

# $E_{10}$ and the BKL-Limit of M Theory

H. Nicolai

AEI, Golm

based on work done with

T. Damour & M. Henneaux

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with thanks to:

A. Feingold

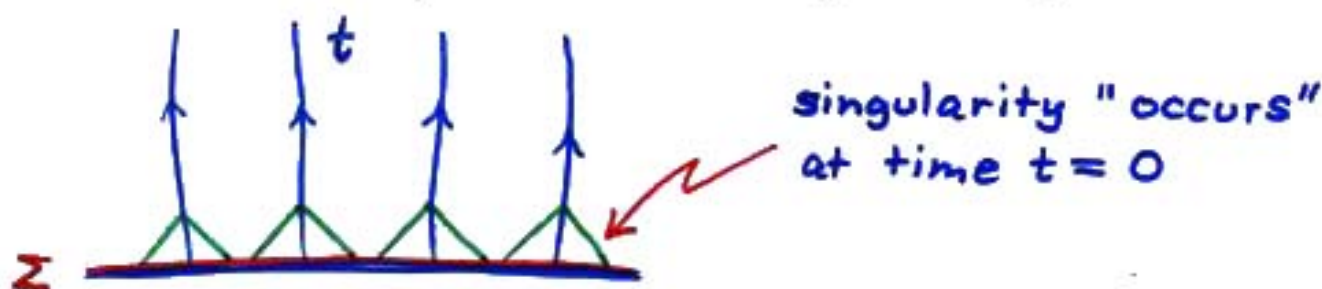
T. Fischbacher

## BKL : the basic picture

Belinskii,  
Khalatnikov,  
Lifshitz

Misner, Chitre, ...

describe generic behavior of EFE's  
near a spacelike singularity



spacelike  $\leftrightarrow$  causal decoupling  
spatial gradients  $\lll$  time derivatives

If true: EFE's reduce to continuous  
superposition of ODE's at each  $\vec{x} \in \Sigma$   
 $\rightarrow$  dimensional reduction of Einstein's  
theory to one dimension (almost!)

"Small tension limit" of Einstein action

$$S = \int dt \int d^d \vec{x} \left[ \rho_b (\partial_t h)^2 - T_b (\partial_x h)^2 \right] + \dots$$

$$\text{take } T_b \rightarrow 0 \Rightarrow c = \sqrt{T_b / \rho_b} \rightarrow 0$$

Recall  $T_s \rightarrow 0$  limit of string theory

$\rightarrow \infty$  many massless states

Hints of (enormous!) underlying  
hidden symmetry of string theory?

(Gross, 1988)

Hamiltonian description

metric ansatz  $ds^2 = -dt^2 + \sum_{a=1}^d e^{-2\beta^a} \omega_a \otimes \omega_a$

$$\omega^d = dx^d$$

$$\omega^{d-1} = dx^{d-1} + N_{d-1,d} dx^d$$

$\vdots$  (Iwasawa)

We seek an "effective low energy theory" for diagonal metric degrees of freedom

$$S = \left( \int d^d \vec{x} \right) \int dt \left\{ \underbrace{N^{-1} G_{ab} \dot{\beta}^a \dot{\beta}^b}_{\text{(a)}} - \underbrace{NV(\beta)}_{\text{(b)}} \right\}$$

- (a) Zeroth order approximation = dimensional reduction of Einstein's theory to one (time) dimension  
 $G_{ab}$  = DeWitt metric in truncated Wheeler-DeWitt superspace  $\mathcal{P}_d \ni \{\beta^a\}$   
 Lorentzian signature  $(- + \dots +)$

- (b) Effective potential  $V(\beta)$  from "integrating out" off-diagonal metric components, spatial inhomogeneities, matter fields, ...

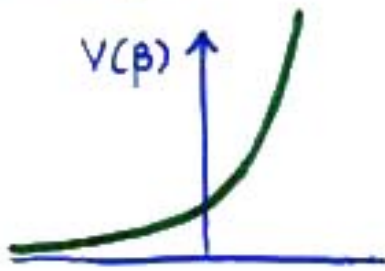
$$\text{Generally: } V(\beta) \sim \sum_A c_A e^{\omega_A(\beta)}$$

where ●  $c_A > 0$  for leading terms

●  $\omega_A(\beta) =$  linear form in  $\beta$

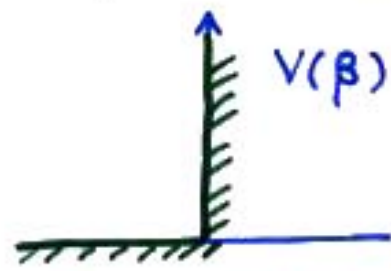


Misner, Chitre: dynamics simplifies as  $t \rightarrow 0$



Toda Wall

$t \rightarrow 0$   
 $\beta^a \rightarrow \pm \infty$

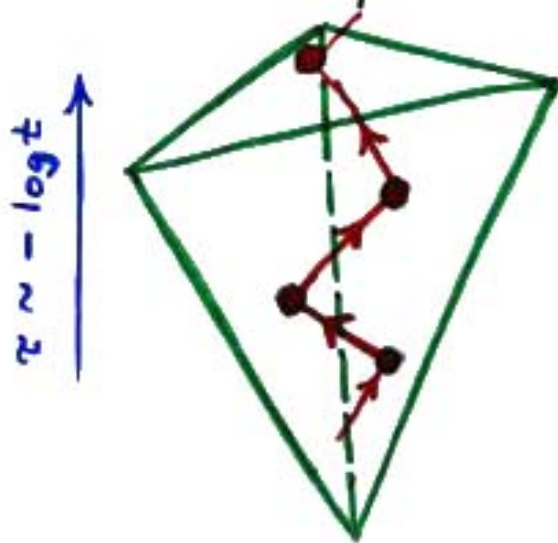


Sharp Wall

Hamiltonian constraint (from  $\delta N$ )

$$G_{ab} \dot{\beta}^a \dot{\beta}^b + V(\beta) = 0$$

In sharp wall limit  $V(\beta) = 0$  or  $\infty$



→ free Kasner flight

● collisions against walls  $\omega_A(\beta) = 0$

→ { Infinitely many collisions:  
 chaotic oscillations à la BKL  
 Finitely many collisions: "AVD"

Motion can be projected onto  $G_{ab} \beta^a \beta^b = -1$



⇒ relativistic ( $m^2 = 0$ )  
 hyperbolic billiard!

**Crucial insight:** dominant walls form fundamental Weyl chamber of some indefinite Kac Moody algebra  $\mathfrak{g}(A)$

Damour, Henneaux (2000)

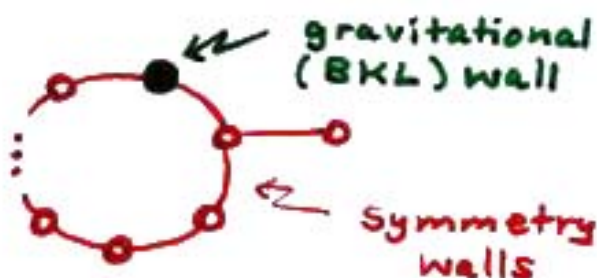
$\Rightarrow$  identify truncated WDW superspace  $\mathcal{P}_d$  with CSA  $\mathfrak{h}$  of  $\mathfrak{g}(A)$ !

- Chaotic oscillations à la BKL if  $\mathfrak{g}(A)$  hyperbolic KM algebra
- AVD if  $\mathfrak{g}(A)$  indefinite, but not hyperbolic (e.g.  $E_n$  for  $n \geq 11$ )

Damour, Henneaux, Julia, N.

### Examples:

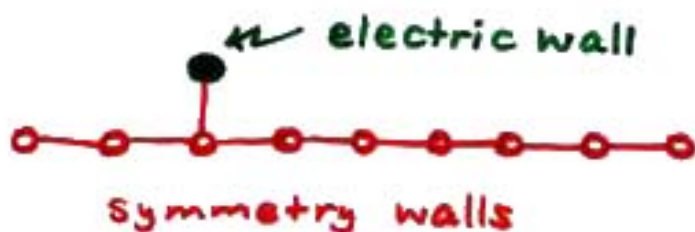
- (i) pure gravity in  $\mathcal{D} = d+1$  dimensions



$A_{E_d}$ :

hyperbolic for  $d \leq 9$

- (ii)  $\mathcal{D} = 11$  supergravity



$E_{10}$ :

maximal rank  
hyperbolic



Our Goal: extend these considerations beyond leading order and beyond CSA

Main Idea: search for map that relates the time evolution of the geometrical **M Theory** data at each spatial point [the fields and all their spatial gradients] to a null geodesic motion on the  $\infty$ -dimensional coset space  $E_{10}/K(E_{10})$ .

cf. conjectured appearance of  $E_{10}$  in  $D=1$  reduction of  $D=11$  SUGRA  
(Julia, 1983)

Results [Damour, Henneaux, N., to appear]

- construct map to low orders & recover Toda wall Hamiltonians
- show that coset space  $E_{10}/K(E_{10})$  can accommodate all geometrical data (and much more).

Or:

- $E_{11}$  ? West, hep-th/0104081
- (truncated) super-Borcherds ?

Labordère, Julia, ...

Proceed formally :  $D = 1$   $\sigma$ -model for

$$U = U(t) \in E_{10} / K(E_{10})$$

↖ "maximal compact" subgroup, see below

$$v(t) := \frac{dU}{dt} U^{-1}(t) \in e_{10} \equiv \text{Lie}(E_{10})$$

The Lagrangian is

$$\mathcal{L}(t) = \frac{1}{n(t)} \langle v_{\text{sym}}(t) | v_{\text{sym}}(t) \rangle \quad (*)$$

where

- $n(t) =$  "lapse"

- $v_{\text{sym}} := v - \omega(v)$

with  $\omega =$  Chevalley involution

$$\omega(h_i) = -h_i, \quad \omega(e_i) = -f_i, \quad \omega(f_i) = -e_i$$

defines maximal "compact" subgroup  $K = \omega(K) \subset E_{10}$

- $\langle \cdot | \cdot \rangle =$  standard invariant bilinear form on KM algebra

There are no other polynomial Casimir invariants on  $E_{10}$  but there do exist transcendental invariants [Kac Peterson] by which (\*) could be modified.

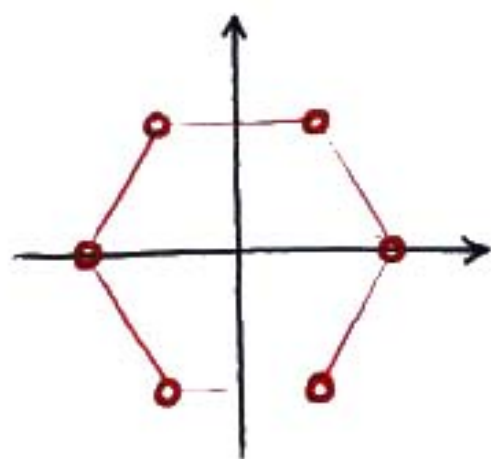
("nonperturbative" effects ?)

Unfortunately, working out the  $\sigma$ -model Lagrangian  $\mathcal{L}$  explicitly is not so easy even in the simplest cases ...

Recall rank 2 KM algebras:

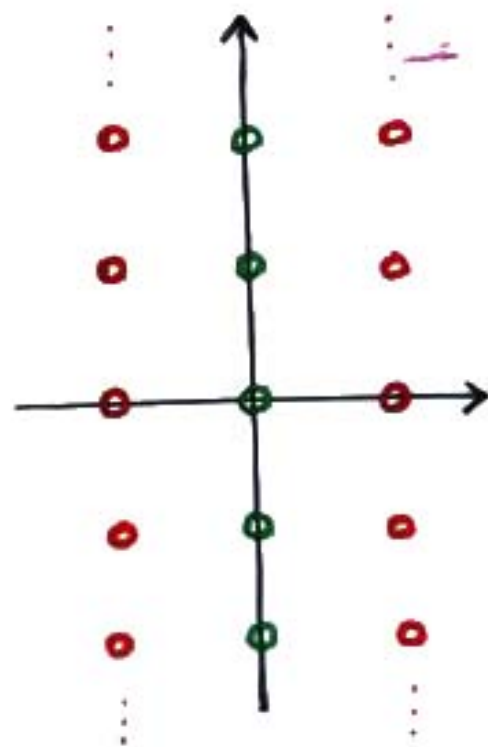
$$A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

$$[ = \widehat{sl(3)} ]$$



$$A = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

$$[ = \widehat{sl(2)}_{c.e.} ]$$



● real roots  $\alpha^2 = 2$

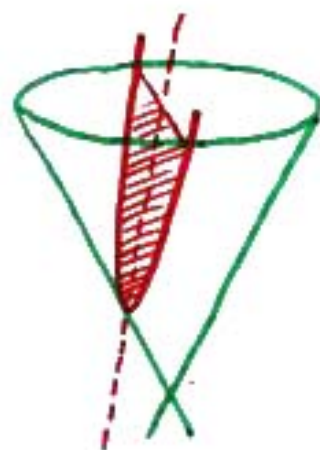
○ null roots  $\alpha^2 = 0$





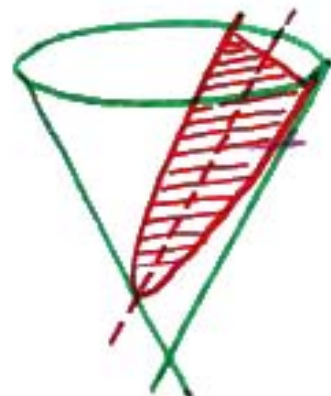
## How to "slice" a hyperbolic KM algebra

w. r. t. hyperbolic subalgebra (with hyperbolic weight diagrams): has not been tried so far, but would be most informative.

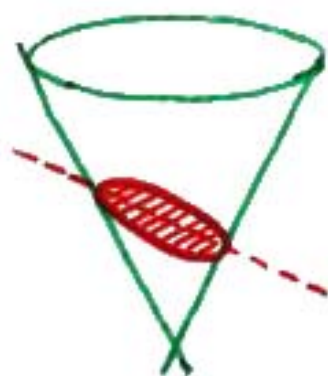


w. r. t. affine subalgebra (with parabolic weight diagrams): only results for affine levels  $0, \pm 1, \pm 2$ .

Feingold, Frenkel (1983)  
Kac, Moody, Wakimoto (1988)



w. r. t. finite subalgebra (finite weight diagrams): the "bottom-up-approach"



**Problem:** must understand structure of algebra "deep inside" forward light cone!





write  $\alpha = l\alpha_0 + \sum_{j=1}^9 m^j \alpha_j$

with  $l \equiv l(\alpha) = \text{"level" w.r.t. } A_9$

Thus, we decompose  $E_{10}$  into irreps of its  $A_9 \equiv \mathfrak{sl}(10)$  sub-algebra for all levels  $l [\rightarrow GL(10)]$ .

Irreps of  $A_9$  are characterized by their highest weights  $\Lambda \equiv \alpha$ , or equivalently their Dynkin labels  $(p_1, \dots, p_9)$  where

$$p_k := (\Lambda, \alpha_k) \geq 0$$

= # (columns with  $k$  boxes in associated Young tableau)

e.g.  $(001000000)$

$(100000010)$

etc.



For  $E_{10}$  the  $A_9$  irreps which can appear at a given level  $\ell$  must obey

$$m^i = S^{i3} \ell - \sum_{j=1}^9 S^{ij} p_j \quad (1)$$

where  $S =$  inverse Cartan matrix of  $A_9$ .  
 Strong constraint because all  $m^i, p_j$  must be non-negative integers!

Also h.w.  $\Lambda$  must be a root (Serre!)

$$\Lambda^2 = \sum_{i,j=1}^9 p_i S^{ij} p_j - \frac{1}{10} \ell^2 \leq 2 \quad (2)$$

Finding a completely explicit description of  $E_{10}$  in terms of infinite towers of  $GL(10)$  irreps is thus reduced to the problem of determining their outer multiplicities  $n_i = n(\ell, \mathcal{R}_i)$ .

In particular,

$$\text{mult}_{E_{10}}(\Lambda) = \sum n_i \text{mult}_{\mathcal{R}_i}(\Lambda)$$

↑  
 multiplicity of  $\Lambda$  as a root of  $E_{10}$

↑  
 outer multiplicity

↑  
 (inner) multiplicity of  $\Lambda$  as a weight of the  $A_9$  irrep  $\mathcal{R}_i$

→ low level irreps

$$l = 0 \quad \text{adjoint of } GL(10) \leftrightarrow K^a_b$$

$$l = 1 \quad (001\ 000\ 000) \leftrightarrow E^{abc}$$

$$l = 2 \quad (000\ 001\ 000) \leftrightarrow E^{a_1 \dots a_6}$$

$$l = 3 \quad (100\ 000\ 010) \leftrightarrow E^{a_1 \dots a_8 | b}$$

up to here: see also West, hep-th/0104081;  
Cremmer, Julia, Lu, Pope (for  $E_8$ )

Clearly,

$$l = 0 \quad \text{Graviton} \leftrightarrow G_{ab} = \underline{G}_{ba}$$

$$l = 1 \quad \text{3-Form} \leftrightarrow A_{abc}$$

$$l = 2 \quad \text{Dual 6-Form} \leftrightarrow A_{a_1 \dots a_6}$$

$$l = 3 \quad \text{"Dual Graviton"} \leftrightarrow A_{a_1 \dots a_8 | b}$$

But we can go further:

$$l = 4 \quad (001\ 000\ 001), (200\ 000\ 000)$$

$$l = 5 \quad (000\ 001\ 001), (100\ 100\ 000)$$

$$l = 6 \quad (100\ 000\ 011), (010010000), \\ (100\ 000\ 100), (000000010)$$

so far: outer multiplicity = 1



## The basic correspondence

choose special frame  $\theta^a(x) = E^a_i(x) dx^i$

$$ds^2 = -N dt^2 + G_{ab} \theta^a \theta^b$$

$$\mathcal{F} = \frac{1}{3!} F_{tabc} dt \wedge \theta^a \wedge \theta^b \wedge \theta^c + \\ + \frac{1}{4!} F_{abcd} \theta^a \wedge \theta^b \wedge \theta^c \wedge \theta^d$$

with  $d\theta^a = \frac{1}{2} C^a_{bc} \theta^b \wedge \theta^c$

$\Rightarrow \mathcal{D} = 11$  SUGRA eqs. of motion  
Cremmer, Julia, Scherk (1978)

$$\partial_t (G^{ac} \partial_t G_{cb}) = \frac{1}{6} G F^{\alpha\beta\gamma\delta} F_{b\beta\gamma\delta} - \\ - \frac{1}{92} G \mathcal{F}^{\alpha\beta\gamma\delta} F_{\alpha\beta\gamma\delta} \delta_b^a - 2 G R^a_b(\Gamma, C)$$

$$\partial_t (G F^{tabc}) = \frac{1}{144} \epsilon^{abcdefghij} F_{tdef} F_{ghij} \\ + \frac{3}{2} G F^{def[ab} C^c]_{de} - G C^e_{de} F^{dabc} - \partial_a (G F^{dabc})$$

$$\partial_t F_{abcd} = 6 F_{te[ab} C^e_{cd]} + 6 \partial_{[a} F_{tbc]d}$$

Demaret, Hanquin, Henneaux, Spindel (1985)

Borel gauge for  $E_{10}/K(E_{10})$   $\sigma$ -model:

$$U = \exp X_h(t) \cdot \exp X_A(t)$$



$$e^a_i \equiv (\exp h(t) \cdot K)^a_i \rightarrow g_{ab} = e_a^i e_{bi}$$

$$X_A(t) = \frac{1}{3!} A_{abc} E^{abc} + \frac{1}{6!} A_{a_1 \dots a_6} E^{a_1 \dots a_6} \\ + \frac{1}{9!} A_{a_1 \dots a_8 | a_9} E^{a_1 \dots a_8 | a_9} + \dots$$

Equations of motion match (keeping first order spatial gradients) with:

- $g_{ab}(t) = G_{ab}(t, x)$  ( $x$  fixed!)
- $\dot{A}_{abc}(t) = F_{tabc}(t, x)$
- $\dot{A}_{a_1 \dots a_6}(t) + 10 A_{[a_1 a_2 a_3]}(t) \dot{A}_{a_4 a_5 a_6]}(t) = \\ = -\frac{1}{4!} \epsilon_{a_1 \dots a_6}^{b_1 \dots b_4} F_{b_1 b_2 b_3 b_4}(t, x)$
- $\dot{A}_{a_1 \dots a_8 | a_9}(t) + 42 A_{\langle a_1 a_2 a_3 \rangle}(t) \dot{A}_{a_4 \dots a_9 \rangle}(t) \\ - 42 \dot{A}_{\langle a_1 a_2 a_3 \rangle}(t) A_{a_4 \dots a_9 \rangle}(t) + \\ + 280 A_{\langle a_1 a_2 a_3 \rangle}(t) A_{a_4 a_5 a_6}(t) \dot{A}_{a_7 a_8 a_9 \rangle}(t) \\ = \frac{3}{2} \epsilon_{a_1 \dots a_8 bc} (C_{a_9}^{bc}(x) + \frac{2}{9} \delta_{a_9}^b C_d^{cd}(x))$

where  $\langle \dots \rangle =$  projector onto (100000010

But how to continue beyond first order in spatial gradients?

[This will be the acid test of the idea!]

Search for "affine" roots ( $m^g = 0$ ) in (1) and (2) reveals existence of three infinite towers of  $E_{10}$  elements:

$$(001\ 000\ 00n) \quad E_{a_1 \dots a_n}{}^{b_1 b_2 b_3}$$

$$(000\ 001\ 00n) \quad E_{a_1 \dots a_n}{}^{b_1 \dots b_6}$$

$$(100\ 0000\ 1n) \quad E_{a_1 \dots a_n}{}^{b_1 \dots b_8 | c}$$

Truncation  $a_1 = \dots = a_n = 1$  and  $b_1, \dots, c \in \{2, \dots, 10\}$  yields all elements of affine subalgebra  $E_g \subset E_{10}$

Comparison with linear system of  $\mathcal{D} = 2$  maximal supergravity suggests that we should associate

$$E_{a_1 \dots a_n}{}^{b_1 b_2 b_3} \longleftrightarrow \partial^{a_1} \dots \partial^{a_n} A_{b_1 b_2 b_3}$$

$$E_{a_1 \dots a_n}{}^{b_1 \dots b_6} \longleftrightarrow \partial^{a_1} \dots \partial^{a_n} A_{b_1 \dots b_6}$$

$$E_{a_1 \dots a_n}{}^{b_1 \dots b_8 | c} \longleftrightarrow \partial^{a_1} \dots \partial^{a_n} A_{b_1 \dots b_8 | c}$$

[dual potentials à la Geroch in reduction to  $\mathcal{D} = 2$ , i.e.  $\partial^a \rightarrow \partial^i$ ]

**BUT THERE IS MUCH MORE...**



Table 3:  $E_{10}$  Root Multiplicities

$\Lambda$	$\Lambda$	$\ell(\Lambda)$	$ht(\Lambda)$	$\Lambda^2$	$mult(\Lambda)$	$\Delta(\Lambda)$
$\delta + \Lambda_8$	[3, 7, 11, 15, 19, 23, 27, 18, 9, 13]	3	145	-16	167116	35064
$5\delta$	[0, 5, 10, 15, 20, 25, 30, 20, 10, 15]	0	150	0	8	0
$3\delta + \Lambda_0$	[1, 5, 10, 15, 20, 25, 30, 20, 10, 15]	1	151	-8	2464	254
$\delta + 2\Lambda_0$	[2, 5, 10, 15, 20, 25, 30, 20, 10, 15]	2	152	-12	22712	3514
$2\Lambda_7$	[4, 8, 12, 16, 20, 24, 28, 18, 8, 14]	4	152	-16	167133	35047
$2\delta + \Lambda_1$	[2, 6, 10, 15, 20, 25, 30, 20, 10, 15]	2	153	-14	63020	11536
$\Lambda_6$	[4, 8, 12, 16, 20, 24, 28, 18, 9, 14]	4	153	-18	425227	101170
$\Lambda_0 + \Lambda_1$	[3, 6, 10, 15, 20, 25, 30, 20, 10, 15]	3	154	-16	167099	35081
$\delta + \Lambda_2$	[3, 7, 11, 15, 20, 25, 30, 20, 10, 15]	3	156	-18	425156	101241
$\Lambda_3$	[4, 8, 12, 16, 20, 25, 30, 20, 10, 15]	4	160	-20	1044218	278125
$3\delta + \Lambda_7$	[2, 7, 12, 17, 22, 27, 32, 21, 10, 16]	2	166	-16	166840	35340
$\delta + \Lambda_0 + \Lambda_7$	[3, 7, 12, 17, 22, 27, 32, 21, 10, 16]	3	167	-20	1043926	278417
$\Lambda_1 + \Lambda_7$	[4, 8, 12, 17, 22, 27, 32, 21, 10, 16]	4	169	-22	2485020	733071
$2\delta + \Lambda_8$	[3, 8, 13, 18, 23, 28, 33, 22, 11, 16]	3	175	-22	2483970	734121
$\Lambda_0 + \Lambda_8$	[4, 8, 13, 18, 23, 28, 33, 22, 11, 16]	4	176	-24	5749818	1862196
$6\delta$	[0, 6, 12, 18, 24, 30, 36, 24, 12, 18]	0	180	0	8	0
$4\delta + \Lambda_0$	[1, 6, 12, 18, 24, 30, 36, 24, 12, 18]	1	181	-10	7704	1026
$2\delta + 2\Lambda_0$	[2, 6, 12, 18, 24, 30, 36, 24, 12, 18]	2	182	-16	166840	35340
$\delta + 2\Lambda_7$	[4, 9, 14, 19, 24, 29, 34, 22, 10, 17]	4	182	-24	5750072	1861942
$3\delta + \Lambda_1$	[2, 7, 12, 18, 24, 30, 36, 24, 12, 18]	2	183	-18	424161	102236
$3\Lambda_0$	[3, 6, 12, 18, 24, 30, 36, 24, 12, 18]	3	183	-18	425058	101339
$\delta + \Lambda_6$	[4, 9, 14, 19, 24, 29, 34, 22, 11, 17]	4	183	-26	12971009	4577911
$\delta + \Lambda_0 + \Lambda_1$	[3, 7, 12, 18, 24, 30, 36, 24, 12, 18]	3	184	-22	2483871	734220
$2\delta + \Lambda_2$	[3, 8, 13, 18, 24, 30, 36, 24, 12, 18]	3	186	-24	5746226	1865788
$2\Lambda_1$	[4, 8, 12, 18, 24, 30, 36, 24, 12, 18]	4	186	-24	5749565	1862449
$\Lambda_0 + \Lambda_2$	[4, 8, 13, 18, 24, 30, 36, 24, 12, 18]	4	187	-26	12970045	4578875
$\delta + \Lambda_3$	[4, 9, 14, 19, 24, 30, 36, 24, 12, 18]	4	190	-28	28592513	10931086
$\Lambda_7 + \Lambda_8$	[5, 10, 15, 20, 25, 30, 35, 23, 11, 17]	5	191	-28	28595548	10928051
$\Lambda_4$	[5, 10, 15, 20, 25, 30, 36, 24, 12, 18]	5	195	-30	61721165	25411831
$4\delta + \Lambda_7$	[2, 8, 14, 20, 26, 32, 38, 25, 12, 19]	2	196	-20	1040664	281679
$2\delta + \Lambda_0 + \Lambda_7$	[3, 8, 14, 20, 26, 32, 38, 25, 12, 19]	3	197	-26	12959290	4589630
$2\Lambda_0 + \Lambda_7$	[4, 8, 14, 20, 26, 32, 38, 25, 12, 19]	4	198	-28	28589025	10934574
$\delta + \Lambda_1 + \Lambda_7$	[4, 9, 14, 20, 26, 32, 38, 25, 12, 19]	4	199	-30	61711591	25421405
$\Lambda_2 + \Lambda_7$	[5, 10, 15, 20, 26, 32, 38, 25, 12, 19]	5	202	-32	130661924	57690454
$3\delta + \Lambda_8$	[3, 9, 15, 21, 27, 33, 39, 26, 13, 19]	3	205	-28	28559052	10964547
$\delta + \Lambda_0 + \Lambda_8$	[4, 9, 15, 21, 27, 33, 39, 26, 13, 19]	4	206	-32	130632964	57719414
$\Lambda_1 + \Lambda_8$	[5, 10, 15, 21, 27, 33, 39, 26, 13, 19]	5	208	-34	271695444	128129588
$7\delta$	[0, 7, 14, 21, 28, 35, 42, 28, 14, 21]	0	210	0	8	0
$5\delta + \Lambda_0$	[1, 7, 14, 21, 28, 35, 42, 28, 14, 21]	1	211	-12	22528	3698
$3\delta + 2\Lambda_0$	[2, 7, 14, 21, 28, 35, 42, 28, 14, 21]	2	212	-20	1040664	281679
$2\delta + 2\Lambda_7$	[4, 10, 16, 22, 28, 34, 40, 26, 12, 20]	4	212	-32	130635596	57716782
$4\delta + \Lambda_1$	[2, 8, 14, 21, 28, 35, 42, 28, 14, 21]	2	213	-22	2474026	744065
$\delta + 3\Lambda_0$	[3, 7, 14, 21, 28, 35, 42, 28, 14, 21]	3	213	-24	5745720	1866294
$\Lambda_0 + 2\Lambda_7$	[5, 10, 16, 22, 28, 34, 40, 26, 12, 20]	5	213	-34	271702532	128122500
$2\delta + \Lambda_6$	[4, 10, 16, 22, 28, 34, 40, 26, 13, 20]	4	213	-34	271618575	128206457
$2\delta + \Lambda_0 + \Lambda_1$	[3, 8, 14, 21, 28, 35, 42, 28, 14, 21]	3	214	-28	28558597	10965002
$\Lambda_0 + \Lambda_6$	[5, 10, 16, 22, 28, 34, 40, 26, 13, 20]	5	214	-36	555652661	278885204
$2\Lambda_0 + \Lambda_1$	[4, 8, 14, 21, 28, 35, 42, 28, 14, 21]	4	215	-30	61699285	25433711
$3\delta + \Lambda_2$	[3, 9, 15, 21, 28, 35, 42, 28, 14, 21]	3	216	-30	61620301	25512695
$\delta + 2\Lambda_1$	[4, 9, 14, 21, 28, 35, 42, 28, 14, 21]	4	216	-32	130630342	57722036
$\delta + \Lambda_0 + \Lambda_2$	[4, 9, 15, 21, 28, 35, 42, 28, 14, 21]	4	217	-34	271609694	128215338



affine levels 0, 1, 2 :

Kac, Moody, Wakimoto



[with T. Fischbacher]

$l = 7$

$1 * (001000002)^+$   
 $1 * (000100100)$   
 $1 * (110000010)$   
 $1 * (001000010)$

$1 * (200000001)$   
 $2 * (010000001)$   
 $1 * (100000000)$

outer multiplicities

$l = 8$

$1 * (000001002)^+$   
 $1 * (000000200)$   
 $1 * (100010010)$   
 $1 * (000001010)$   
 $1 * (011000001)$

$2 * (100100001)$   
 $2 * (000010001)$   
 $1 * (210000000)$   
 $2 * (101000000)$   
 $2 * (000100000)$

$l = 9$

$1 * (100000012)^+$   
 $1 * (010000110)$   
 $1 * (100000020)$   
 $1 * (001010001)$   
 $1 * (200001001)$   
 $2 * (010001001)$   
 $4 * (100000101)$

$3 * (000000011)$   
 $1 * (110100000)$   
 $1 * (001100000)$   
 $1 * (200010000)$   
 $3 * (010010000)$   
 $4 * (100001000)$   
 $4 * (000000100)$

etc. [+ : "affine" irreps]

... a glimpse of  $E_{10}$  ...

→ suggests the existence of a huge and hitherto unknown nonlocal symmetry of Einstein's theory and/or its generalizations (transcending Geroch).



Horatio: ... but this is wondrous strange!

Hamlet: And therefore as a stranger give it welcome. There are more things in heaven and earth, Horatio, Than are dreamt of in your philosophy.

(Hamlet, Act I, Scene V)