# STRINGS IN TIME DEPENDENT ORBIFOLDS II

Continuation of G. Moore's talk

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Related works: Fabinger + McGreevy
Horowitz + Polchinski

- What are the observables?
- How to compute?
- What happens at space like or null singularities? These can be present at tree level or can be created or enhanced by backreaction (see e.g. H+P). Can we answer it at tree level or do we need $g_s$ corrections?
- Important applications to cosmology and black holes

# Interesting examples

- locally flat
- SUSY
- no CTC

Typical example: null-brane $\mathbb{R}^{1,3}/\mathbb{Z}$ (Figueroa-O'Farrill + Simon)

$$\begin{pmatrix} x^+ \\ x \\ x^- \\ z \end{pmatrix} \sim \begin{pmatrix} x^+ \\ x + 2\pi x^+ \\ x^- + 2\pi x + \frac{1}{2}(2\pi)^2 x^+ \\ z + 2\pi R \end{pmatrix}$$

$R=0$ – "parabolic orbifold," singular (Horowitz + Steif)

$R\neq 0$ – smooth

# Functions on the orbifold

Look for eigenfunctions of the Laplacian and the Killing vectors

$$(-2\partial_{x^+}\partial_{x^-} + \partial_x^2 + \partial_z^2)\psi = m^2\psi$$

$$\hat{J}\psi = -i(x^+\partial_x + x\partial_{x^-})\psi = J\psi$$

$$\hat{p}^+\psi = i\partial_{x^-}\psi = p^+\psi$$

$$\hat{k}\psi = -i\partial_z\psi = k\psi$$

$$\psi_{p^+Jkm^2} = \sqrt{\frac{p^+}{ix^+}}\, e^{-ip^+x^- - i\frac{m^2}{2p^+}x^+ + i\frac{p^+}{2x^+}\left(x + \frac{J}{p^+}\right)^2 + ikz} =$$

$$= \int \frac{dp}{\sqrt{2\pi}}\, e^{ip\frac{J}{p^+}}\, e^{-ip^+x^- - ip^-x^+ + ipx + ikz}$$

$$p^- = \frac{p^2 + k^2 + m^2}{2p^+}$$

Orbifold constraint $J + kR = n \in \mathbb{Z}$

Focusing

$$\lim_{x^+\to 0} \psi = \frac{1}{2\pi} e^{-ip^+x^- + ikz}\, \delta\left(x + \frac{J}{p^+}\right)$$

Infinite energy density at $x^+=0$ $\Rightarrow$
danger of large back reaction
(many physicists)

In $\int dp$ high energy is not suppressed $\Rightarrow$ large coupling to gravity

# Tree amplitudes

Start in $\mathbb{R}^{1,3} \times \mathbb{R}^{22}$

$$\phi_{p^+ p k \vec{p}} = \int d^2\sigma \, e^{-ip^+x^- - ip^-x^+ + ipx + ikz + i\vec{p}\cdot\vec{x}}$$

$$p^- = \frac{p^2 + k^2 + \vec{p}^{\,2} - \frac{4}{\alpha'}}{2p^+}$$

$\langle \prod_i \phi_i \rangle =$ standard S-matrix $= A_{\text{flat space}}$

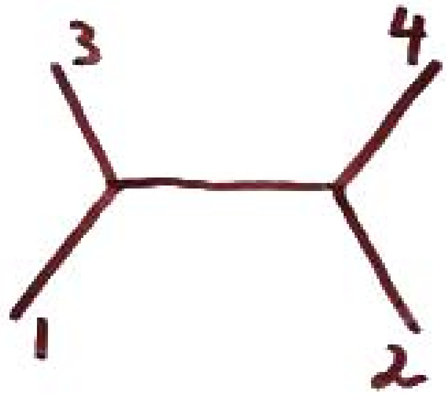
J eigenfunctions

$$V_{p^+ J k \vec{p}} = \int \frac{dp}{\sqrt{2\pi}} e^{ip\frac{J}{p^+}} \phi_{p^+ p k \vec{p}}$$

For $n = J + kR \in \mathbb{Z}$ they are complete basis on the orbifold

$$\Rightarrow \quad S_{\text{orbifold}} = \int \prod_i dp_i \, e^{i\sum p_i \frac{J_i}{p_i^+}} A_{\text{flat space}}$$

(inheritance)



$$S = \int dp_1\, dp_2\, dp_3\, dp_4\, e^{i(\Sigma \ldots)} A = \int ds\, A(s, t(s))\, e^{i(\ldots)}$$

Finite except $p_3^+ - p_1^+ = J_3 - J_1 = 0$

$$(\vec{p}_3 - \vec{p}_1)^2 + (k_3 - k_1)^2 < \frac{4}{\alpha'}$$

(not only $= 0$)

Divergence in t-channel (near IR) due to large energy (UV) in s-channel

Interpret: large back reaction and perturbation theory is not valid

In the null brane ($R \neq 0$), but not in the parabolic orbifold, we can avoid the problem by replacing $V_{p^+ J k \vec{p}}$ with

$$V_{p^+ f(J)\, n\, \vec{p}} = \int dJ\, f(J)\, V_{p^+, J, k = \frac{n-J}{R}, \vec{p}}$$

$f(J)$ of rapid decrease

These functions on spacetime (wave packets of $\hat{J}$ and $\hat{k}$ eigenfunctions) are finite and suppress high energy.

$\Rightarrow$

$\langle \prod_i V_i \rangle$ is better behaved

$$\langle \prod_i V_i \rangle = \text{finite}$$

Except $p_3^+ - p_1^+ = n_3 - n_1 = \vec{P}_3 - \vec{P}_1 = 0$, i.e. only IR.

The standard $\frac{1}{(\vec{P}_1 - \vec{P}_3)^2}$ IR singularity in the $t$-channel is enhanced by the UV region of the $s$-channel to

$$\frac{1}{|\vec{P}_1 - \vec{P}_3|^5}$$

Intuitively, incoming particles are accelerated by the contraction and couple more strongly to the exchanged graviton

We expect these IR divergences to be harmless (as in $QED_{d\geq 4}$) except when there are no noncompact dimensions - no $\vec{P}$.

## Conclusions

Some time dependent backgrounds can be analyzed perturbatively.

Others, including the parabolic orbifold need nonperturbative treatment.

Many of our results are more general than these models