Supersymmetric completion of the R^4 term in IIB SUGRA

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based on work with

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Motivation

The effective description of the massless modes of string theories at low energies is given to leading order by supergravity theories.

String theory implies specific higher derivative corrections to leading order supergravity theories.

In type II theories

$$S = \int d^{10}x \left[R + \alpha'^3 (R^4 + \ldots) \right]$$

Since these terms represent the leading stringy effects at low energies, it is important to understand in detail their structure as well as their implications.

Implications

- These terms are important in studying dualities beyond the leading supergravity approximation.

- In AdS/CFT they capture $1/N$ and 't Hooft coupling corrections to leading results.

- Lead to stringy corrections to supergravity solutions

  → Stringy corrections to black holes and their properties

- May allow for de Sitter compactifications

- May stabilize moduli (cf compactifications with flux)

Effective actions

One may compute the effective action b

- σ-model computations
- scattering amplitude computations

In type II string theories the leading corrections arise from

- 4-loop contribution to β-function computation [Grisaru et.al. '86]
  ⋮
- 4-pt graviton scattering amplitudes and other n-pt functions [Gross-Witten, '86]
  ⋮

Despite much work, the full set of terms that appear at leading order α'^3 is not yet available.

The purpose of this work is to provide all terms that are related to R^4 terms by SUSY.

We do this by constructing the R^4 term in IIB superspace.

Results

IIB Effective action

$$S = S_0 + \alpha'^3 S_3 + \ldots$$

- The freedom to do field redefinitions imply that S_3 is only defined up to lowest order field eqns

$$\phi^I \to \phi^I + \alpha'^3 f^I(\phi^I)$$

$$S^3 \to S^3 + \frac{\partial S_0}{\partial \phi^I} f^I$$

We eliminate the ambiguity by working with on-shell superspace

- IIB superpotential

$$S_3 = \int d^{10}x \, d^{16}\theta \, \Delta \, W(\Phi)$$

Δ: Supersymmetric chiral measure: $\Delta = \sqrt{g} + \theta \psi^* + \ldots$

$W(\Phi)$: Superpotential

• Dilaton superfield

The physical scalar parametrize the coset spase. $SU(1,1)/U(1)$

$$\mathcal{V} = \begin{pmatrix} U & V \\ V^* & U^* \end{pmatrix}$$

anti-chiral superfields

$D_\alpha U = D_\alpha V^* = 0$

chiral superfields

$D^*_\alpha V = D_\alpha U^* = 0$

• All ~~component~~ x-space fields of IIB SUGRA are in the components of V, U^*

• $$W = (U^*)^8 \sum_n \underbrace{c_n}_{\text{constants}} \left(\frac{V}{U^*}\right)^n$$

$$= \underbrace{c_4 (U^* V)^4}_{\text{containing } R^4 \text{ terms}} + \dots + \underbrace{c_n (U^*)^{8-2n} (U^* V)^n}_{\text{terms that violate } U(1) \text{ by } 2n-8 \text{ units}}$$

containing R^4 terms

terms that violate $U(1)$ by $2n-8$ units

- Higher derivative terms in x-space

  - We provide all necessary formulae in order to obtain the component action

  - Bosonic terms

$$S_3 = \int d^{10}x \, d^{16}\theta \; \Delta \, (U * V)^4$$

$$= \int d^{10}x \Big[t_8 t_8 \, \mathcal{R}^4$$

+ terms that vanish when all fields but g_{mn} and F_5 are zero (contain F_3, F_3^* terms)$\Big]$

$$\mathcal{R} = \underbrace{C + DF_{(5)}}_{\text{terms present in linearized superfield of Howe-West}} + F_{(5)}^2 + F_3 F_3^*$$

C: Weyl tensor

- New terms contribute to 4-pt functions in AdS/CFT correspondence

$\underline{\text{II B SUGRA}}$

[Schwarz '83
Howe, West '84]

Field content:

g_{mn}, complex a_{mn}, real a_{mnpq},
complex ψ_m, complex λ
complex a

The theory is invariant under
$SU(1,1) \simeq SL(2,\mathbb{R})$ transformations

$\leadsto$ becomes $SL(2,\mathbb{Z})$ at the quantum level

a parametrizes the coset $SU(1,1)/U(1)$

To realize linearly the $SU(1,1)$ invariance we introduce an ~~extra~~ scalar ϕ and an extra LOCAL $U(1)$ invariance

$$V = \frac{1}{\sqrt{1-aa^*}}\begin{pmatrix} e^{-i\phi} & a e^{i\phi} \\ a^* e^{-i\phi} & e^{i\phi} \end{pmatrix} = \begin{pmatrix} u & v \\ v^* & u^* \end{pmatrix}$$

$$uu^* - vv^* = 1$$

We work in the gauge invariant formulation.

The transformation of the fields under the local $U(1)$ and the rigid $U(1)$ subgroup of $SU(1,1)$ are

Fields	g_{mn}	ψ_m	λ	a_{mn}	a_{mnrs}	u	v	u^*	v^*	ξ
local $U(1)$	0	1/2	3/2	0	0	−1	1	1	−1	1/2
rigid $U(1)$	0	0	0	1	0	1	1	−1	−1	0

The physical scalar a is neutral under the local $U(1)$ and has charge 2 under the rigid $U(1)$

The dilaton and axion are related to a by

$$\tau = i\,\frac{1-a}{1+a}$$

- τ is invariant under local $U(1)$
- It transforms non-linearly under rigid $U(1)$

Type IIB Superspace

[Howe, West '84]

- For every field we introduce a superfield

$$u \longrightarrow U(z) \quad \text{s.t.} \quad U|_{\theta=0} = u$$

$$z^M = \{x^m, \theta^\alpha, \theta^{*\alpha}\}$$

etc.

- Superspace geometry is encoded in algebra of supercovariant derivatives

$$[D_A, D_B\} = -T_{AB}{}^C D_C + \tfrac{1}{2} R_{ABC}{}^D L_D{}^C + 2i H_{AB} K$$

SO(1,9) generators

U(1) generator

- The theory is invariant under LOCAL $SO(1,9) \times \underbrace{U(1)}$ symmetry.

identified with local U(1) of SU(1,1)/U(1)

- Appropriate covariant constraints are imposed s.t. the only independent component fields are that of IIB supergravity multiplet

- In the presence of constraints the Bianchi identities become non-trivial equations.

  Solving them one obtains:

  - supergeometry data: torsions, curvatu

  - fermionic derivatives of all fields:

    e.g. $D_\alpha V = -2 U \Lambda_\alpha$

  - field eqns (on-shell superspace)

- Component fields are obtained using projections

  $s = S|_{\theta=0}$ $\quad s_\alpha = D_\alpha S|_{\theta=0}$ $\quad$ etc

- Supersymmetry rules

  $$\delta S| = \zeta^\alpha D_\alpha S| - \zeta^{*\alpha} D^*_\alpha S|$$

$$\{\nabla_\alpha, \nabla^*_\beta\} \sim \nabla_c$$ ← flat superspace

$$\{\nabla_\alpha, \nabla_\beta\} \sim \underbrace{\Lambda^*}_{= T_{\alpha\beta}{}^{\bar\gamma}} \nabla^*_\gamma$$

Λ : dilatino

$$[\nabla_\alpha, \nabla_\beta] \sim F_3^* \nabla^*_\gamma + F_{(5)} \nabla_\gamma$$

F_3^*: complex field strength of a_{mn}

$F_{(5)}$: 5-form field strength

$$[\nabla_a, \nabla_b] \sim \Psi_{ab}{}^\gamma \nabla_\gamma$$

$\Psi_{ab}{}^\gamma$: field strength of gravitino

Bianchi identities

- Super-Jacobi identities

$$[D_A, \{D_B, D_C\}\} + \text{graded cyclic} = 0$$

$$\Downarrow$$

Bianchi identities

$$D_A T_{BC}{}^D + T_{AB}{}^E T_{EC}{}^D - R_{ABC}{}^D = 0$$

$$\vdots$$

- Field strengths are closed

The dilaton superfield

In flat superspace one can impose the linear constraint

$$\boxed{D^*_\alpha \phi = 0}$$

("chirality" or analyticity condition)

In curved spacetime such constrained superfields generically do NOT exist

$$0 = \{ D^*_\alpha , D^*_\beta \} \Phi = - T_{\bar\alpha \bar\beta}{}^\gamma \, D_\gamma \phi$$

If $T_{\bar\alpha\bar\beta}{}^\gamma \neq 0$ then generically $\to D_\gamma \Phi = 0$

$\rightsquigarrow \phi = \text{const}$

In type IIB superspace $T_{\bar\alpha\bar\beta}{}^\gamma \neq 0$:

$$T_{\bar\alpha\bar\beta}{}^\gamma = (\gamma^a)_{\alpha\beta} (\gamma_a)^{\gamma\delta} \Lambda_\delta - 2\, \delta_{(\alpha}{}^\gamma \Lambda_{\beta)}$$

$$\Rightarrow \quad \Lambda_{(\alpha} D_{\beta)} \phi = 0$$

$$\Rightarrow \quad \boxed{D_\alpha \Phi \sim \Lambda_\alpha}$$

Only fields that vary into dilatino can be the leading component of a chiral superfield

$\Rightarrow$ $W(V, U^*)$ is a chiral superfield

non-derivative function

superpotential

Supersymmetric actions

Chiral superfields should be integrated over half of superspace.

In supergravity theories the measure det e trasforms under SUSY, so one need to construct the appropriate chiral measure Δ

Superaction

$$S = \int d^{10}x \, d^{16}\theta \; \Delta \; W(\Phi)$$

$$d^{16}\theta \equiv \frac{1}{16!} \epsilon^{\alpha_1 \dots \alpha_{16}} D_{\alpha_1} \cdots D_{\alpha_{16}}$$

$$\Delta | = \det e$$

- We construct Δ by requiring that S is supersymmetric for any $W[\Phi]$

- Δ cannot be a chiral scalar superfield since

$$D_\alpha \Delta \sim (\gamma^a \psi_a)_\alpha$$

Chiral measure

$$S \sim \int d^{10}x \left[\Delta D^{16} W + D\Delta D^{15} W + D^2 \Delta D^{14} W + \cdots \right]$$

We require $\delta S = 0$

One can systematically do this by starting with the terms proportional to $D^{16} W$, then move to ones with $D^{15} W$, etc

- Cancellation of terms proportional to J uniquely determine ALL projections of Δ

$$D^n \Delta| \sim \left(\delta D^{n-1} \Delta| \right)_J$$

+fermion bilinears

- Cancellation of terms proportional to J^* should be automatic

We checked that this is indeed the case in the first two orders.

- $d^{16}\theta$ carries -8 units of $U(1)_{Local}$ charge

$\Rightarrow$ Gauge invariance requires that $W[\Phi]$ has $U(1)_{Local}$ charge $= +8$

$$\rightsquigarrow \quad W[\Phi] = \underbrace{(U^*)^8}_{U(1)\text{ compensator}} \; \tilde{W}[\underbrace{A}_{A = \frac{V}{U^*}}]$$

- $A| = a$

- Linearizing all components, A reduces to the Howe-West superfield

The form of $\tilde{W}[A]$ is determined by matching with known results from scattering amplitude / β-function computations.

- The rigid $U(1)$ symmetry ~~does~~ is not a symmetry of the superstring. The terms in the effective action, however, still organize themselves according to their $U(1)$ ~~value~~ charge

$$\tilde{W} = \sum_n c_n A^n$$

A has $U(1)$ charge 2

$$W = (U^*)^8 \, \tilde{W}$$
$$= \sum_n c_n \, (U^*)^{8-2n} \, \Phi^n$$

$\Phi = U^* V$ → The linearized version of this superfield was used by M. Green and collaborators

Components

To work out the component action we need to know all projections of V, V^*

$$D_\alpha V^* = \frac{V^*}{V} D_\alpha V$$

so it is sufficient to determine the projections of V.

These are determined by Bianchi's.

$$V| = v \xrightarrow[\tau_2 \to \infty]{} \sqrt{\tau_2}$$

$$D_\alpha V| \sim u \lambda_\alpha$$

$$D_\alpha D_\beta V| \sim \gamma^{abc}_{\alpha\beta} \hat{f}_{3\,abc}$$

$$D_\alpha D_\beta D_\gamma V| \sim \psi^{c}_{ab} + \cdots$$

$$D_\alpha D_\beta D_\gamma D_\delta V| \sim R_{abcd} + (DF_{(5)})_{abcdef} + F_5^2 + F_2 F_3^* + \cdots$$

No new field appears at higher components

All $D^n V$ are determined iteratively.

SL(2, Z) vs manifest susy

The superinvariant we constructed is NOT SL(2, Z) invariant.

SL(2,Z) invariance requires the introduction of non-holomorphic modular forms as coefficients of component fields.

[Green-Gutperle
⋮
Green-Sethi]
⋮

Supersymmetry of the effective action requires that the susy rules are modified

$$\delta S = 0 \quad \Rightarrow \quad \delta_0 S_3 + \delta_3 S_0 = 0$$

$$\delta = \delta_0 + \alpha'^3 \delta_3 + \ldots$$

and IIB superspace is modified

The construction of the superinvariant should generalize to an SL(2, Z) invariant formulation once the α' corrections to IIB superspace are worked out

Conclusions

- We constructed the superpotential of IIB SUGRA in IIB on-shell superspace. These terms are the leading stringy corrections to IIB SUGRA

- Construction should generalize to IIA and M-theory

- Terms that appear at order α'^5 may also have a superspace representation in terms of integral over 3/4 of superspace

- Applications
  - AdS/CFT
  - compactifications
  - Black hole physics

  etc etc