Gauge Invariant Words and the Fifth Dimension

Two problems :

1) Quark Confinement

Origin of Space-Time.

United by the String theory

quarks

Gauge Fields

Alphabet

$(\nabla^k F_{\mu\nu} \dots)$

Words

$\mathrm{Tr}(D^k F\, D^l F \dots)$

Space-Time

Strings

warped Liouville

closed string states

The origin of the fifth dimension

Flux lines in 4d form a world sheet

$$x^\mu = x^\mu(\xi^1, \xi^2).$$

The action:

$$S = \int \sqrt{g}\left(g^{ab}\, \partial_a x^\mu \partial_b x^\mu + \lambda\right) d^2\xi$$

$$g_{ab} = e^\varphi \delta_{ab}.$$

$$S_{eff} \sim \int (\partial\varphi)^2 + (\partial\vec{x})^2 + \dots$$

All string fields, describing closed strings (gluonium) are 5d :

$$\Psi = \Psi(\vec{x}, \varphi).$$

Warped Liouville

For $c > 1$ Flat $(\varphi, \bar{x})$ geometry is unstable.

The effective action becomes

$$S_{eff} = \int ((\partial\varphi)^2 + a^2(\varphi)(\partial\bar{x})^2 \, d^2\sigma$$

↑ running string tension

+ fermions)

(we will need NSR string)

The warp factor is determined by the Weyl symmetry on the world sheet (β-function equations)

In $\mathcal{N}=4$ Y.M. the general scheme simplifies

$$\mathcal{L}_{\gamma} = \frac{1}{2\alpha_0}\left\{\frac{(\partial\bar{x})^2+(\partial y)^2}{y^2} + (\partial\vec{n})^2\right\} + \ldots =$$

$$\boxed{\begin{gathered}\alpha_0 = 1/\sqrt{\lambda}\\ \lambda = g^2_{YM}N\end{gathered}}$$

$$= \frac{1}{2\alpha_0}\left\{(\nabla\vec{N})^2 + (\nabla\vec{n})^2\right\} + \ldots$$

($AdS_5 \times S_5$ sigma model).

Here: $\vec{n} \in S^5 \quad \vec{n}^2 = 1$

$N \in AdS^5$

$$\boxed{N_+ N_- - N_\mu^2 = 1}$$

$$\begin{cases} N_- = 1/y \\ N_+ = 1/y\,(y^2 + \bar{x}^2) \\ N_\mu = 1/y\, x_\mu \end{cases}$$

Vertex operators

$\Leftrightarrow$ Gauge invariant words: if we have a (1, 1) operator on the W.S. : $\Psi_n[x(\xi), y(\xi)]$

Then: a gauge theory operator $\mathcal{O}_n(\vec{x})$ is given by

$$\boxed{\mathcal{O}_n(\vec{x}) = \int \Psi_n(\vec{x}(\xi) - \vec{x}, y(\xi)) d^2\xi}$$

Example:

$$\Psi = \left(n_+^J \; N_+^{-\Delta}\right) {}^{(2)}R(g)\sqrt{g}$$

$$\delta = \frac{\alpha_0}{2}\left[J(J+4) - \Delta(\Delta-4)\right]$$

$= 0$ / $S^5 \to AdS_5$ by $J \to -\Delta$ / $\alpha_0 \to -\alpha_0$

$$\mathcal{O} = \mathrm{Tr}\left(F_{\mu\nu}^2 \, \phi_+^J\right)$$

Dynamics

Words are not independ.
There are relations
coming from
$\nabla_\mu F_{\mu\nu} = 0$ and $F_{\mu\nu} = [\nabla_\mu, \nabla_\nu]$
These relations reflect
Y.-M. dynamics.
What they mean in the
string language?
Example: one of the relations
is $\partial_\mu T_{\mu\nu} = 0$.
On the string side
the graviton is
$h_{MN}\, a^{+}_{1M}\, a^{+}_{1N} |0, p\rangle$
$\Rightarrow (\ldots) + L_{-1} |f\rangle$ The null state

The L_{-1} on the world-sheet means

$$\delta h_{\mu\nu} = \nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu .$$

A stringy variational principle

$$\langle V_{NULL} \rangle = 0 \Rightarrow$$

$\Rightarrow$ Y. M. equations on the gauge side.

Demonstration: Take a D-brane action $S_D[y^i(x) \ldots]$ (where y^i is transverse coord.)

D-brane located at $y^i = 0$

Insertion of $V_{NULL} = L_{-1}V$ generates $y^i \to y^i + \epsilon^i$ If Y. M. eq. satisfied:

$$\langle V_{NULL} \rangle \sim \int \frac{\delta S_\theta}{\delta y^i} \epsilon^i = 0$$

so

$\langle V_{NULL} \rangle = 0 \Leftrightarrow$ relations between the words.

Another example of L_{-1}-relation:

$$\partial_\mu \mathrm{Tr}(X^i F_{\mu\nu}) = \mathrm{Tr}(F_{\mu\nu} D_\nu X^i) + \dots$$

coming from:

$$\delta B_{\mu\nu} = D_\mu \epsilon_\nu - D_\nu \epsilon_\mu$$

Semiclassical lim

(Large dimensions, but weak couplings)

General idea:

if $\alpha_0 \to 0$, but Δ, J, S, ... etc are large, then

$$\delta = \frac{1}{\alpha_0} f(\alpha_0 J, \alpha_0 S, \alpha_0 \Delta, \ldots)$$

The function f can be calculated as a mass of the conformal soliton

Large R charge revisited

We have to calculate:

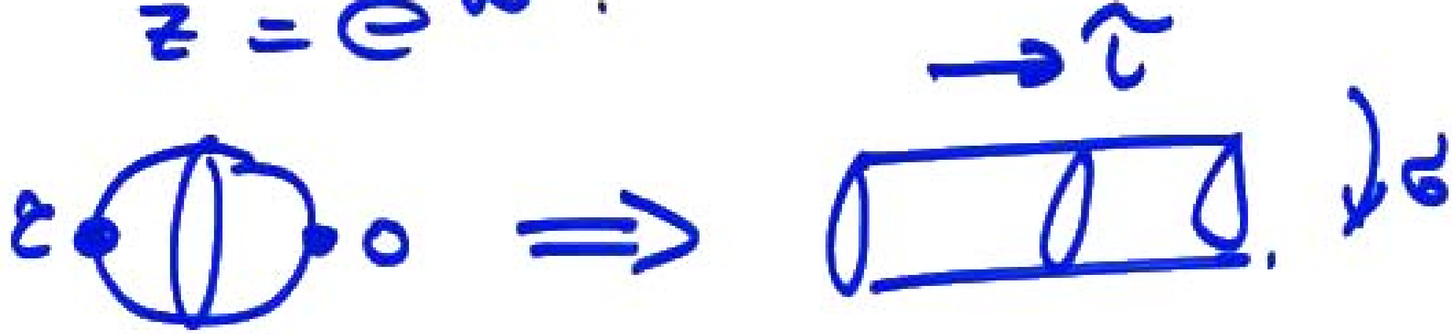
$$\int \mathcal{D}\vec{n}\, e^{-\frac{1}{\lambda_0}\int (\nabla \vec{n})^2}\, n_+^J(\tau)\, n_-^J(0)$$

$$\sim \tau^{-2\delta(J)}$$

When J is large, the integral is dominated by the classical config.

Radial quantization:

$z = e^w$.



$$\boxed{\delta = \text{Energy on the cylinder}}$$

For the small α_0 this energy is (semi) classical

To fix J consider

$$\tilde{H} = H - \omega J$$

$$\mathcal{L} = \frac{1}{2}\left[(\nabla\theta)^2 + \cos^2\theta\,(\nabla\varphi)^2\right]$$

Classical solution!

$\varphi = i\omega\tau$ $\qquad \theta = 0$ (equator)

/ i - comes from

$$n_+^J \sim e^{iJ\varphi} \ldots$$ /.

$$E_{cl} = -\frac{1}{2\alpha_0}\omega^2. \qquad J = \frac{\partial E}{\partial \omega}$$

$$\delta_{cl} = \frac{\alpha_0}{2} J^2.$$

For large quantum numbers, δ can be calculated classically.

1) Leading Regge Trajectory.

In flat space - vertex op of the type:

$$V \sim \partial_z X_{\{\mu_1} \ldots \partial_z X_{\mu_j} \partial_{\bar z} X_{\nu_1} \ldots \partial_{\bar z} X_{\nu_j\}} \times e^{ipx}$$

$M^2 \geq J$. ($\partial_z^2 X_\mu$ - will increase the mass).
(lower bound)

For large J, $\boxed{M^2 = J}$

state is described by the rotation of the folded string

The strong coupling limit

As $\alpha_0 \to \infty$ and $\lambda \longrightarrow 0$ we expect to get an integer spectrum once again.

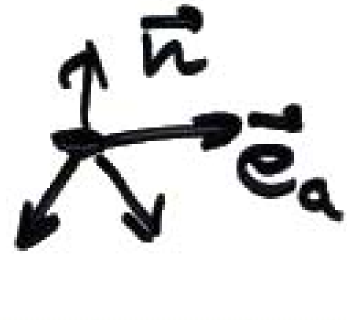
$$\mathcal{L} = \frac{1}{2\alpha_0}\left[(\partial\vec{n})^2 + (\partial\vec{N})^2\right]$$

$$+ \bar{\Psi}\,\sigma_\alpha(\partial_\alpha + A_\alpha)\Psi \; +$$

$$+ \frac{\alpha_0}{4}(\bar{\Psi}\sigma_\alpha\Psi)^2 \; +$$

$$+ \bar{\chi}\,\sigma_\alpha(\partial_\alpha + B_\alpha)\chi + \frac{\alpha_0}{4}(\bar{\chi}\sigma_\alpha\chi)^2$$

$$+ f\big(\Sigma(n)\,\Sigma(N)\,\Sigma_{gh}\big).$$



$$A_\alpha^{ab} = (\vec{e}_a\,\partial_\alpha\,\vec{e}_b)$$

SPIN CONNECTION

We are looking for the ops which have a finite limit as $\alpha_0 \to \infty$.

Without RR terms, that gives the „zero current" constraint

$$\bar{\Psi} \sigma_\alpha \Psi \approx 0 \ ; \ \bar{\chi} \sigma_\alpha \chi \approx 0$$

The theory is empty: only the ground state remains.

When Σ - terms are added, the constraint is non-trivial:

$$\bar{\Psi} \sigma_\alpha \Psi + \frac{\delta}{\delta A_\alpha} (\Sigma_{tot}) \approx 0$$

Solution?

The RR problem

The supercurrent of NSR $G(z) = (\psi_\mu \partial_z X_\mu)$ is not local with respect to Σ : $G(z)\,\Sigma(0) \sim \frac{1}{\sqrt{z}}(\ldots$

So, when we have a term $f \cdot \int \Sigma\, d^2z$ (a magnetic field in the Ising model) we can't define $G(z)$.

A natural resolution : keep gravitino alive :

$$S \sim \int d^2z\,(\chi G) + f \cdot \int \Sigma\, \Sigma_\chi$$

is well defined.

Itinerant picture changing

Better treatment of spin

The wave eqs in general

$$(\Gamma_\mu P_\mu)\Psi = m\Psi$$

$$[M_{\mu\nu}, \Gamma_\lambda] = \delta_{\mu\lambda}\Gamma_\nu - \delta_{\nu\lambda}\Gamma_\mu$$

a solution: take $SO(D+1)$ representation and identify $M_{M,N}$

$$\Gamma_\mu = M_{\mu, D+1}.$$

String theory

Replace the Dirac-Ramond operator with

$$D(z) = \Gamma_\mu(z) P_\mu(z)$$

/Right Hausdorf dimension/

$$P_\mu(z+\xi/2)\, P_\nu(z-\xi/2) =$$

$$= \frac{\delta_{\mu\nu}}{\xi^2} + \dots$$

$$\Gamma_\mu(z) \in \frac{SO_k(\mathcal{D}+1)}{SO_{k+1}(\mathcal{D})}$$

$$\mathcal{D}(z+\xi/2)\,\mathcal{D}(z-\xi/2) =$$

$$= \frac{k\cdot d}{\xi^4} + \frac{1}{\xi^2}\left[k\, T^{(x)} + (k+\mathcal{D}-1)\, T^{(r)} \right]$$

For $k=1$ consistent

For higher k ?

~~Als~~ Supersymm. $\Longrightarrow$ K.-M. algebra

Cosmology

Consider a metric

$$ds^2 = dt^2 + a^2(t)\,(d\vec{x})_3^2$$

and the dilaton $\phi(t)$.

At one loop there are solutions $a(t) \sim t^{\alpha}$

We expect that

$$\int \mathcal{D}\vec{x}\,\mathcal{D}t\, e^{-\int (\partial t)^2 + a^2(t)(\partial \vec{x})^2 + \dots}$$

$$\sim \log \int \mathcal{D}A_\mu\, e^{-\int F_{\mu\nu}^2(\vec{x})\, d^3\vec{x}}$$

The „Big Bang" singularity

$\Rightarrow$ IR divergence of the 3d Y.M. (which is harmless)

In cosmology „Naturalness"
may be unnatural.
We have to speak of
„stability." E.g. flatness
simply means that $\{\gamma_i, M_i\}$
is well defined.
On the other hand,
we must not have
stable Minkowsky space.
This condition may be
inconsistent with super-
symmetry (?).

How to discuss stability in string theory? Tunneling is necessarily off-shell. That means that we have to add an extra Liouville time. For stable b.g. Liouville is sleeping:

$$\mathcal{L} = (\partial x)^2 + \underbrace{(\partial \varphi)^2 + i\sqrt{\tfrac{1}{12}} \int R\varphi}_{c_L = 0}$$

When we jump from one b.g. to another it occurs in the „Liouville time"