Intersecting Brane Worlds on Calabi-Yau Orientifolds

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hep-th/0206038, to appear in JHEP

Strings 2002, Cambridge July 2002

I) Introduction

One of the main goals in string theory:

Find a string realization of the Standard Model!

Key properties of the SM:

- Non-Abelian (product) gauge group
- 3 generations of chiral matter

Intersecting D-brane worlds provide a promising avenue of approach to this goal.

Origin of chirality and Non-Abelian gauge groups in string theory:

- Heterotic string

- Type II superstring

- M-theory

Origin of chirality and Non-Abelian gauge groups in string theory:

- Heterotic string

Different treatment of left- and rightmoving string modes
⟶ charged, chiral spectrum in closed string sector

- Type II superstring

D-branes (and orientifolds) ⟶ charged, chiral spectrum in open string sector on D-branes

- M-theory

Put M-theory on a singular space (e.g. intervall $S^1/\mathbb{Z}_2$)
⟶ charged, chiral spectrum on end of the world branes

Origin of chirality **and** Non-Abelian gauge groups **in string theory:**

Compactification to 4D on an internal space $\mathcal{M}^6$ ($\mathcal{M}^7$):

- Heterotic string

Chirality $\longleftrightarrow$ Topology of $\mathcal{M}^6$ with vectorbundle V:

$$N_F = c_3(V)/2 \stackrel{V=T}{=} \chi(\mathcal{M}^6)/2$$

- Type II superstring

Chirality $\longleftrightarrow$ D-brane intersections on $\mathcal{M}^6$:

$$N_F = I_{ab} \equiv \#(\pi_a \cap \pi_b)$$

- M-theory

Chirality $\longleftrightarrow$ Singularities of $\mathcal{M}^7$, e.g. $\mathcal{M}^7 \simeq \mathbb{R}_+ \times Y_6$:

$$N_F = \chi(Y_6)/2$$

Origin of chirality and Non-Abelian gauge groups in string theory:

- Heterotic string

Microscopic closed string consistency: world sheet modular invariane ⟶ target space anomaly freedom

- Type II superstring

Microscopic open string consistency: cancellation of RR charges ⟶ target space anomaly freedom

- M-theory

No microscopic consistency checks! But local anomaly cancellation by anomaly inflow on singularities

I) Introduction

Further plan of the talk:

- Intersecting brane worlds (IBW's) on Calabi-Yau spaces
- M-theory lift via G_2 manifolds ($\mathcal{N}=1$ models)

II) IBW's on Calabi-Yau spaces

- Closed string background: $(\mathcal{N} = 8, 4, 2)$

$$\mathcal{M}^{10} = \mathbb{R}^{9-2d,1} \times \mathcal{M}^{2d}, \quad d = 1, 2, 3$$

- Orientifold $O(9-d)$-plane:
  Consider orbifold $\mathcal{M}^{10}/(\Omega\overline{\sigma})$ with Ω world sheet parity and $\overline{\sigma}$ an anti-holomorphic involution.
  The fixed point locus of $\overline{\sigma} \longrightarrow d$-cycle π_O in $\mathcal{M}^{2d}$.

- $D(9-d)$-branes: space-time filling, wrapped around homology d-cycles π_a in $\mathcal{M}^{2d}$.

Massless spectrum:

- Supergravity in the 10D bulk $(\mathcal{N} = 4, 2, 1)$

- Gauge fields localized on the $10-d$ dimensional world-volume of the D-branes $(\mathcal{N} = 4, 2, 1)$

- Chiral matter localized on the $10-2d$ dimensional intersection locus of two D-branes $(\mathcal{N} = 2, 1, 0)$
  (Berkooz, Douglas, Leigh, hep-th/9606139)

  The number of chiral fermions is given by the topological intersection numbers between two D-branes.

Models studied so far:

- $\mathcal{M}^6 = T^6$: search for the Standard Model: $G = SU(3) \times SU(2)_L \times U(1)_Y$, 3 generations of q, l!

  (Blumenhagen, Görlich, Körs, Lüst, hep-th/0007024)
  (Angelantonj, Antoniadis, Dudas, Sagnotti, hep-th/0007090)
  (Aldazabel, Franco, Ibáñez, Rabadán, Uranga, hep-th/0011073, hep-ph/0011132)
  (Blumenhagen, Körs, Lüst, hep-th/0012156)
  (Ibáñez, Marchesano, Rabadan, hep-th/0105155)
  (more papers: Aldazabel, Bailin, Cremades, Förste, Garcia-Bellido, Honecker, Ibáñez, Kokorelis, Kraniontos, Love, Marchesano, Schreyer)

- $\mathcal{M}^6 = \frac{T^6}{\mathbf{Z}_2 \times \mathbf{Z}_2}$: supersymmetric IBW's.

  (Cvetic, Shiu, Uranga, hep-th/0107143, hep-th/0107166)
  (Cvetic, Langacker, Shiu, hep-th/0205252, hep-th/0206115)

- $\mathcal{M}^6 = \frac{T^6}{\mathbf{Z}_3}$: GUT models.

  (Blumenhagen, Körs, Lüst, Ott, hep-th/0107138)
  (Ellis, Kanti, Nanopoulos, hep-th/0206087)

- Study of the potential $\longrightarrow$ hybrid inflation in IBW's.

  (Blumenhagen, Körs, Lüst, Ott, hep-th/0202124)
  (Burgess, Majumdar, Nolte, Quevedo, Rajesh, Zhang, hep-th/0105204)

Generalization of IBW's to K3 and Calabi-Yau spaces:

(i) $O(9-d)$ orientifold planes:
Choose the anti-holomorphic involution in local coordinates to be $\overline{\sigma} : z_i \to \overline{z}_i$. The orientifold plane π_O, i.e. the fixed locus Fix($\Omega\overline{\sigma}$), is a sLag d-cycle, implying

$$\imath^* J = \imath^* \Im(\Omega_d) = 0, \quad \imath^* \Re(\Omega_d) = \imath^* d\text{vol}$$

(ii) Introduce N_a D(9-d) branes wrapped on sLag homology d-cycles π_a and their $\Omega\overline{\sigma}$ images π_a'.
For π_a only require $\imath^* \Re(e^{i\theta_a}\Omega_d) = \imath^* d\text{vol}$

Weak (individual) $\mathcal{N}=1$ SUSY conservation.
(Cremades, Ibáñez, Marchesano, hep-th/0201205)

Effective U(N) gauge theory on N D$(9-d)$-branes

$$\begin{aligned} \mathcal{S}_{\text{eff}} &= \int_{\mathbb{R}^4 \times \mathcal{M}^6} dx^4 d\xi^6 \; \mathcal{L}_{\text{gravity}}(g, B_2, \phi, C_{10-d}) \\ &+ \int_{\mathbb{R}^4 \times \mathcal{W}^{6-d}} dx^4 d\zeta^{6-d} \; (\underbrace{\mathcal{L}_{\text{DBI}}(g, \mathcal{F}, \phi)}_{\text{Tension}} + \underbrace{\mathcal{L}_{\text{CS}}(\mathcal{F}, C_{10-d})}_{\text{Charge}}) \end{aligned}$$

- RR-charge cancellation ($p = 9 - d$):

$$\mathcal{S}_{\mathrm{CS}}^{(\mathrm{D}p)} = \mu_p \int_{\mathrm{D}p} \mathrm{ch}(\mathcal{F}) \wedge \sqrt{\frac{\hat{\mathcal{A}}(\mathcal{R}_T)}{\hat{\mathcal{A}}(\mathcal{R}_N)}} \wedge \sum_q C_q \stackrel{\mathrm{sLag}}{\longrightarrow} \mathrm{rk}(\mathcal{F})\ C_{p+1}$$

$\longrightarrow$ Topological tadpole cancellation conditions:

$$\sum_a \mathrm{N}_a(\pi_a + \pi_a') - 2^{5-d}\pi_{\mathrm{O}} = 0$$

- The disc level scalar potential (tension of the branes) can be deduced from the DBI action for the D-branes (D-term potential):

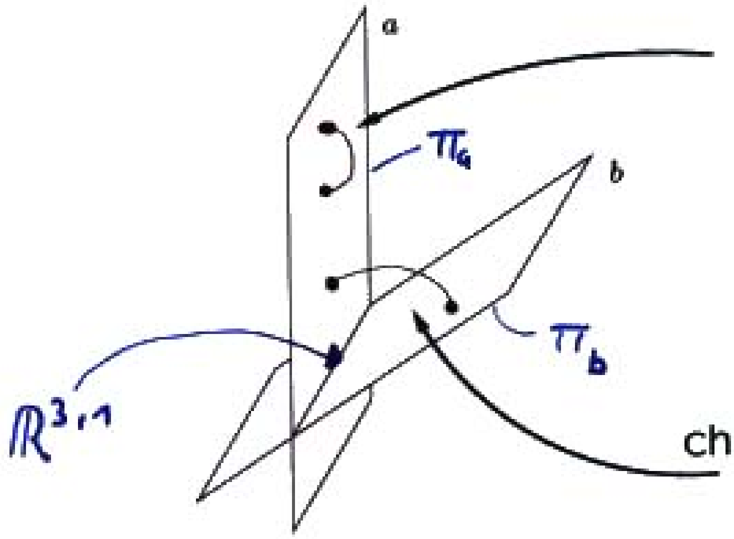
$$\mathcal{V} = T_p\, e^{-\phi_{10-2d}} \left(\sum_a N_a \left| \int_{\pi_a} \widehat{\Omega}_d \right| + \sum_a N_a \left| \int_{\pi_a'} \widehat{\Omega}_d \right| - 2^{5-d} \left| \int_{\pi_{\mathrm{O}p}} \widehat{\Omega}_d \right| \right).$$

$\mathcal{N} = 1$ SUSY: all branes are calibrated with respect to the same d-form as the Op-plane, so that

$$\mathcal{V} = T_p\, e^{-\phi_{10-2d}} \left(\sum_a N_a \int_{\pi_a + \pi_a'} \Re(\widehat{\Omega}_d) - 2^{5-d} \int_{\pi_{\mathrm{O}p}} \Re(\widehat{\Omega}_d) \right),$$

which vanishes due to the RR-tadpole condition. Compact $\mathcal{M}^6 \stackrel{\mathrm{SUSY}}{\Rightarrow}$ Orientifold Op-planes needed.

Consider type IIA IBW's on 6-dim. CY spaces $(d = 3)$:



- 7-dim. $U(N_a)$ gauge bosons on the D6-branes wrapped around sLag 3-cycle π_a $(codim = 3)$
- 4-dim. chiral fermions on the intersections of the D6-branes $(codim = 6)$:

Representation	Intersection #
$(A_a)_L$	$\frac{1}{2}(\pi_a \circ \pi_a' + \pi_a \circ \pi_{O6})$
$(S_a)_L$	$\frac{1}{2}(\pi_a \circ \pi_a' - \pi_a \circ \pi_{O6})$
$(\bar{N}_a, N_b)_L$	$\pi_a \circ \pi_b$
$(N_a, N_b)_L$	$\pi_a \circ \pi_b'$

$(I_{ab} = -I_{ba})$

Example: **Fermat quintic CY_3:**

$$P(z_i) = z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0 \subset \mathbb{CP}^4$$

- O6-plane: sLag $\mathbb{RP}^3$, $\bar{\sigma}$-fixed set $\pi_{O6} = \pi_{0,0,0,0}$:

  $$P(x_i) = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 = 0 \subset \mathbb{RP}^4$$

- D6-branes: Use $\mathbb{Z}_5^4$, $z_i \mapsto \omega^{k_i} z_i$, $\omega = e^{2\pi i/5}$, $k_i \in \mathbb{Z}_5$
  $\longrightarrow$ sLag 3-cycle $\pi_a = \pi_{k_2,k_3,k_4,k_5}$:

  $$x_1^5 + \Re(\omega^{k_2} z_2)^5 + \Re(\omega^{k_3} z_3)^5 + \Re(\omega^{k_4} z_4)^5 + \Re(\omega^{k_5} z_5)^5 = 0$$

  $5^4 = 625$ sLag $\mathbb{RP}^3$'s, calibrated with $\Re(\prod_i \omega^{k_i} \Omega_3)$.
  (125 sLag's are $\Re(\Omega_3)$ calibrated $\longrightarrow \mathcal{N} = 1$ SUSY)

- Intersection numbers of π_{k_2,k_3,k_4,k_5} and π_{O6} from
  Brunner, Douglas, Lawrence, Römelsberger, hep-th/9906200

  (The 125 SUSY D6-branes have zero intersection #'s $\longrightarrow$ no $\mathcal{N} = 1$ chiral models from the quintic!)

Non-supersymmetric Standard Model from the quintic:

- Introduce four stacks of D6-branes with $N_a = 3$, $N_b = 2$ and $N_c = N_d = 1 \longrightarrow$ gauge group:

$$G = U(3) \times U(2) \times U(1)^2$$

- Choose the following "wrapping numbers":

$$\pi_a = |0,0,3,1\rangle, \quad \pi_b = |4,3,0,3\rangle,$$
$$\pi_c = |3,0,1,1\rangle - 2|4,3,0,3\rangle,$$
$$\pi_d = |4,2,4,4\rangle - 2|0,0,3,1\rangle$$

  This produces the intersection numbers of the standard model with

  3 generation of quarks and leptons

  cfr. talk by Luis Ibáñez

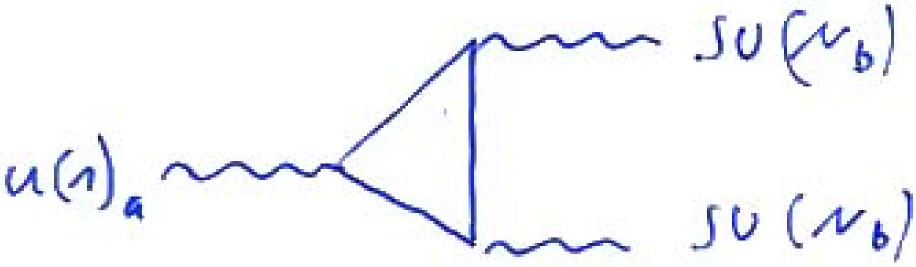
- Anomaly-free hypercharge is

$$U(1)_Y = \frac{1}{3}U(1)_a - U(1)_c + U(1)_d$$

- GS couplings to cancel $U(1) - SU(\mathrm{N})^2$ anomalies.

- An invisible sector needed for tadpole cancellation.

$U(1)_a \times SU(N_b)^2$ anomaly:



$$A_{ab} = \frac{N_a}{2}\left(-\pi_a + \pi_a'\right) \circ \pi_b$$

This is cancelled by the GS-mechanism:

$$\mathcal{L}_{CS} \sim \int_{\mathbb{R}^{1,3}\times\pi_b} C_3 \wedge \mathrm{Tr}\,(F_b \wedge F_b) + \int_{\mathbb{R}^{1,3}\times\pi_a} C_5 \wedge \mathrm{Tr}\, F_a$$

$$\pi_a = e^a_i\, \alpha^i + m^i_a\, \beta_i$$

$$\phi_i = \int_{\alpha^i} C_3 \quad , \quad B^i = \int_{\beta_i} C_5$$

Hodge dual in 4D!

III) M-theory on G_2 manifolds

The following singularities play an important role in M-theory:

- Codimension one singularities:

  e.g. M-theory on $S^1/\mathbb{Z}_2 \times \mathcal{M}^{10}$. This is known as heterotic M-theory with $G \subseteq E_8$ on the end of the world orbifold fixed planes $\mathcal{M}^{10}$

  (Horava, Witten, hep-th/9603142, 9510209)

  (Generalization to 6D and 4D: T^5/Γ, T^7/Γ orbifolds with intersecting orbifold planes.)

  (Faux, Ovrut, D.L., hep-th/9903028)
  Doran, Faux, Ovrut, hep-th/0108078)

- Codimension four singularities:

  e.g. M-theory on A_n-singularity $\mathbb{R}^4/\mathbb{Z}_{n+1} \times \mathcal{M}^7$. This is the M-theory lift of $n+1$ D6-branes with $U(n+1)$ gauge bosons on $\mathcal{M}^7$.

- Codimension seven singularities:

  e.g. M-theory on $(\mathbb{R}_+ \times Y_6) \times \mathcal{M}^4$. This is the M-theory lift of intersecting D6-branes with chiral fermions on $\mathcal{M}^4$.

M-theory on a G_2 manifold X_7 ($d\phi^{(3)} = d\star\phi^{(3)} = 0$):

Type IIA reduction:
assume that X_7 has a S^1 isometry, which means that we are considering a circle fibration π: $S^1 \longrightarrow X_7 \longrightarrow B_6$ with metric

$$g = \alpha \otimes \alpha + \pi^* \hat{g}\,,$$

where $\hat{g}$ is the metric of the IIA (non Ricci-flat) base B_6 and α the RR 1-form potential.

Singularity structure:

(Atiyah, Witten, hep-th/0107177)
(Acharya, Witten, hep-th/0109152)

- S^1 action has fixed points in codimension four $\longleftrightarrow$ $D6$ branes which are wrapped on the 3-dim. fixed point set $L \subset X_7$.
- Isolated, conical singularaties of codimension 7 (colliding singularities) $\longleftrightarrow$ intersecting D6-branes.

  $$ds^2_{X_7} = dr^2 + r^2 ds^2_{Y_6}$$

  The (non Ricci-flat) space Y_6 must have weak $SU(3)$ holonomy, i.e. $d\Im(\Omega^{(3)}) = -2\omega \wedge \omega$, $d\omega = 3\Re(\Omega^{(3)})$.

A class of metrics of non-compact X_7 spaces can be explicitly constructed as an $\mathbb{R}^3$ bundle over a 4-dim. quaternionic space $\mathcal{Q}$ (Y_6 is an $U(1)^2$ bundle over $\mathcal{Q}$):

(Bryant,Salamon;Gibbons,Page,Pope;Cvetic,Gibbons,Lu,Pope; Atiyah,Maldacena,Vafa;Brandhuber,Gomis,Gubser,Gukov)

$$\begin{aligned} ds_7^2 &= \frac{1}{\sqrt{2\kappa|u|^2+u_0}}(du^i+\epsilon^{ijk}A^j u^k)^2+\sqrt{2\kappa|u|^2+u_0}\, ds_4^2 \\ &\stackrel{u_0\to 0}{\longrightarrow} dr^2+r^2 ds_{Y_6}^2 \end{aligned}$$

Known examples of cohomogeneity one metrics:

- $\mathcal{Q}=S^4$, $Y_6=\mathbb{C}P^3$: $G=U(1)^2$, 2 intersecting D6-branes
- $\mathcal{Q}=\mathbb{C}P^2$, $Y_6=SU(3)/U(1)^2$: $G=U(1)^3$, 3 intersecting D6-branes

This can be generalized to get matter charged under $SU(n)$ and $SO(2n)$ gauge groups.

(Atiyah, Witten, hep-th/0107177)
(Berglund, Brandhuber, hep-th/0205184)

III) M-theory on G_2 manifolds (continued)

Other generalization: consider quaternionic spaces $\mathcal{Q}$ with only two $U(1)$ isometries:

(Behrndt, Dall'Agata, Mahapatra, D.L., hep-th/0207117)
(see also: Anguelova, Lazaroiu, hep-th/0205070)

$$
\begin{aligned}
ds_4^2 &= \frac{p^2-q^2}{P}dp^2 + \frac{p^2-q^2}{Q}dq^2 + \\
&\quad + \frac{P}{p^2-q^2}(d\tau + q^2 d\sigma)^2 + \frac{Q}{p^2-q^2}(d\tau + p^2 d\sigma)^2 \\
P &= -\kappa(p-r_1)(p-r_2)(p-r_3)(p-r_4) \\
Q &= \kappa(q-r_1)(q-r_2)(q-r_3)(q-r_4)
\end{aligned}
$$

(Plebanski, Demianski)

Dimensional reduction of X_7 over $U(1)$ isometry $k = \beta_1\partial_\tau - \beta_2\partial_\sigma$ (β_1/β_2 is a root of P (resp. Q))
$\longrightarrow$ 2 types of fixed point sets:

(a) $P=0$ and $Q=0$: NUT on $\mathcal{Q}$
(a) $P=0$ or $Q=0$: Bolt on $\mathcal{Q}$

Both fixed point sets correspond to codimension four singularities on $X_7 \longrightarrow$ 3 intersecting D6-branes

E.g. killing vector $k = \beta \partial_\tau - \partial_\sigma$

$$\beta = \frac{1}{2}\left[r_3(r_2 - r_4) + r_2(r_2 + r_4)\right]$$

$\Rightarrow$ fixed points : $p = r_3,\ q = r_4$ and vice versa

Set $r_1 = r_2 = n$

$r_3 = m$

$(\Rightarrow\ r_4 = -2n - m)$

$\Rightarrow$ 2 D6 brane loci

$D6_1$: $p = r_4 = -2n - m,\quad q = r_3 = m$

$D6_2$: $p = r_3 = m,\quad q = r_4 = -2n - m$

$\hat{=}$ 3 stacks of D6-branes with

$G = SU(N) \times SU(N) \times SU(M)$

This might just correspond to the case.

with $Q = W\mathbb{CP}^2_{N,N,M}$ (Acharya, Witten)

IV) Summary

- Intersecting brane worlds with D6-branes on Calabi-Yau spaces provide an interesting framework in order to construct models with attractive phenomenological properties.

  The chiral spectrum is determined by the topological intersection numbers.

- G_2 manifolds provide an natural M-theory lift of $\mathcal{N} = 1$ supersymmetric IBW's.

  However it is very hard to construct explicit models with compact G_2 manifolds.

Further interesting aspects of IBW's

- Higgs effect by open string tachyon condensation and D-brane recombination.
- Mirror description of IBW's on the mirror Calabi-Yau space with D9-branes and magnetic gauge field fluxes.
- Problem of stabilty:

  (i) D-term potential due to FI-terms; this vanishes for supersymmetric D-brane configuations.

  (ii) F-term superpotential from disc and RP^2 instantons.

$$w(s, T^k) = \sum_{k,n,\vec{m}} \frac{1}{n^2} N_{k,\vec{m}} \exp(n[ks - \vec{m} \cdot \vec{T}])\,.$$

  These might destabilize the background. For certain simple non-compact CY threefolds using mirror symmetry such superpotentials have been explicitly computed.

  (Aganagic, Vafa, hep-th/0012041)
  (Aganagic, Klemm, Vafa, hep-th/0105045)
  (Acharya, Aganagic, Hori, Vafa, hep-th/0202208)