SOME STRINGY ASPECTS OF THE ADS/CFT DUALITY

IGOR R. KLEBANOV

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THE SUPERGRAVITY APPROXIMATION TO TYPE IIB STRING THEORY ON $AdS_5 \times X^5$

may be used to extract many properties of dual conformal gauge theories at large 't Hooft coupling λ.

As λ is reduced, the α' corrections to SUGRA become important.

Hence, even in the 't Hooft large N limit we need to solve classical string theory in a Ramond-Ramond background, on $AdS_5 \times X_5$.

This is a well-known and hard problem.

HOWEVER, EVEN AS $\lambda \to \infty$ THERE ARE IMPORTANT STRINGY EFFECTS WHICH ARE CRUCIAL FOR THE CONSISTENCY OF ADS/CFT DUALITY.

THESE EFFECTS TYPICALLY INVOLVE OPERATORS OF HIGH DIMENSION.

Some field theories contain operators with dimension $\Delta \sim N$.

They appear, for example, in the $SU(N) \times SU(N)$ gauge theory with bifundamental chiral superfields A_1, A_2; B_1, B_2, each one with R-charge $\frac{1}{2}$.

This $\mathcal{N}=1$ SCFT is dual to IIB superstring on $AdS_5 \times T^{11}$ IK, Witten Morrison, Plesser

It contains "dibaryon" operators

like $\det A_1$

which have R-charge $\frac{N}{2}$ and hence

$$\Delta = \frac{3}{2} R = \frac{3N}{4}.$$

Their dual stringy description is by a D3-brane wrapping an $S^3 \subset T^{11}$. IK Gubser

Recently the matching between baryons and D-branes has been further shown for their BPS excitations. Berenstein, Ik, Herzog.

Certain waves running on the wrapped D3-branes are BPS states dual to operators of R-charge $\frac{N}{2} + k$ obtained by replacing $A_1 \to A_1 B_j A_2$ k times in $\det A_1$.

Thus, for $\Delta \sim N$ we observe a stringy effect $\Rightarrow$ the appearance of D-branes. For a more general discussion, see a very recent paper by Beasley.

In fact, Δ does not need to be this large. Stringy effects generically appear for high-dimension operators, for $\boxed{\Delta \sim \sqrt{\lambda}}$. Polyakov; Berenstein, Maldacena, Nastase.

Using local scalar fields in AdS_5 gives the standard formula for Δ:

$$\Delta_\pm = 2 \pm \sqrt{4 + (mL)^2}\ ; \quad L^2 = \alpha' \sqrt{\lambda}.$$

For a massive string state, $m^2 = \frac{2K}{\alpha'}$,

$$\Delta_+ \longrightarrow \sqrt{2K}\, \lambda^{1/4} \text{ for large } \lambda.$$

Recently it has become clear that this formula is violated for very large K, $\boxed{K \sim \sqrt{\lambda}.}$

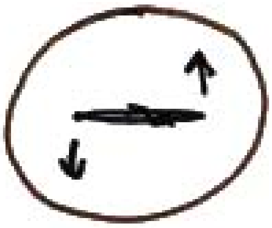
This effect is easiest to study if the massive string state carries a large quantum number, such as R-charge (angular momentum on S^5) or Lorentz spin.

$$\mathrm{Tr}\left[\Phi^i \nabla_{(\mu_1} \nabla_{\mu_2} \dots \nabla_{\mu_S)} \Phi^i\right] + \dots$$

They correspond to closed strings on the leading Regge trajectory:

$$m^2 = \frac{2}{\alpha'}(S-2); \quad S = 2, 3, \dots$$

(lowest m^2 for spin S)



When $S \gg 1$, there is a simple classical picture: a folded closed string spinning around its center.

Consider analogous picture in global AdS_5 metric   Gubser, IK, Polyakov

$$ds^2 = L^2\left(-dt^2 \cosh^2\rho + d\rho^2 + \sinh^2\rho \, d\Omega_3^2\right)$$

Energy E on $S^3 \times R$ gives the conformal dimension Δ.

Pick the static gauge in the Nambu action

$\tau = t\,; \quad \sigma = \sigma(\rho)$.

Induced metric $\gamma_{\alpha\beta} = G_{\mu\nu} \dfrac{\partial X^\mu}{\partial \sigma^\alpha} \dfrac{\partial X^\nu}{\partial \sigma^\beta}$.

$$\mathcal{L} = -\frac{1}{2\pi\alpha'} \int_0^{2\pi} d\sigma \sqrt{-\det \gamma} = -\frac{4L^2}{2\pi\alpha'} \int_0^{\rho_0} d\rho \sqrt{ch^2\rho - \dot{\phi}^2 sh^2\rho}\,.$$

$$d\Omega_3^2 = d\alpha^2 + \sin^2\alpha \left(d\beta^2 + \sin^2\beta\, d\phi^2 \right)$$

The string spins at $\alpha = \beta = \frac{\pi}{2}$; $\boxed{\phi = \omega t}$

$\rho = \rho_0$ is the turnaround point; $\boxed{\coth \rho_0 = \omega}$

$$E = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \dot{\phi} - \mathcal{L} = 4 \frac{L^2}{2\pi\alpha'} \int_0^{\rho_0} d\rho \frac{ch^2\rho}{\sqrt{ch^2\rho - \omega^2 sh^2\rho}}$$

$$S = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = 4 \frac{L^2}{2\pi\alpha'} \int_0^{\rho_0} d\rho \frac{\omega\, sh^2\rho}{\sqrt{ch^2\rho - \omega^2 sh^2\rho}}\,.$$

These eqns specify dimension E as a function of spin S.

THERE ARE TWO SIMPLE LIMITS:

1). SHORT STRINGS: $\omega >> 1$.

$\rho_0 \approx \frac{1}{\omega}$.

The leading form of energy and spin is

$$E = \frac{L^2}{\alpha'\omega}; \quad S = \frac{L^2}{2\alpha'\omega^2} \Rightarrow E^2 = L^2\frac{2S}{\alpha'}.$$

In this regime, $1 \ll S \ll \sqrt{\lambda}$, we thus find agreement with $\Delta = 2 + \sqrt{4 + (mL)^2} \approx mL$.

2). LONG STRINGS: $\omega = 1 + 2y$; $y \ll 1$.

$$\rho_0 \to \frac{1}{2}\ln(1/y)$$

$$E = \frac{L^2}{2\pi\alpha'}\left(\frac{1}{y} + \ln(1/y) + \ldots\right)$$

$$S = \frac{L^2}{2\pi\alpha'}\left(\frac{1}{y} - \ln(1/y) + \ldots\right)$$

In this regime, $S >> \sqrt{\lambda}$, and

$$E - S = \frac{\sqrt{\lambda}}{\pi}\ln\frac{S}{\sqrt{\lambda}} + \mathcal{O}(S^0).$$

Compare with perturbation theory.

For large S and small λ, it was found that the anomalous dimension

$$\Delta - S = (a_1 \lambda + a_2 \lambda^2 + \ldots) \ln S$$

Remarkably, at 2 loops, $\ln^3 S$ and $\ln^2 S$ terms cancel! Gonzalez-Arroyo + Lopez (1980)

Thus, the natural interpolating formula is

$$\boxed{\Delta - S = \tilde{f}(\lambda) \ln S + \mathcal{O}(S^0)}$$

as $S \to \infty$.

What guarantees the $\ln S$ growth?

What is the interpolating function $\tilde{f}(\lambda)$?

Leading correction at large λ was recently calculated by Frolov + Tseytlin

A similar interpolating function appears in a study of the $SU(N)$ $\mathcal{N}=4$ SYM at finite temperature, T.

In the 't Hooft large N limit, we expect the free energy to be of the form

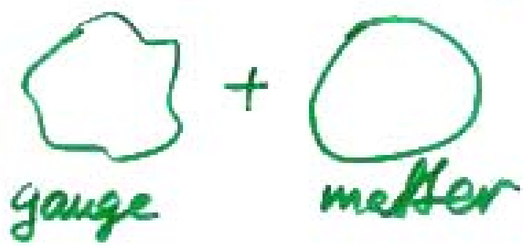
$$F = -\frac{\pi^2}{6} N^2 f(\lambda)\, T^4 V$$

For small λ, Feynman graph calculations give

$$f(\lambda) = 1 - \frac{3}{2\pi^2}\lambda + \frac{3+\sqrt{2}}{\pi^3}\lambda^{3/2} + \ldots$$

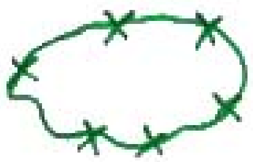
Fotopoulos + Taylor
Vazquez-Mozo
Kim + Rey
Nieto + Tytgat

The leading term comes from a free field calculation



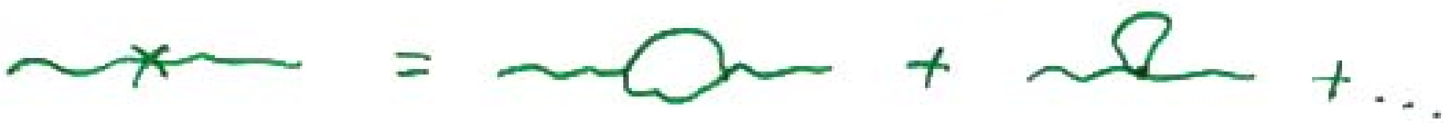
$\mathcal{O}(\lambda)$ correction comes from 2-loop graphs

A more surprising $\mathcal{O}(\lambda^{3/2})$ term is due to a resummation of graphs, needed since an



IR divergence appears at $\mathcal{O}(\lambda^2)$.

with insertions of the thermal mass
$m^2 \sim \lambda T^2$
which is generated at 1 loop

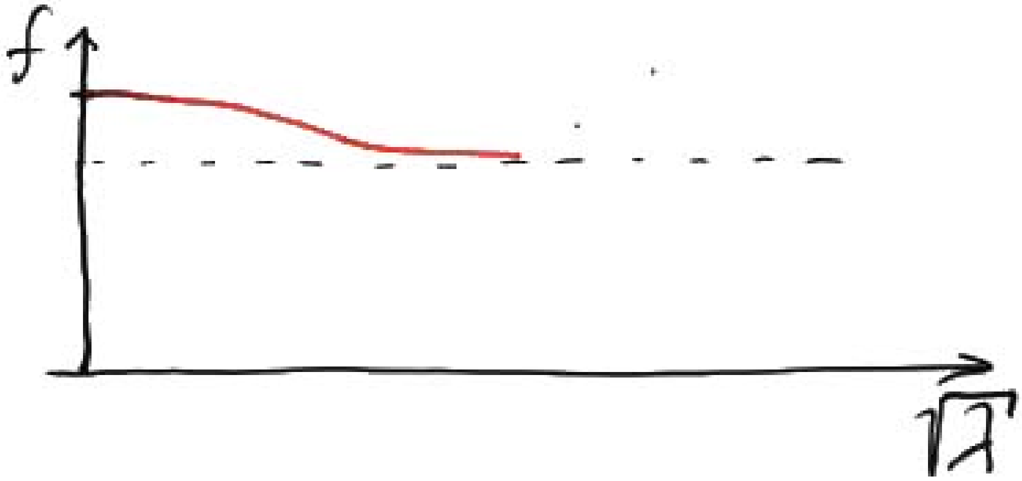


For large λ, stringy α' corrections give the following prediction for $f(\lambda)$:

$$f(\lambda) = \frac{3}{4} + \frac{45}{32}\zeta(3)\lambda^{-3/2} + \dots$$

The leading correction to $\frac{3}{4}$ comes from the $\alpha'^3 R^4$ term in the effective action.

It is plausible that $f(\lambda)$ smoothly interpolates between 1 and $3/4$:



Finding the precise form of $f(\lambda)$ remains a challenge.

Recently, an analogous problem was solved using type IIB strings on a R-R charged pp-wave background.

String theory gives an explicit formula for certain operator dimensions: Berenstein Maldacena Nastase

$$\Delta = J + \sum_{n=-\infty}^{\infty} N_n \sqrt{1 + \frac{\lambda n^2}{J^2}}$$

where J is the R-charge of the operator.

This formula has been checked using planar diagram summation in gauge theory. Santambrogio Zanon

For more complicated quantities there is evidence that at small λ', the interpolating functions contain odd powers of $\sqrt{\lambda'}$; $\lambda' = \frac{\lambda}{J^2}$. IK, Spradlin, Volovich

Interaction vertex in light-cone SFT

$$|V\rangle = \exp\left[\frac{1}{2}\sum_{r,s=1}^{3}\sum_{m,n=-\infty}^{\infty} a^{I+}_{m(r)} N^{(rs)}_{mn} a^{I+}_{n(s)}\right]|0\rangle$$

To calculate N, need to invert

$$\Gamma_+ = \Gamma_0 - H$$

IK, Spradlin, Volovich

$$[\Gamma_0]_{mn} = 2\frac{\sqrt{m^2+\mu^2}}{m}\delta_{mn};$$

$$H_{mn} = \frac{8}{\mu^2\pi^2}(-1)^{m+n}\sqrt{mn}\,\sin(\pi m y)\sin(\pi n y)$$

$$\int_1^\infty dz\,\frac{F(z)\sqrt{z^2-1}}{\left(z^2+\frac{m^2}{\mu^2}\right)\left(z^2+\frac{n^2}{\mu^2}\right)}; \qquad y = \frac{p^+_{(1)}}{p^+_{(3)}};$$

$$F(z) = \frac{1}{2}\left[\coth(\pi\mu y z) + (y \to 1-y)\right].$$

Expand for large $\mu \sim \frac{1}{\sqrt{\lambda'}}$ => weak effective coupling.

Γ_0 has only odd powers of μ;

H has only even powers.

$$\mu[\Gamma_+^{-1}]_{mn} = \left[\frac{m}{2} - \frac{m^3}{4}\lambda' + \mathcal{O}(\lambda'^2)\right]\delta_{mn} + \lambda'^{3/2} R_{mn} + \mathcal{O}(\lambda'^{5/2}).$$

Some elements of $N^{(rs)}_{mn}$ have similar structure.

Could the appearance of terms like $\lambda'^{3/2}$, $\lambda'^{5/2}$, etc. be related to IR divergences in planar graphs, as in the thermal case?

SUMMARY

I have discussed 3 correspondences that go beyond the basic SUGRA limit of AdS/CFT.

1). Between baryonic operators and wrapped D3-branes.

2). Between operators with high Lorentz spin and spinning folded strings in AdS_5.

3). Between correlators of BMN operators of large R-charge and string field theory in the pp-wave background.

These correspondences are clearly relevant at large λ. Extrapolation to $\boxed{\text{small } \lambda}$ is a crucial problem.

In case 3) small $\lambda' = \frac{\lambda}{J^2}$ can perhaps give small λ results.