimura and Sugino

Kawai, Kawamoto, Kuroki, Matsuo and Shinohara

We now apply the improved perturbation theory to the IIB matrix model. We add and subtract the most general U(N)-invariant quadratic terms to the action:

$$\frac{S}{N} = -\frac{\lambda}{4} Tr\,([A^{\mu}, A^{\nu}]^2) - \frac{\lambda^{\frac{1}{2}}}{2} Tr\,(\overline{\Psi}\Gamma^{\mu}[A^{\mu}, \Psi])$$
$$-\frac{\lambda}{2} M_{\mu\nu} Tr\,(A^{\mu} A^{\nu}) - \frac{\lambda}{2} m_{\alpha\beta\gamma} Tr\,(\overline{\Psi}\Gamma^{\alpha\beta\gamma}\Psi)$$
$$+\frac{1}{2} M_{\mu\nu} Tr\,(A^{\mu} A^{\nu}) + \frac{1}{2} m_{\alpha\beta\gamma} Tr\,(\overline{\Psi}\Gamma^{\alpha\beta\gamma}\Psi)$$

Here we have chosen the scale such that $g^2 N = 1$ in the action, and we have

$$\frac{S}{N} = -\frac{1}{g^2 N} Tr(\frac{1}{4}[A^{\mu}, A^{\nu}]^2 + \frac{1}{2}\overline{\Psi}\Gamma^{\mu}[A^{\mu}, \Psi])$$
$$= -\frac{1}{4} Tr([A^{\mu}, A^{\nu}]^2) - \frac{1}{2} Tr(\overline{\Psi}\Gamma^{\mu}[A^{\mu}, \Psi]) \cdot$$

We then have introduced the formal expansion parameter λ which counts the number of loops and is set to 1 at the end.

Here $M_{\mu\nu}$ and $m_{\alpha\beta\gamma}$ correspond to $m_0{}^2$ in the previous example. What we have to do now is the following.

(i) Calculate the free energy of the massive IIB matrix model $f(\lambda, M, m)$ by the naïve perturbation theory.

$$\frac{S}{N} = -\frac{\lambda}{4} Tr\,([A^\mu, A^\nu]^2) - \frac{\lambda^{\frac{1}{2}}}{2} Tr\,(\overline{\Psi}\Gamma^\mu [A^\mu, \Psi]) + \frac{1}{2} M_{\mu\nu} Tr\,(A^\mu A^\nu) + \frac{1}{2} m_{\alpha\beta\gamma} Tr\,(\overline{\Psi}\Gamma^{\alpha\beta\gamma}\Psi)$$

(ii) Replace $M_{\mu\nu} \to (1-\lambda) M_{\mu\nu}$ and $m_{\alpha\beta\gamma} \to (1-\lambda) m_{\alpha\beta\gamma}$ in $f(\lambda, M, m)$, and reexpand it with respect to λ to obtain the improved series.

$$f_{improved}(\lambda, M, m) = f(\lambda, (1-\lambda)M, (1-\lambda)m)$$

↓ Expand with respect to λ up to order n and set $\lambda = 1$.

$$f_{improved}{}^{(0)}(M, m),\quad f_{improved}{}^{(1)}(M, m),\quad f_{improved}{}^{(2)}(M, m), \cdots$$

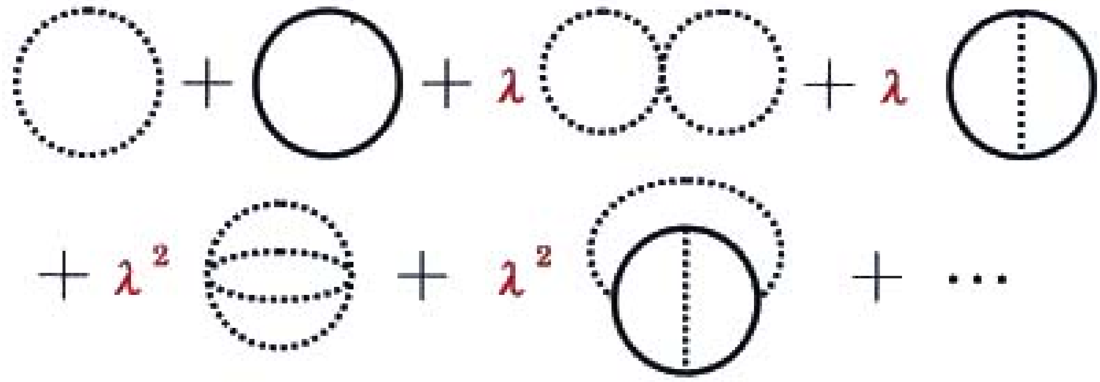
(iii) Search plateaus in the space of $M_{\mu\nu}$ and $m_{\alpha\beta\gamma}$, and determine the physical quantities.

Step(i) and (ii)

In principle the first step is only a matter of summing up Feynman diagrams, and the second step is a simple manipulation. But in order to reduce the calculation in the first step we have used the following fact:

> Suppose $G(C)$ is the sum of the 2PI diagrams expressed in terms of the exact propagator C,
>
> and $f(M)$ is the sum of all the vacuum diagrams expressed in terms of the mass M. Then $G(C)$ and $f(M)$ are related by the Legendre transformation.

Sum of the 2PI diagrams

$G =$ 

$$= -\frac{1}{2}\log(\det(C)) + \frac{1}{2}\log(\det(\not{u}))$$
$$+\lambda\left(-\frac{1}{2}tr(C^2) - \frac{1}{2}(tr(C))^2) - \frac{1}{2}C_{\mu\nu}Tr(\not{u}\Gamma^{\mu}\not{u}\Gamma^{\nu})\right).$$
$$+\cdots$$

Here $C_{\mu\nu}$ **and** $\not{u} = u_{\alpha\beta\gamma}\Gamma^{\alpha\beta\gamma}$ **are the exact propagators of** A_μ **and** Ψ**, respectively.**

Legendre Transformation

$$\frac{1}{2}M_{\mu\nu} = \frac{\partial}{\partial C^{\mu\nu}}G$$
$$\frac{1}{2}m_{\alpha\beta\gamma} = -\frac{\partial}{\partial u_{\alpha\beta\gamma}}G$$

free energy as a function of $M_{\mu\nu}$ **and** $m_{\alpha\beta\gamma}$

$$f = G - \frac{1}{2}M_{\mu\nu}C_{\mu\nu} + \frac{1}{2}m_{\alpha\beta\gamma}u_{\alpha\beta\lambda}$$

In this way we have calculated the following diagrams, and obtained the free energy up to the 5th order level.

2PI diagrams

- Extra factors:
  - −1 for each fermion loop.
  - N for each color loop.

E Planar Feynman graphs

Here we show all the Feynman graphs from the zeroth order to the fifth order. The number below each graph is the symmetry factor. The value of each graph is given by

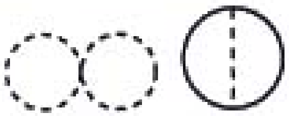
$$(-1) \times \frac{1}{\text{symmetry factor}} \times (\text{factor from Feynman rules}). \tag{E.1}$$

The first factor of minus one originates from the minus sign of the definition of the free energy, $F = -\log Z$.

E.0 zeroth order

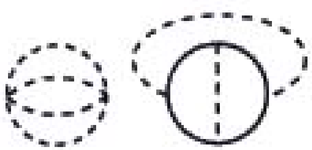
— —

E.1 first order



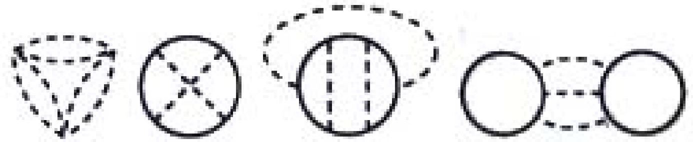
2 2

E.2 second order



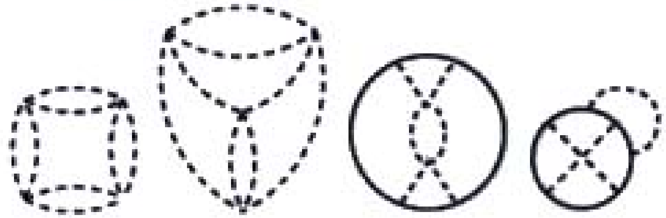
8 4

E.3 third order

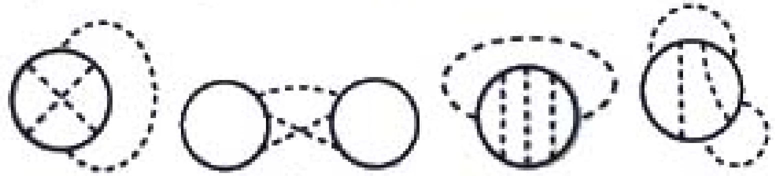


6 4 2 6

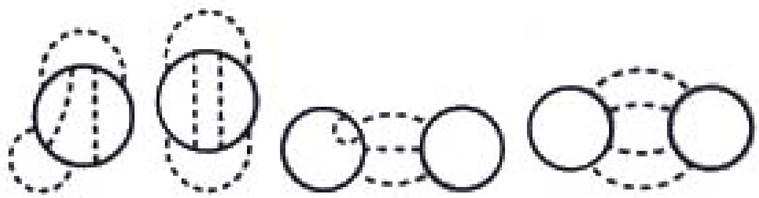
E.4 fourth order



8 4 2 1

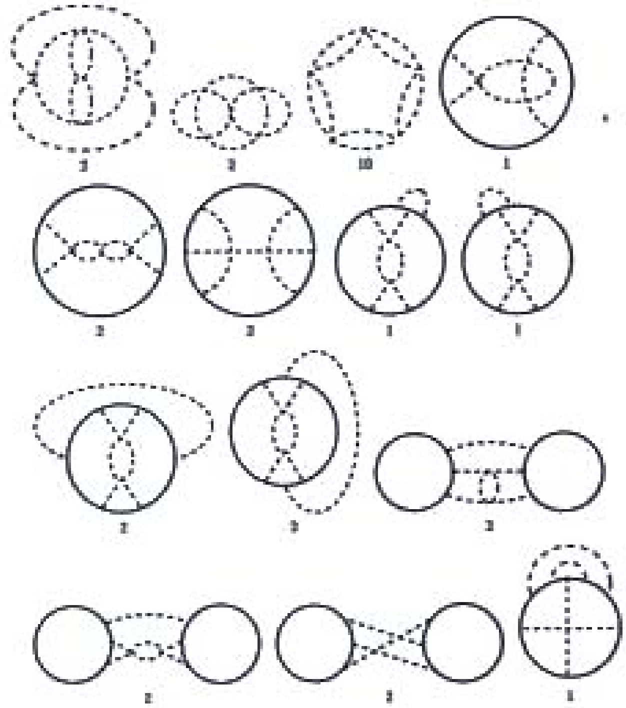


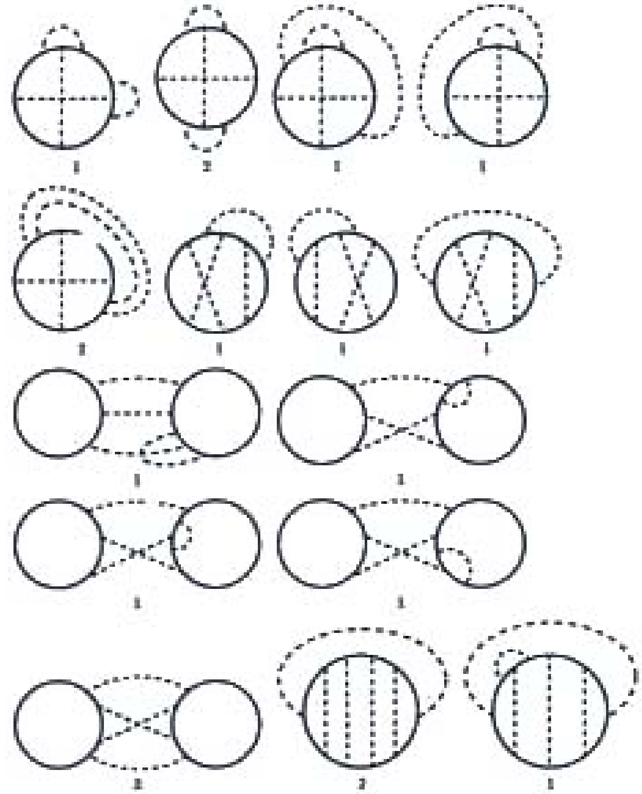
2 2 2 1

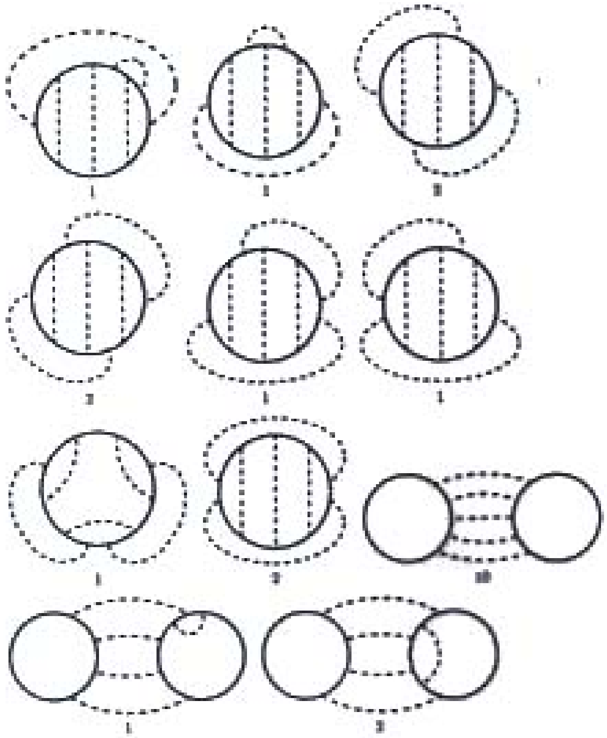


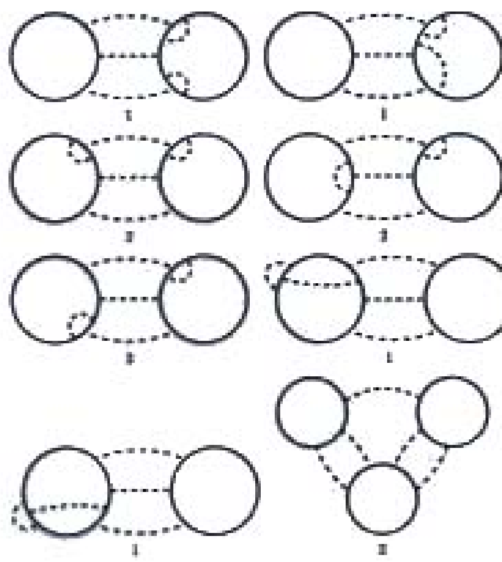
1 4 1 8

E.5 fifth order



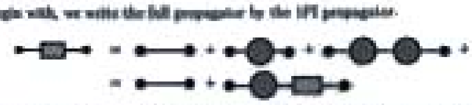






F Derivation of SDE for full propagator

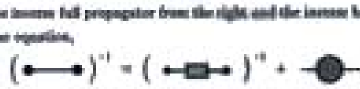
To begin with, we write the full propagator by the 1PI propagator.



(F.1)

(F.2)

By multiplying the inverse full propagator from the right and the inverse bare propagator from the left, we get the equation,



(F.3)

Step(iii)

As for searching plateaus, we have too many parameters $M_{\mu\nu}$ and $m_{\alpha\beta\gamma}$, which makes the search very difficult. In order to reduce the number of parameters we impose various symmetries on $M_{\mu\nu}$ and $m_{\alpha\beta\gamma}$. Here we discuss the following three symmetries as typical examples.

(1) SO(7)×SO(3) (We refer to this as SO(7) ansatz.)

We decompose the 10 dimensions into 7+3 and impose SO(7)×SO(3) symmetry, which reduces the parameters to

$m_{8\,9\,10}$, $M_{11} = M_{22} = \cdots = M_{77}$ and $M_{88} = M_{99} = M_{1010}$.

(2) SO(4)×SO(3)×SO(3) ×Z2 (SO(4) ansatz)

We decompose the 10 dimensions into 4+3+3, and in addition to the rotational symmetry in each block we impose an extra Z_2 symmetry that permutes the last two blocks. The non-zero parameters are

$m_{5\,6\,7} = m_{8\,9\,10}$, $M_{11} = \cdots = M_{44}$ and $M_{55} = \cdots = M_{1010}$.

(3) SO(1)×SO(3)×SO(3)×SO(3) ×Z3 (SO(1) ansatz)

We decompose the 10 dimensions into 1+3+3+3, and in addition to the rotational symmetry in each block we impose an extra Z_3 symmetry that permutes the last three blocks. The non-zero parameters are

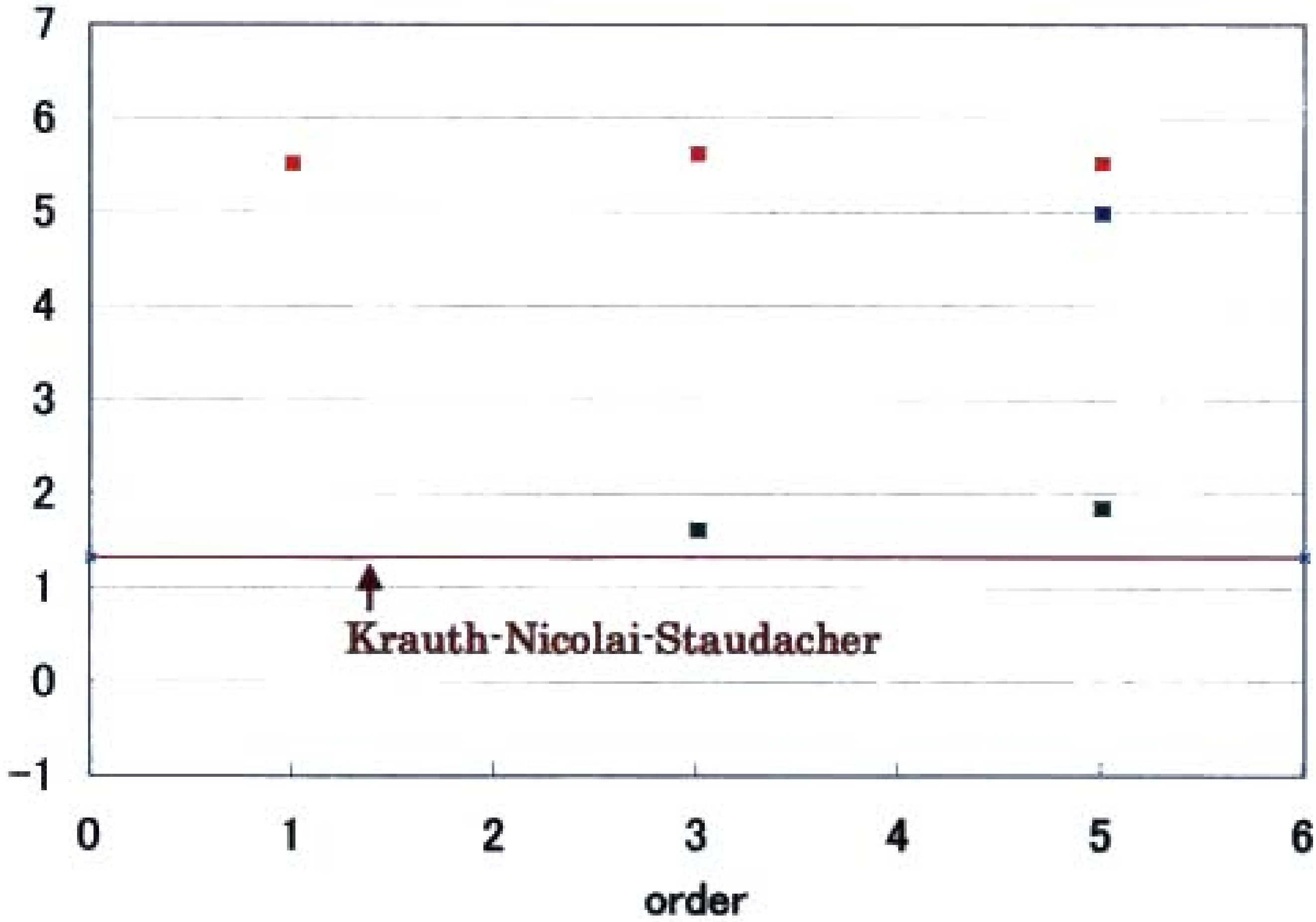
$m_{234} = m_{567} = m_{8910}$, M_{11} and $M_{22} = \cdots = M_{1010}$.

Another problem for the plateau search is that the orders we are considering (up to 5th order) are not high enough to observe clear plateaus. Instead we list up all the extrema for each order. For each extremum we evaluate the square size of the eigenvalue distribution $<\frac{1}{N}Tr(A^\mu A^\nu)>$, which is proportional to the exact propagator. At the present order it is not clear how these extrema will accumulate to a plateau as we go to higher orders, but we can see clear differences among the above three ansatz.

First the SO(1) ansatz gives no extremum at any order of the perturbation theory.

Secondly, the SO(7) ansatz gives one extremum at the 1st order, two at the 3rd order and three at the 5th order. Two of them are close to the value of Krauth-Nicolai-Staudacher, but at this stage it is not clear where extrema will accumulate to form a plateau as we go to higher orders.

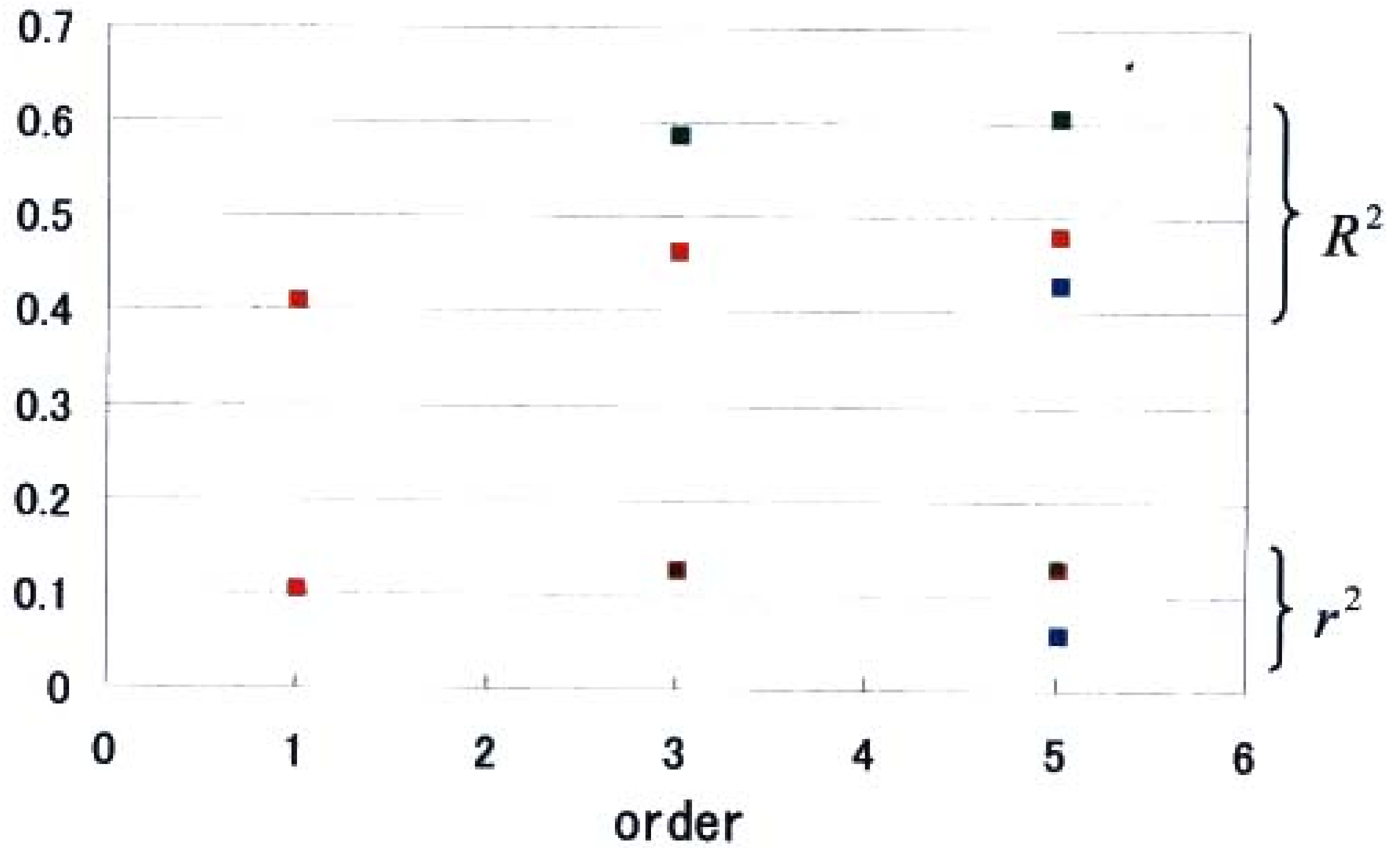
extrema of free energy for SO(7) ansatz



Square size for each extremum for SO(7) ansatz

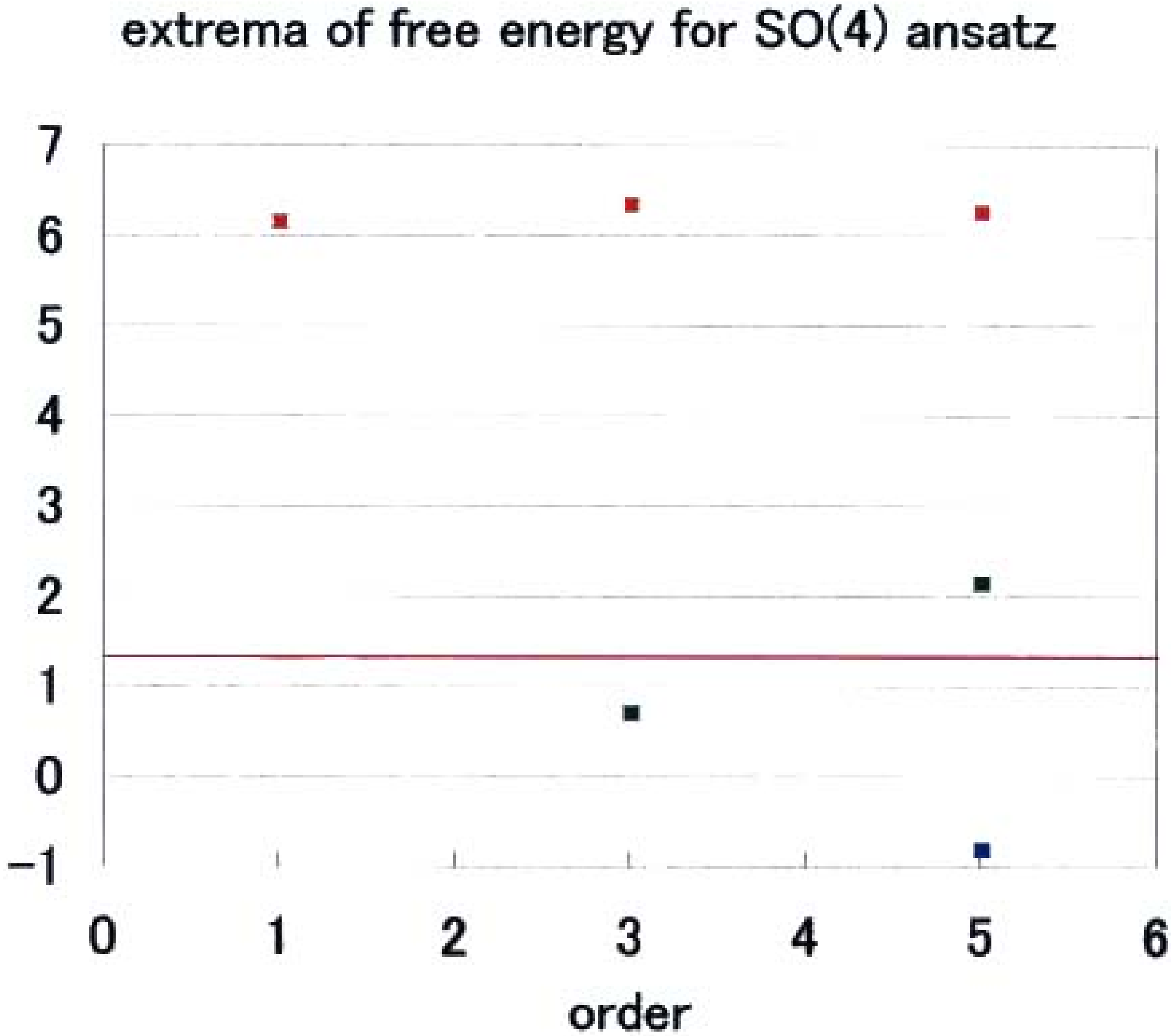
$$R^2 = <\frac{1}{N}Tr(A^1A^1)> = \cdots = <\frac{1}{N}Tr(A^7A^7)>,$$

$$r^2 = <\frac{1}{N}Tr(A^8A^8)> = \cdots = <\frac{1}{N}Tr(A^{10}A^{10})>.$$



It seems that extremum with smaller free energy has larger rotational asymmetry. But this tendency is not very strong compared to the SO(4) ansatz.

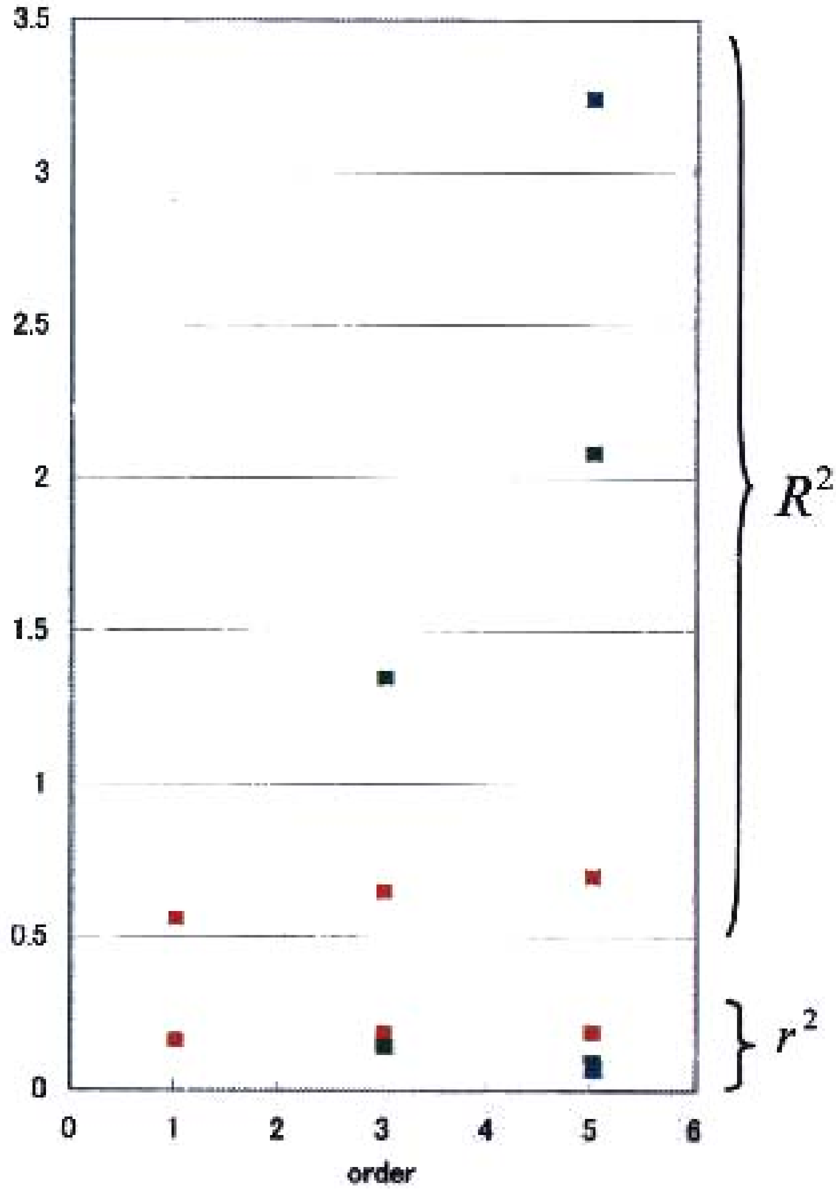
Thirdly, the SO(4) ansatz gives the same number of extrema as the SO(7) ansatz. The free energy seems to be stable as in the SO(7) ansatz. Again it is not clear where extrema will accumulate to form a plateau as we go to higher orders.



But the behavior of the square size is different from that of the SO(7) ansatz. The larger component seems to grow in higher orders. Also the smaller component tends to shrink.

Square size for each extremum for SO(4) ansatz

$$R^2 = \frac{1}{N}Tr(A^1A^1) = \cdots = \frac{1}{N}Tr(A^4A^4),\ \ r^2 = \frac{1}{N}Tr(A^5A^5) = \cdots = \frac{1}{N}Tr(A^{10}A^{10}).$$

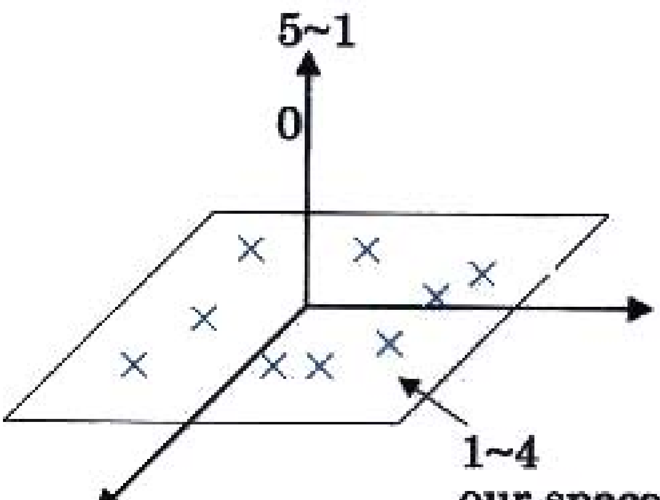


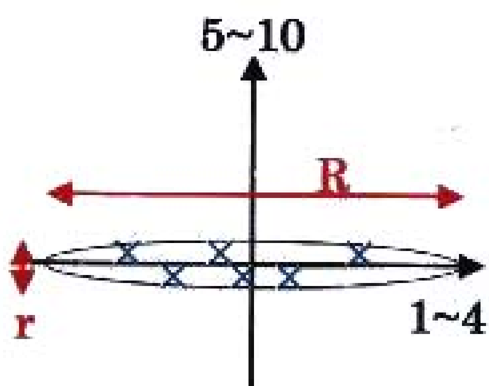
3. Breaking of rotational symmetry and spontaneous formation of space-time

These results are consistent with the following conjecture we made some time ago: Aoki, Iso, Kawai, Kitazawa and Tada

The rotational symmetry is spontaneously broken to SO(4). The eigenvalues of A^μ spread along a four dimensional hyperplane, which we can regard as our space-time. In the large-N limit the size of the eigenvalue distribution becomes infinite in four directions, while it stays finite in the other directions.

$$\frac{R}{l_{string}} \to \infty, \quad \frac{r}{l_{string}} < \infty \quad (N \to \infty).$$





If it is true, we expect that the ratio $\frac{R}{r}$ calculated by the improved perturbation theory grows to infinity as we go to higher orders, which is consistent with what we have seen.

We have seen the ratio $\frac{R}{r}$ tends to grow in higher orders in SO(4) ansatz.

Understanding the symmetry breaking

There would be several ways to understand this symmetry breaking. Among them the following two are simple and seem to be realistic.

(1) phase of the fermion determinant

One is by Nishimura and Vernizzi. They pointed out that the phase of the fermion determinant plays an important role. The phase is generically non-zero, but it is zero if some of the A^{μ}'s are linearly dependent. Therefore in the path integral lower dimensional configurations are enhanced in the sense of the stationary phase.

(2) purely bosonic model

The other is by Vernizzi and Wheater. Recently they have constructed a class of solvable models that show the symmetry breaking explicitly. Their model consists of D bosonic matrices A^{μ} (μ =1~D), and the action is given by

$$S = N\ f(T)\,,$$

where $T^{\mu\nu}$ is the D-dimensional symmetric tensor defined by $T^{\mu\nu} = \frac{1}{N} Tr(A^{\mu} A^{\nu})$, and f is an O(D) invariant function of $T^{\mu\nu}$. This model is solvable in the large-N limit, and shows various patterns of O(D) symmetry breaking.

4. Conclusion

The improved perturbation theory seems promising. Although we need to go to higher orders to say something more definite, the results at the present order are consistent with the spontaneous breaking of the rotational symmetry. Furthermore they seem to support our conjecture for having four-dimensional space-time in the IIB matrix model.