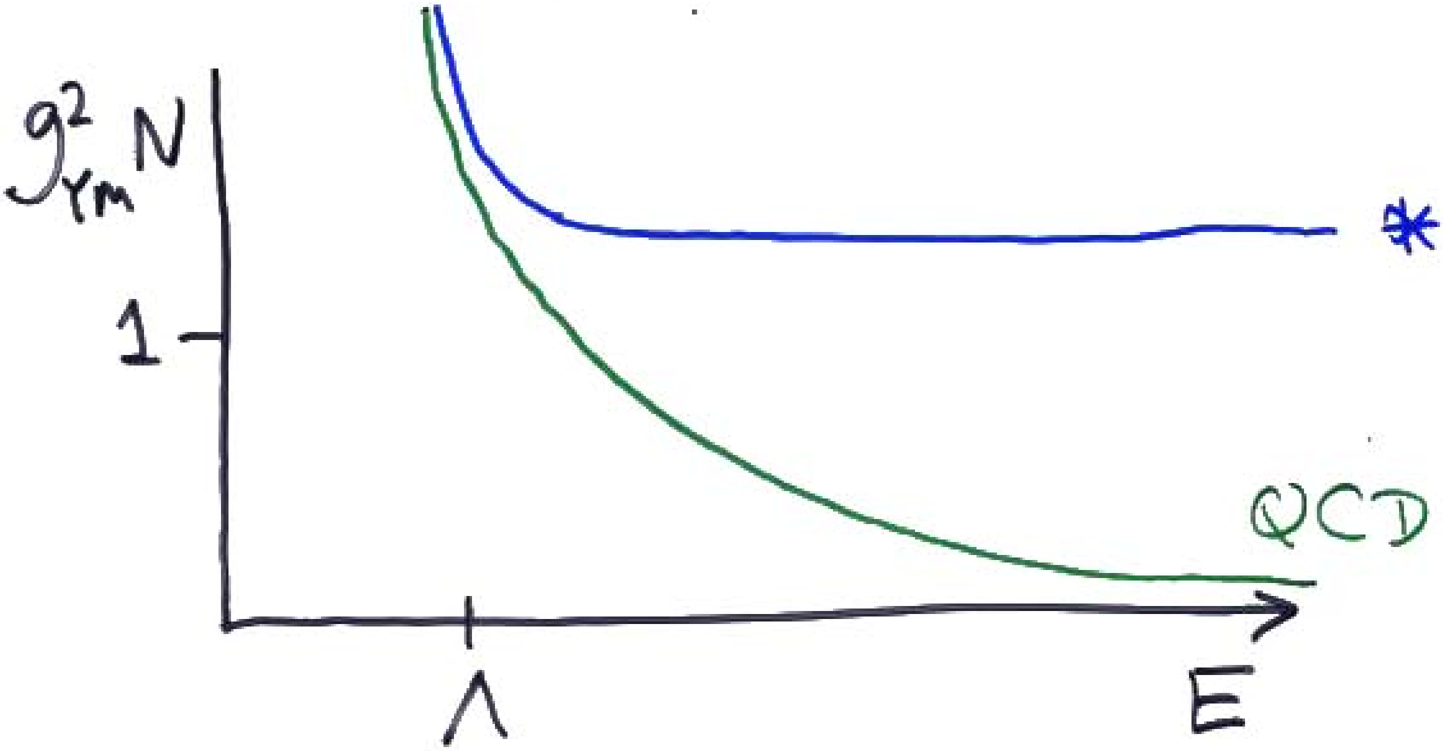
Rutherford Scattering in String Theory



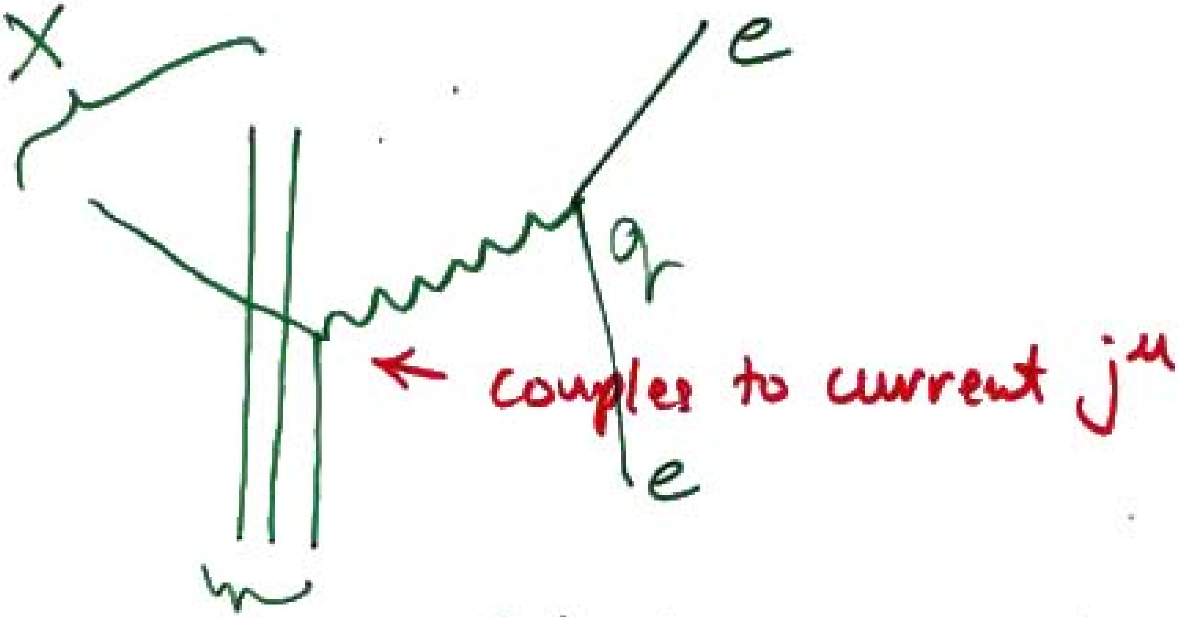
e.g. $\mathcal{N}=4$ softly broken, $N \gg 1$

dual = $AdS_5 \times M_5$ with small-r cutoff

- alternate history
- experimental probe: deep inelastic scattering (DIS)

w/ Matt Strassler, forthcoming

Review of DIS



$|h\rangle$, initial hadron, momentum P

- total cross section, summed over X with given $M_X^2 = -P_X \cdot P_X$
- P^2 negligible so two parameters, q^2 and M_X^2
- Bjorken: work with q^2 and

$$x = \frac{q^2}{q^2 + M_X^2}, \qquad 0 < x < 1$$

(note: $M_X^2 \to \infty$ as $x \to 0$, fixed q^2)

$$\text{Rate} \propto \sum_X \langle h | j^\mu | X \rangle \langle X | j^\nu | h \rangle$$

$$= \text{Im} \langle h | T(j^\mu j^\nu) | h \rangle$$

$$= F_1(x, q^2) \left(g_{\mu\nu} - q_\mu q_\nu / q^2 \right)$$

$$+ \frac{2x}{q^2} F_2(x, q^2) \left(P_\mu + \frac{q_\mu}{2x} \right) \left(P_\nu + \frac{q_\nu}{2x} \right)$$

In the parton model:

$$F_2(x, q^2) = 2x F_1(x, q^2) = \sum_i Q_i^2 f_i(x)$$

x = momentum fraction of struck parton

Q_i = charge of parton type i.

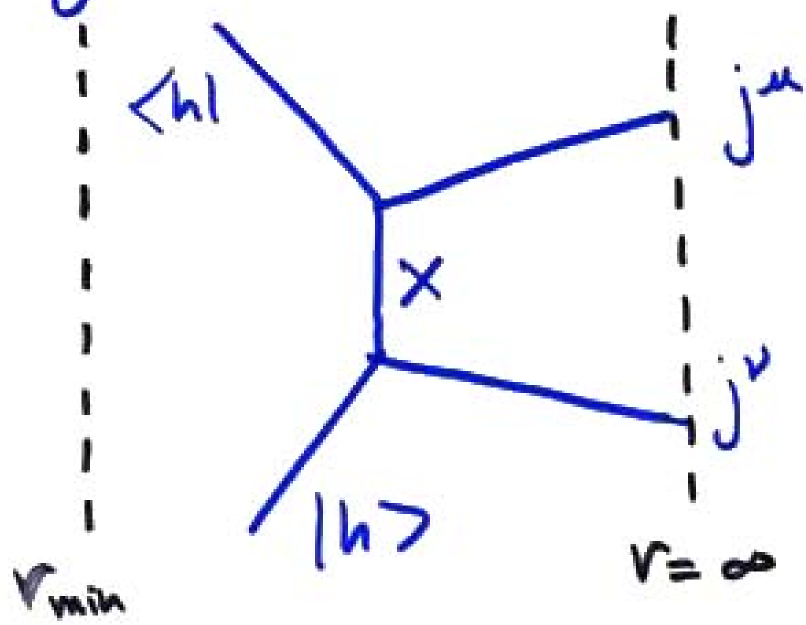
- independent of q^2.
- in QCD: slow evolution to small x w/ increasing q^2 (parton splitting)

Need to calculate $\langle h | T(j^\mu j^\nu) | h \rangle$.

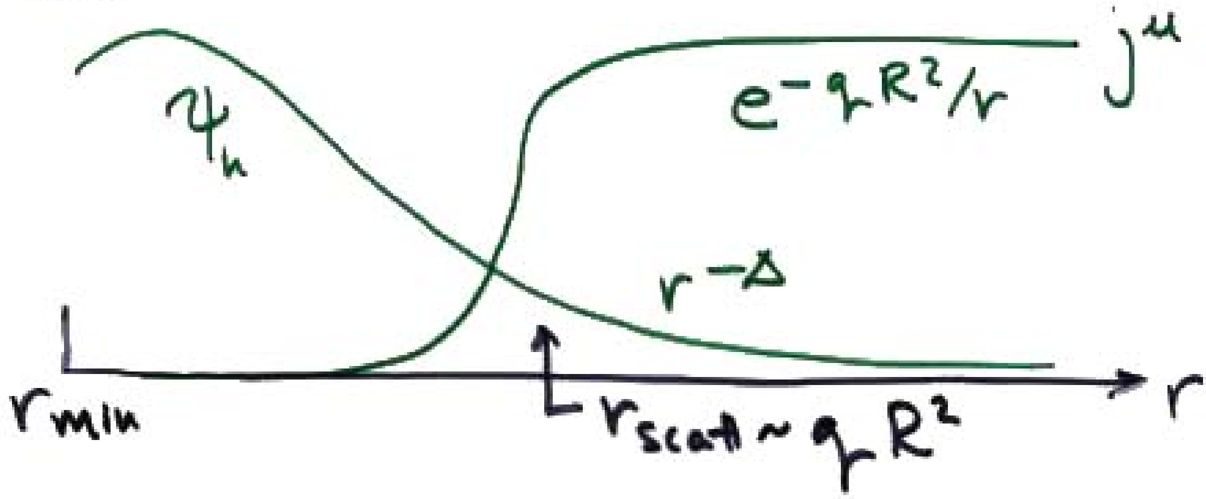
Straightforward application of AdS/CFT dictionary (Gubser, Klebanov, Polyakov; Witten)

$|h\rangle$ = normalizable mode in cutoff AdS space

j^μ excites non-normalizable mode



X = single string (leading 1/N).



Three regimes:

$x = O(1)$: only produce massless 10-d states so sugra calculation.

$x = O(1/\sqrt{g_{YM}^2 N})$: produce excited strings; fold flat-spacetime string amplitude into local external wavefunctions

$x = O(e^{-\sqrt{g_{YM}^2 N}})$: produce excited strings as large as AdS spacetime, must do string calculation in curved spacetime.

• note: leading $1/N$ only

$x = O(1)$: $F_2 \propto x^{\Delta+1}(1-x)^{\Delta-2}\left(\Lambda^2/q^2\right)^{\Delta-1}$

F_1 similar, depends on spin

String interpretation: falls with q^2, so no hard partons, but doesn't fall exponentially - due to warped geometry.

(picture)

Field theory interpretation: operator of twist τ ($\equiv$ dimension − spin) in $j^\mu j^\nu$ OPE contributes $F_{1,2} \sim q^{2-\tau}$.

At weak coupling $\bar{\psi}\gamma^\mu \partial^\nu \partial^\lambda \ldots \partial^\rho \psi$ all have $\tau = 2 + O(g_{YM}^2 N)$,

but at strong coupling $\tau \sim (g_{YM}^2 N)^{1/2}$.

GKP

Dominant operators double trace, $\mathcal{O}^\dagger \mathcal{O}$, where $\langle h | \mathcal{O} | 0 \rangle \neq 0$

$\tau = 2\Delta$

Need to calculate $\langle h | T(j^\mu j^\nu) | h \rangle$.

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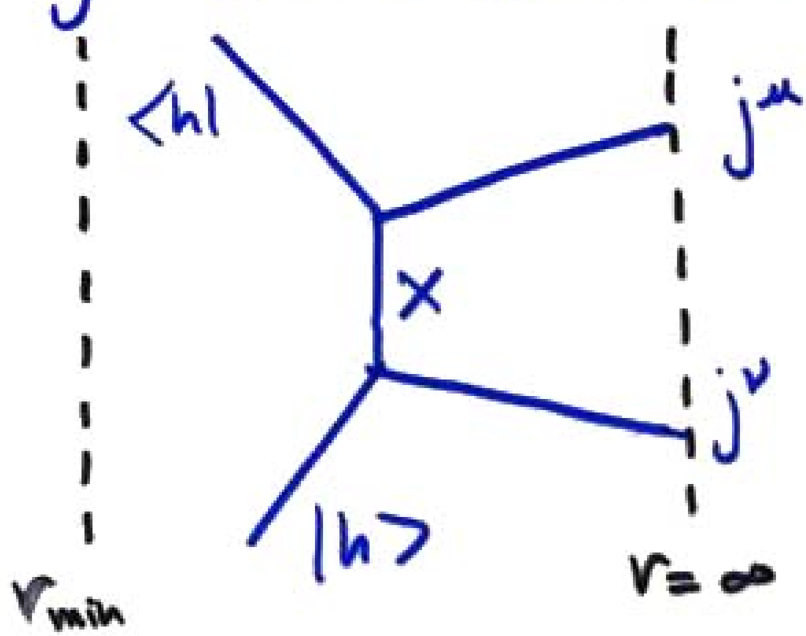

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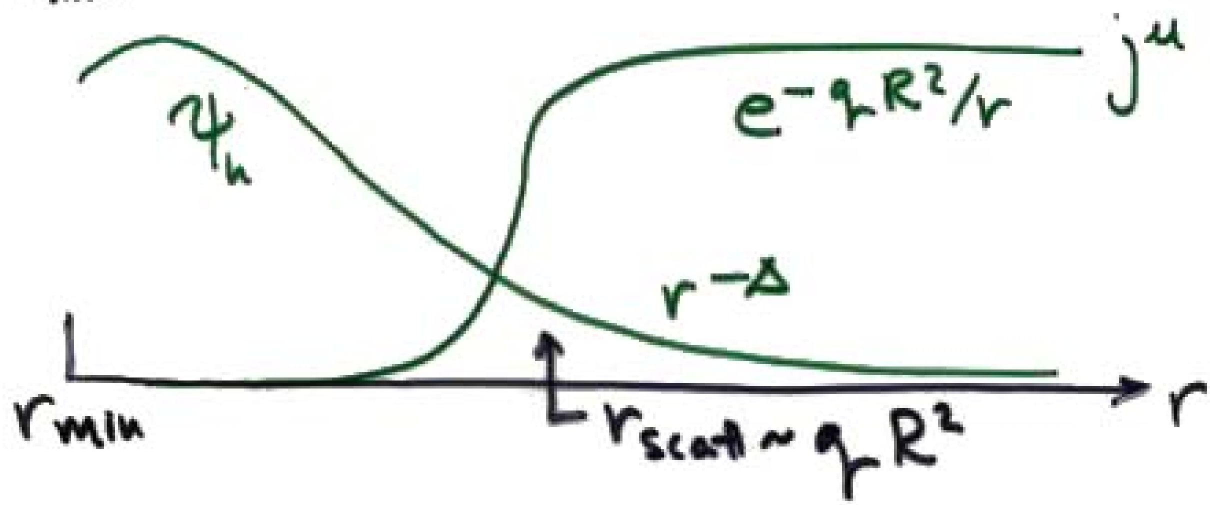
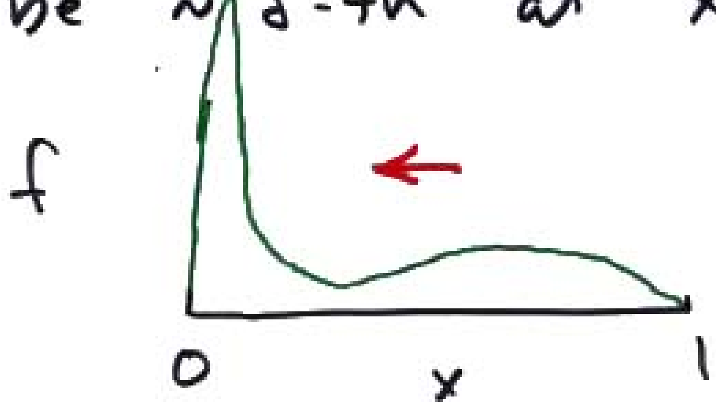


One more thing:

$$\int_0^1 x f(x, q^2)\, dx \text{ is conserved}$$

(momentum sum rule) so there must be ~ δ-fn at $x = 0$.

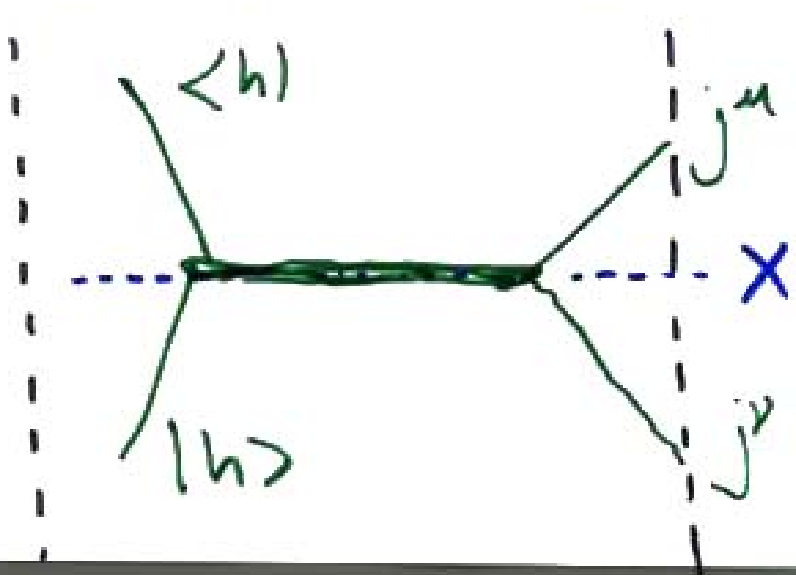


(Kogut-Susskind) 1973

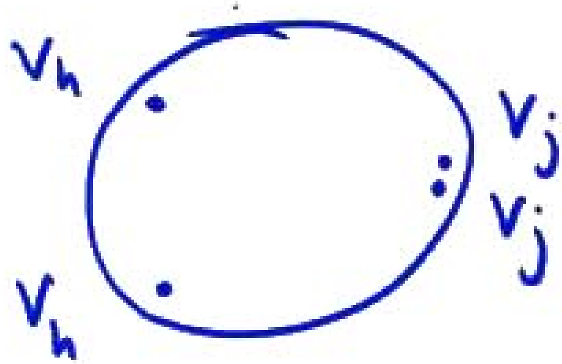
Inclusion of excited strings almost fixes this, $f \sim 1/x^2 \cdot q^{2-2\Delta}$

but momentum sum diverges at small x.

What cuts it off? Growth of strings: ↑ $\sqrt{\ln s}$



Need full string calculation, but its very simple. In Regge regim



dominated by separation $\sim x \ll 1$.

So:

A) Use $V_j V_j$ OPE (keep leading Regge trajectory).

* B) Use RNG from world-sheet scale x to scale 1. (Inserts $x^{\alpha' \nabla^2_{t\text{-channel}}}$)

C) Evaluate final 3-pt function.

* sums all orders in $\frac{1}{\sqrt{g_{YM}^2 N}} \ln x$

Future Directions

- Are there other contexts where the high-energy growth of strings in curved spacetime is relevant? Black holes?

- At large $g_{YM}^2 N$,

At small $g_{YM}^2 N$

How is this reflected in the strongly coupled world-sheet CFT. E.g., unusual properties of operator

$\vec{X}(\sigma, \tau)$ (JP + Susskind)